Inflation, Horizon, and Flatness as Incomplete-History Effects

Causal and Geometric Tests of the Triple Bounce Framework

Paper II of the Triple Bounce Cosmology Series

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Replacement Thesis and Claim-Scope Edition

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Status statement. This manuscript inherits the one-singularity architecture and the Version 1.1.0 replacement thesis of Paper I. Paper II directly tests the proposition that the complete noninflationary three-leg history can replace the separate primordial inflationary stage and the single-direction expansion reconstruction used to infer it. Horizon, flatness, transition matching, perturbation recovery, thermal handoff, and historical identification are independent claim families: success or failure in one does not propagate to another without a derived dependency. No completed validation is claimed. Version 1.1.0 supersedes Version 1.0.1 by converting one-sided benchmark language into reciprocal model testing and by scoping every negative verdict to the exact counterclaim, mechanism, realization, or model class actually tested.

Abstract

Standard inflationary cosmology addresses the observed large-scale uniformity, near-flat geometry, and primordial perturbation spectrum by introducing an early interval of accelerated expansion within the reconstructed history of the observable universe. The Triple Bounce Cosmological Framework proposes a different interpretive possibility: the observable hot expanding universe may represent primarily the third outward leg of a longer directional history, preceded by an outward *Electro* leg, an inward *Electro* return, and two sector-specific reversals occurring while the larger cosmological background may remain globally expansive. The framework contains one singular opening. The outer and central reversals are sector-specific *Electro* transitions rather than additional singularities, and their causes remain unresolved.

This paper does not identify the first outward *Electro* leg with inflation. It investigates whether inflationary inference may partly result from reconstructing a three-leg history as though only the observable third leg existed. The central question is whether causal contact, curvature suppression, and acceptable primordial perturbations can arise from the accumulated pre-third-leg history without requiring the first leg to behave as conventional inflation.

The analysis distinguishes observational success from historical uniqueness. It introduces a resolved post-singularity raw conformal interval, a third-leg-only conformal interval, correlation-retention factors, transition matching conditions, curvature-transfer maps, and a central matching operator connecting the inherited two-pass state to third-leg perturbation initial data. The framework is required to reproduce the established successes of inflationary ΛCDM, including the cosmic microwave background acoustic structure, near-flatness, large-scale statistical isotropy, and an acceptable primordial perturbation spectrum. Paper II distinguishes observational recovery from historical identification: a model may recover the measured third-leg observables yet remain degenerate with a suitably chosen inflationary or other pre-hot-history model. A longer history is not considered a viable candidate resolution unless the relevant causal integrals, correlation-retention laws, curvature evolution, primordial transfer, and cross-observable relations are explicitly calculated.

The primary thesis tested here is replacement: the complete Triple Bounce history is proposed to perform the causal, geometric, perturbative, and thermal functions for which a separate primordial inflationary stage is ordinarily invoked. This is a reciprocal test. A successful frozen noninflationary counterexample falsifies the corresponding claim that inflation is necessary for that function; failure of that counterexample does not prove inflation necessary and does not reject the triple directional history or any other independent Triple Bounce claim. Empirical analysis may force a restricted, complementary, degenerate, or rejected outcome, but those are outcome classifications rather than the starting thesis. Joint evaluation requires one finite parameter ledger, one shared background, predeclared priors, synthetic parameter-recovery tests, a calibration–validation split, explicit complexity control, and claim-specific failure conditions.

Replacement Thesis, Reciprocal Falsification, and Independent Claims

What is being replaced

The observed third-leg expansion remains part of the model. The replacement target is the conjunction

$$\mathcal{S} = \left\{ \begin{array}{c} \text{the measured outward expansion exhausts the resolved directional history,} \\ \text{and a separate primordial inflationary stage is necessary to prepare that history} \end{array} \right\}. \tag{1}$$

Paper II advances the complete outward–inward–outward history as the primary alternative to this conjunction. The first leg is not renamed inflation. The proposed replacement is the full causal and geometric history, including both finite reversals and the central conversion into the hot third leg.

Parallel counterclaims

The replacement program is not one indivisible proposition. It contains parallel counterclaims:

$$\begin{aligned} T_H &: \text{prior-leg history supplies sufficient retained causal preparation,} && (2) \\ T_K &: \text{boundary and transfer dynamics produce acceptable curvature,} && (3) \\ T_P &: \text{central conversion generates acceptable primordial perturbations,} && (4) \\ T_T &: \text{the central transition supplies an acceptable hot thermal state,} && (5) \\ T_I &: \text{observations identify a nontrivial prior history,} && (6) \\ T_3 &: \text{the ordered outward–inward–outward trajectory occurred.} && (7) \end{aligned}$$

These claims may fit together and jointly support a maximal replacement package, but none is automatically equivalent to another. In particular,

$$\boxed{\mathrm{Fail}(T_H) \not\Rightarrow \mathrm{Fail}(T_3), \qquad \mathrm{Fail}(T_K) \not\Rightarrow \mathrm{Fail}(T_3).} \tag{8}$$

Likewise, failure to identify a historical residue does not establish that the hidden history was physically impossible.

Reciprocal necessity tests

For each function $X \in \{H, K, P, T\}$, let

$$N_{\text{inf}}^{X}: \quad \text{inflation is necessary for function } X. \tag{9}$$

A stable, conserving, frozen Triple Bounce realization that performs X without an independent inflationary interval is a counterexample:

$$\boxed{T_X \text{ passes} \Longrightarrow N_{\text{inf}}^{X} \text{ is falsified as a necessity claim.}} \tag{10}$$

The converse is forbidden:

$$\boxed{T_X \text{ fails} \not\Longrightarrow N_{\text{inf}}^{X} \text{ is proven.}} \tag{11}$$

A failed proposed alternative does not establish the unique necessity or historical occurrence of one competitor.

Verdict vocabulary

Every test must identify both the claim family and the rejection level. Permitted verdicts include parameter exclusion, mechanism rejection, realization rejection, model-class rejection, failure of historical identification, observational degeneracy, incompleteness, underdetermination, and—only when the defining postulates force the conflict—claim-family or architecture-level falsification. The laws, parameter domain, priors, transfer operators, calibration data, validation data, and numerical tolerances must be frozen before a decisive verdict. Changing them defines a new model version rather than rescuing the failed one.

Keywords: inflation; cosmological horizon; flatness problem; conformal time; primordial perturbations; bounce cosmology; cosmic initial conditions; Triple Bounce; incomplete-history hypothesis; cosmic microwave background.

Contents

1 Introduction

Modern cosmology reconstructs cosmic history by combining general relativity, astronomical distance measurements, the thermal history of the early universe, the cosmic microwave background, and the statistical distribution of matter.

Within the standard interpretation, the observed universe developed from an early hot, dense state. Inflation is placed before the ordinary radiation-dominated hot expansion to account for several otherwise difficult features:

1. the large-scale causal uniformity of the observable universe;
2. the small measured magnitude of spatial curvature;
3. the dilution of unwanted relics in many high-energy scenarios;
4. the generation of coherent primordial perturbations; and
5. the near scale invariance of the scalar perturbation spectrum.

These achievements make inflation a powerful theoretical framework (Starobinsky, 1980; Guth, 1981; Linde, 1982; Albrecht and Steinhardt, 1982).

Inflation is not, however, a single unique model. It is a broad class of accelerated-expansion scenarios whose microphysical realization, initial conditions, duration, energy scale, and relation to a more complete theory remain active areas of research (Liddle and Lyth, 2000; Mukhanov, 2005; Baumann, 2009).

The Triple Bounce framework introduces a different question.

Suppose the observable hot expanding universe is not the complete cosmic history.

Suppose instead that the state ordinarily treated as the beginning of the observable expansion is the output of a prior directional sequence:

$$\boxed{\text{first outward leg} \longrightarrow \text{second inward leg} \longrightarrow \text{third outward leg}}. \tag{12}$$

The first and second legs could then contribute causal history, structural memory, and geometric evolution before the observable third-leg plasma developed, provided that physically defined carriers propagate correlations and the transitions preserve them.

The inference of a separate inflationary stage might consequently depend, at least in part, on an incomplete reconstruction of the full history.

This possibility is called the *incomplete-history hypothesis.*

Hypothesis 1.1 (Incomplete-history hypothesis)**.** *Some phenomena attributed to a primordial inflationary interval may instead arise from causal, geometric, and structural evolution occurring during the hidden first and second legs of the Triple Bounce history.*

The hypothesis does not state that the observational data supporting inflationary cosmology are incorrect.

It states that the same third-leg observations may not uniquely identify the earlier physical history that produced them.

2 Purpose of Paper II

Paper I defined the foundational Triple Bounce architecture.

It separated the global scale factor

$$a(t) \tag{13}$$

from the effective physical position of the active *Electro* sector,

$$R_E(t) = a(t)\chi_E(t). \tag{14}$$

This distinction permits the second-leg condition

$$\dot{a}(t) > 0, \qquad \dot{R}_E(t) < 0. \tag{15}$$

The inward second leg is therefore not automatically a contraction of the entire Friedmann–Lemaître–Robertson–Walker background.

Paper I also proposed that:

- *Electro* pathways and a distributed dark-matter-associated scaffold co-form during the first outward leg;
- the scaffold preserves inherited pathway structure after the active *Electro* front changes direction;
- the second inward passage adds counter-directed history;
- at the central transition, *Electro* returns to the post-singularity region or topology, and the third outward leg begins with candidate magnetic coupling;
- third-leg matter occupies, transforms, and modifies the inherited scaffold; and
- the Gravity Triune operates through binding, scaffolding, and chasing.

Paper II has a narrower purpose.

Assumption 2.1 (Controlling Paper I architecture)**.** Throughout Paper II, Triple Bounce denotes one complete outward–inward–outward history containing one singular opening. The outer and central reversals are internal changes in the effective *Electro* trajectory, not additional singularities. Their physical causes remain unresolved. At the central transition, *Electro* returns to the post-singularity region or topology before the third outward leg begins.

All proposed links to inflation, causal contact, near-flatness, and primordial perturbations are candidate resolutions until one continuous background and perturbation history satisfies the full mathematical and observational test set.

It tests whether this extended history can perform the explanatory work usually assigned to inflation.

The four principal problems are:

I. **Causal contact:** Can the first and second legs provide sufficient conformal history to correlate regions visible in the cosmic microwave background?

II. **Near-flatness:** Can the pre-singularity starting conditions, singular-opening map, and three-leg curvature transfer produce or preserve small spatial curvature without imposing an unexplained third-leg initial value?

III. **Perturbation generation:** Can the central transition map inherited structure into an acceptable primordial scalar, tensor, and isocurvature spectrum?

IV. **Observational equivalence and distinction:** Can the model reproduce inflation's successful predictions while also producing at least one calculable difference?

3 Scope and Exclusions

This paper focuses on causal geometry and perturbative initial conditions.

It does not attempt to complete every part of the Triple Bounce model.

The primary scope includes:

- the definition of the full classical conformal history;
- the third-leg-only causal horizon;
- the extension supplied by the first and second legs;
- matching across the outer and central transitions;
- curvature propagation through the three-leg sequence;
- the relation between balanced pre-singularity ordering and near-flatness;
- the central mapping into third-leg perturbations;
- comparison with the functions of inflation; and

- explicit success and failure conditions.

The following subjects remain outside the main derivation:

- the microscopic identity of *Electro*;
- a final quantum theory of the pre-singularity state or singular opening;
- the full Gravity Triune field equations;
- galaxy-scale dark-matter phenomenology;
- the detailed baryogenesis mechanism;
- the complete electromagnetic precursor theory;
- the geological origin of the packet hierarchy;
- the full packet reconstruction and numerical interpretation; and
- the complete third-leg expansion fit.

Those subjects belong primarily to Papers III and IV or to later microphysical work.

4 The First Outward Leg Is Not Inflation

A central distinction of this paper is

$$\boxed{\text{first outward } \textit{Electro} \text{ leg} \neq \text{standard inflation.}} \tag{16}$$

Standard inflation is defined by accelerated expansion of the cosmic scale factor.

A common condition is

$$\ddot{a} > 0. \tag{17}$$

Using the Hubble slow-roll parameter,

$$\epsilon_H = -\frac{\dot{H}}{H^2}, \tag{18}$$

inflation requires

$$\epsilon_H < 1. \tag{19}$$

The accumulated inflationary expansion is conventionally expressed through the number of e-folds,

$$N = \ln \left[\frac{a(t_{\text{end}})}{a(t_{\text{begin}})} \right]. \tag{20}$$

The first Triple Bounce leg is defined instead by the outward motion of the active *Electro* sector:

$$\dot{R}_E^{(1)} > 0. \tag{21}$$

This condition does not imply

$$\ddot{a}^{(1)} > 0. \tag{22}$$

The first leg may involve rapid sector propagation, early geometric evolution, or scaffold formation without satisfying the standard inflationary conditions.

The incomplete-history hypothesis therefore does not rename the first leg as inflation.

It asks whether the *combined consequences* of the first leg, second leg, and central transition can reproduce the functions ordinarily assigned to inflation.

Definition 4.1 (Functional inflation-equivalence)**.** A Triple Bounce background is functionally inflation-equivalent over a specified observational domain if it reproduces the causal, geometrical, and perturbative predictions ordinarily obtained from inflation without requiring the first outward leg itself to satisfy the standard inflationary expansion condition.

Functional equivalence would not imply physical identity. It would also not follow from one matching background quantity. The complete model must recover the required causal-domain overlap, curvature evolution, primordial statistics, hot-state handoff, and downstream observables within one consistent history.

Two different histories may produce similar third-leg observables.

5 What Is an Incomplete-History Effect?

5.1 Observable history versus total history

Let

$$\mathcal{H}_3 \tag{23}$$

denote the reconstructable third-leg history, and let

$$\mathcal{H}_{123} \tag{24}$$

denote the full three-leg history.

The incomplete-history hypothesis states

$$\mathcal{H}_3 \subset \mathcal{H}_{123}. \tag{25}$$

The distinction is analogous to reconstructing an entire trajectory from only its final segment.

A highly accurate model of the final segment need not uniquely identify the dynamics preceding it.

5.2 History compression

Third-leg observations constrain an effective boundary state near the central transition:

$$\mathcal{I}_3 = \{a, H, \rho_i, p_i, \delta_i, \theta_i, \mathcal{R}, h_{ij}, \dots\}_{t=t_{\mathrm{C}}^+}. \tag{26}$$

Many prior histories may, in principle, map into similar third-leg boundary data:

$$\{\mathcal{H}_A, \mathcal{H}_B, \mathcal{H}_C, \dots\} \longrightarrow \mathcal{I}_3. \tag{27}$$

This creates a history-compression degeneracy.

Definition 5.1 (History-compression degeneracy)**.** A history-compression degeneracy occurs when physically distinct pre-third-leg histories generate observationally similar third-leg initial data.

The scientific task is not merely to show that the Triple Bounce history can be made compatible with $\mathcal{I}_3$.

It must identify a residual feature that retains information about the earlier legs.

5.3 Inference under truncated history

Suppose an observer reconstructs the history using only third-leg variables.

The inferred model may require an early process

$$\mathcal{F}_{\mathrm{inf}} \tag{28}$$

to generate causal contact, near-flatness, and coherent perturbations.

The full-history model instead proposes

$$\mathcal{F}_{\mathrm{full}} = \mathcal{T}_{\mathrm{C}} \circ \mathcal{U}_2 \circ \mathcal{T}_{\mathrm{O}} \circ \mathcal{U}_1, \tag{29}$$

where:

- $\mathcal{U}_1$ is first-leg evolution;
- $\mathcal{T}_\mathrm{O}$ is the outer transition;
- $\mathcal{U}_2$ is second-leg evolution; and
- $\mathcal{T}_\mathrm{C}$ is the central transition.

The composition symbol records chronological operator ordering; it does not assume that the four maps commute. The question is whether

$$\mathcal{O}\left[\mathcal{F}_{\text{full}}\right] \approx \mathcal{O}\left[\mathcal{F}_{\text{inf}}\right] \tag{30}$$

for the established observables, while remaining distinguishable in at least one additional prediction.

6 Baseline Cosmological Geometry

6.1 Classical FLRW regime

Once a classical homogeneous and isotropic geometry exists, the background line element is written as

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right], \tag{31}$$

where:

- $a(t)$ is the scale factor;
- k is the normalized spatial-curvature parameter; and
- $\mathrm{d}\Omega^2$ is the metric on the unit two-sphere.

Wherever factors of c are displayed explicitly, ρ_i denotes mass-equivalent energy density, $\rho_i = \varepsilon_i/c^2$.

The background Friedmann equation is

$$H^2 = \frac{8\pi G}{3}\rho_{\text{tot}} - \frac{kc^2}{a^2}. \tag{32}$$

The acceleration equation is

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{\text{tot}} + \frac{3p_{\text{tot}}}{c^2}\right). \tag{33}$$

6.2 First Resolved Post-Singularity Boundary

The pre-singularity state and singular opening are not assumed to possess the classical metric of equation (31).

Therefore, the resolved classical conformal-time integral does not begin until the first resolved post-singularity boundary,

$$t = t_{\text{res}}. \tag{34}$$

The relationship between the unresolved pre-singularity state and the first resolved post-singularity hypersurface is represented by a candidate singular-opening map,

$$\mathcal{T}_{\text{g}} : \mathcal{S}_{\text{pre}} \longrightarrow \{a, H, k, \rho_i, p_i, \delta_i\}_{t=t_{\text{res}}^+}. \tag{35}$$

Paper II does not assume that classical time, classical distance, or ordinary causal propagation extend unchanged through the pre-singularity state or singular opening. No pre-singularity conformal contribution is counted unless a later theory defines both its causal measure and its matching to the resolved post-singularity history.

Limitation 6.1. The full-history conformal calculation begins at the first resolved post-singularity boundary unless a later theory supplies a mathematically defined pre-singularity causal measure and transition rule.

7 Three-Leg Time Structure

Let the relevant classical intervals be

$$\mathcal{I}_1 = \left[t_{\text{res}}, t_{\text{O}}^-\right], \tag{36}$$

$$\mathcal{I}_{\text{O}} = \left[t_{\text{O}}^-, t_{\text{O}}^+\right], \tag{37}$$

$$\mathcal{I}_2 = \left[t_{\text{O}}^+, t_{\text{C}}^-\right], \tag{38}$$

$$\mathcal{I}_{\text{C}} = \left[t_{\text{C}}^-, t_{\text{C}}^+\right], \tag{39}$$

$$\mathcal{I}_3 = \left[t_{\text{C}}^+, t_0\right], \tag{40}$$

where t_0 denotes the present cosmic time.

The directional *Electro* conditions are

$$\dot{R}_E^{(1)} > 0, \tag{41}$$

$$\dot{R}_E^{(2)} < 0, \tag{42}$$

$$\dot{R}_E^{(3)} > 0. \tag{43}$$

The global-expansion hypothesis permits

$$H(t) > 0 \tag{44}$$

through all three classical legs.

No exact equality among the leg durations is assumed:

$$T_1 \neq T_2 \neq T_3 \tag{45}$$

in the general formulation.

Approximate equal-duration models may be studied later as restricted subcases.

8 The Horizon Question

8.1 Third-leg conformal history

Conformal time is defined by

$$\mathrm{d}\eta = \frac{\mathrm{d}t}{a(t)}. \tag{46}$$

For a signal propagating locally at speed c, the third-leg comoving causal interval before an event at time t is

$$\chi_{\mathrm{causal}}^{(3)}(t) = \int_{t_{\mathrm{C}}^{+}}^{t} \frac{c\,\mathrm{d}t'}{a_3(t')}. \tag{47}$$

If the third leg is treated as the complete history, this interval limits the regions that could have exchanged ordinary causal signals.

8.2 Full Resolved Post-Singularity Raw Causal Reach

The Triple Bounce framework permits a larger raw comoving propagation reach across the resolved post-singularity history:

$$\chi_{\text{raw}}^{(123)}(t,k) = \sum_j \int_{\mathcal{I}_j} \frac{c_{\text{eff}}^{(j)}(t',k)}{a_j(t')} \mathrm{d}t', \tag{48}$$

where the sum includes the first leg, outer transition, second leg, central transition, and the relevant portion of the third leg. This raw reach is not yet the retained common-cause domain: correlation can be degraded or converted at either transition.

The effective propagation speed

$$c_{\text{eff}}^{(j)}(t) \tag{49}$$

is not assumed to be identical for every pre-electromagnetic degree of freedom.

After ordinary electromagnetism is established,

$$c_{\text{eff}}^{(3)} = c \tag{50}$$

for electromagnetic signals in vacuum.

Earlier propagation laws require derivation.

8.3 History-extension ratio

Define the raw history-extension ratio

$$\mathfrak{H}_{\text{raw}}(t,k) = \frac{\chi_{\text{raw}}^{(123)}(t,k)}{\chi_{\text{causal}}^{(3)}(t)}. \tag{51}$$

A hidden prehistory enlarges the available classical causal interval when

$$\mathfrak{H}_{\text{raw}}(t,k) > 1. \tag{52}$$

This inequality shows only that the resolved prehistory adds raw reach. It is neither necessary for every nontransport common-state mechanism nor sufficient to establish a horizon candidate resolution.

8.4 Causal success criterion

Consider two points on the last-scattering hypersurface separated by spatial geodesic distance

$$d_k(\chi_*, \theta)\,. \tag{53}$$

Let the retained backward causal-preparation radius of either point be

$$\chi_{\rm ret}^{(123)}(t_*, k) = \sum_j \mathcal{E}_{j\to 3}(k)\chi_j(k), \tag{54}$$

where $\mathcal{E}_{j\to 3}$ is an explicitly derived transfer or retention factor rather than an adjustable rescue parameter. In a symmetric background, causal-domain overlap requires

$$2\chi_{\rm ret}^{(123)}(t_*, k) \geq d_k(\chi_*, \theta) \tag{55}$$

for every scale and angular separation whose common preparation is being claimed. The general condition is nonempty intersection of the relevant backward causal domains.

A shared origin, an inherited web, added elapsed duration, or a larger raw conformal integral does not replace this calculation.

Proposition 8.1 (Horizon-test requirement)**.** *The incomplete-history hypothesis addresses the horizon problem only if a physically derived common-cause domain is large enough, survives both transitions, and enters the third-leg state before the observed correlations are fixed.*

9 The Flatness Question

9.1 Curvature density parameter

The curvature density parameter is

$$\Omega_k(t) = -\frac{kc^2}{a^2(t)H^2(t)}. \tag{56}$$

Near-flatness requires

$$|\Omega_k(t_0)| \ll 1. \tag{57}$$

In a single-fluid Friedmann–Lemaître–Robertson–Walker regime with approximately constant equation of state w,

$$|\Omega_k| \propto a^{1+3w} \tag{58}$$

when the relevant fluid dominates.

For ordinary radiation,

$$|\Omega_k| \propto a^2, \tag{59}$$

and for pressureless matter,

$$|\Omega_k| \propto a. \tag{60}$$

An early accelerated phase with

$$w < -\frac{1}{3} \tag{61}$$

suppresses the relative curvature contribution.

This is one of inflation's central explanatory successes.

9.2 Triple Bounce curvature-transfer problem

The Triple Bounce candidate resolution must distinguish a provisional cross-boundary prescription at the singular opening from ordinary classical evolution and from the two later, nonsingular reversal maps. The complete curvature history is represented schematically by

$$\mathcal{S}_{\text{pre}} \xrightarrow{\mathcal{B}_{\text{res}}} \Omega_k(t^+_{\text{res}}) \xrightarrow{\mathcal{U}_{k,1}} \Omega_k(t^-_{\text{O}}) \xrightarrow{\mathcal{T}_{k,\text{O}}} \Omega_k(t^+_{\text{O}}) \xrightarrow{\mathcal{U}_{k,2}} \Omega_k(t^-_{\text{C}}) \xrightarrow{\mathcal{T}_{k,\text{C}}} \Omega_k(t^+_{\text{C}}) \xrightarrow{\mathcal{U}_{k,3}} \Omega_k(t_0). \tag{62}$$

Here $\mathcal{B}_{\text{res}}$ is not a classical curvature flow through the singularity. It is a provisional boundary prescription that maps pre-singularity ordering data to the earliest resolved post-singularity geometric data. The operators $\mathcal{T}_{k,\text{O}}$ and $\mathcal{T}_{k,\text{C}}$ instead describe curvature transfer across the outer and central reversals, neither of which is a singularity.

The model cannot simply choose

$$\Omega_k^{(3)} \approx 0 \tag{63}$$

as an unexplained central-transition boundary condition.

The small third-leg value must arise from:

- the pre-singularity ordering;

- first-leg evolution;
- outer-transition matching;
- second-leg evolution;
- central-transition matching; or
- a controlled combination of these effects.

9.3 Balanced pre-singularity ordering

Paper I proposed that the pre-singularity state may be dominated by a balanced internal mode.

This does not mean that the pre-singularity state is literally two-dimensional or already described by zero-curvature Euclidean space.

It means that the dominant ordering may constrain the asymmetry of the earliest resolved post-singularity geometry. Paper II does not claim that geometry itself originates at that boundary.

Let

$$\epsilon_{\text{asym}} = \frac{\|\delta\Psi\|}{\|\Psi_0\|} \tag{64}$$

measure departures from the dominant balanced mode.

A schematic full-history curvature map is

$$\Omega_k(t_0) = \mathcal{K}_{\text{full}}\left[\mathcal{B}_{\text{res}}, \epsilon_{\text{asym}}, \mathcal{U}_{k,1}, \mathcal{T}_{k,\text{O}}, \mathcal{U}_{k,2}, \mathcal{T}_{k,\text{C}}, \mathcal{U}_{k,3}\right]. \tag{65}$$

The near-flatness hypothesis requires a non-negligible basin satisfying

$$|\mathcal{K}| < \Omega_{k,\max} \tag{66}$$

without extreme adjustment of ϵ_{asym}.

Hypothesis 9.1 (Inherited near-flatness)**.** *The observed near-flat third-leg geometry may descend from a singular-opening boundary prescription constrained by balanced pre-singularity ordering, followed by controlled curvature evolution through the first two legs and regular transfer across both nonsingular reversals.*

10 The Perturbation Question

10.1 Inflationary benchmark

Inflation provides a mechanism for generating coherent primordial perturbations from quantum fluctuations (Mukhanov and Chibisov, 1981; Mukhanov, 2005).

A successful alternative must reproduce the observed properties of the scalar perturbation spectrum, including:

- small amplitude;
- near scale invariance;
- predominantly adiabatic initial conditions;
- near-Gaussian statistics;
- phase coherence;
- small statistical anisotropy; and
- acceptable tensor and isocurvature contributions.

The scalar curvature spectrum is conventionally parameterized as

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \left[1 + \Delta_{\mathrm{TB}}(k)\right], \tag{67}$$

where:

- A_s is the scalar amplitude;
- n_s is the scalar spectral index;
- k_* is the pivot scale; and
- $\Delta_{\mathrm{TB}}(k)$ is a possible Triple Bounce contribution.

The function $\Delta_{\mathrm{TB}}(k)$ must be derived rather than selected separately for each feature.

10.2 Central matching map

The first two legs may leave a correlated inherited state immediately before the central transition. The following state vector is a bookkeeping representation of the information that a concrete model would have to evolve:

$$\mathcal{I}_{\mathrm{C}}^{-} = \left\{\mathcal{W}_{12}, \boldsymbol{\mathcal{P}}_{12}, \mathcal{M}_{ij}^{(12)}, \rho_i, p_i, f_i, g_{\mu\nu}\right\}_{t=t_{\mathrm{C}}^{-}}. \tag{68}$$

A viable central-transition theory must specify a physical transfer from this inherited state into third-leg perturbation data:

$$\mathcal{T}_{\mathrm{C}} : \mathcal{I}_{\mathrm{C}}^{-} \longrightarrow \{\mathcal{R}_{\boldsymbol{k}}, S_{ij,\boldsymbol{k}}, h_{\boldsymbol{k}}, \delta_i, \theta_i, \Phi, \Psi\}_{t=t_{\mathrm{C}}^{+}} . \tag{69}$$

Here:

- $\mathcal{R}_{\boldsymbol{k}}$ is the curvature perturbation;
- $S_{ij,\boldsymbol{k}}$ denotes relative-entropy or isocurvature perturbations;
- $h_{\boldsymbol{k}}$ denotes tensor modes;
- δ_i denotes sector density contrasts;
- θ_i denotes velocity-divergence perturbations; and
- Φ and Ψ are scalar metric potentials.

This operator is an effective stand-in for transition dynamics that must be derived; it is not a freely adjustable fitting map. Constructing it from a finite, stable central-transition model is one of the central mathematical burdens of Paper II.

10.3 Inherited web versus primordial randomness

The inherited web introduces structured pre-third-leg information.

A viable model must show how this structure remains compatible with the observed approximate statistical homogeneity and isotropy.

The third-leg curvature perturbation may be decomposed as

$$\mathcal{R}_{\boldsymbol{k}} = \mathcal{R}_{\boldsymbol{k}}^{\mathrm{base}} + \mathcal{R}_{\boldsymbol{k}}^{\mathrm{inherit}} + \mathcal{R}_{\boldsymbol{k}}^{\mathrm{transition}}. \tag{70}$$

Here, the baseline term denotes any statistically homogeneous component not assigned to inherited-web transfer or transition sourcing. This is bookkeeping rather than a claim that three independent microscopic sources have already been identified.

The inherited and transition contributions must satisfy observational bounds. If they generically produce excessive preferred direction, phase locking, non-Gaussianity, or isocurvature, the model fails.

11 Inflation as an Observationally Successful Benchmark

The Triple Bounce framework should not be tested against an artificially weak version of inflation.

The proper comparison is with inflationary models that reproduce the observed universe using a consistent perturbation and thermal history.

The benchmark includes:

1. sufficient causal correlation;
2. suppression of the relative curvature contribution;
3. a nearly scale-invariant scalar spectrum;
4. coherent acoustic oscillations;
5. acceptable tensor modes;
6. small non-Gaussianity;
7. acceptable reheating or hot-state transition;
8. successful nucleosynthesis;
9. successful recombination; and
10. agreement with the observed cosmic microwave background and matter spectra.

The Triple Bounce framework gains no evidential advantage merely by noting that inflation has unresolved microphysics.

It must provide its own complete causal and perturbative account.

Remark 11.1. Inflation may remain a viable physical stage even if the observable third leg has a longer prehistory. The incomplete-history hypothesis and inflation are not logically exclusive until the full dynamics are derived.

12 Primary Replacement Thesis and Empirical Outcome Classes

The declared scientific thesis of Paper II is functional replacement: the complete noninflationary Triple Bounce history is proposed to perform the relevant causal, geometric, perturbative, and thermal work without a separate primordial inflationary stage. The data and completed dynamics may nevertheless force narrower outcome classifications. Those classifications do not change the starting thesis, and none is allowed to merge independent claims.

12.1 Outcome A: replacement counterexample survives

For a specified function X, a frozen Triple Bounce realization supplies a replacement counterexample when it performs X without an independent inflationary stage:

$$\mathcal{F}^X_{\mathrm{TB}} \approx_{\mathrm{obs}} \mathcal{F}^X_{\mathrm{required}}, \qquad \mathcal{F}^X_{\mathrm{inf}} \text{ not independently required.} \tag{71}$$

Its physical history remains distinct from an inflationary realization:

$$\mathcal{F}^X_{\mathrm{TB}} \neq_{\mathrm{phys}} \mathcal{F}^X_{\mathrm{inf}}. \tag{72}$$

Success falsifies inflationary necessity for that function. It does not automatically establish the other Triple Bounce counterclaims or prove that inflation could not occur in any conceivable history.

12.2 Outcome B: restricted or forced complement

A completed comparison may show that a derived Triple Bounce contribution survives while an additional accelerated stage is still required by the tested joint model. This is a forced empirical outcome rather than the primary thesis. It must specify which function is supplied by each stage and may not use an unconstrained inflationary interval to conceal a failed transfer law.

$$\mathcal{F}_{\mathrm{full}} = \mathcal{F}_{\mathrm{additional}} \circ \mathcal{F}_{\mathrm{TB,pre}}. \tag{73}$$

This result would restrict the replacement package while leaving independently successful Triple Bounce claims standing.

12.3 Outcome C: claim-specific rejection

A horizon, flatness, perturbation, thermal, or identification mechanism may fail within its frozen admissible domain. The correct verdict is rejection of that named mechanism, realization, model class, or claim family. Such failure removes the tested noninflationary counterexample for that function; it does not prove inflation necessary, validate the single-direction history, or reject the triple directional trajectory unless the relevant dependency has been derived independently.

The paper does not predetermine the empirical outcome, but it does predetermine the logic by which each outcome is classified.

13 Core Hypotheses of Paper II

The analysis is organized around six principal hypotheses.

Hypothesis 13.1 (Extended causal history)**.** *The first and second legs provide a positive, physically defined raw causal contribution before third-leg recombination for at least the modes relevant to the horizon test.*

Hypothesis 13.2 (Causal sufficiency)**.** *The retained backward causal domains overlap across the regions and scales whose uniformity is observed in the cosmic microwave background.*

Hypothesis 13.3 (Inherited near-flatness)**.** *A non-finely-tuned singular-opening map together with stable three-leg curvature transfer produces a naturally small third-leg curvature parameter.*

Hypothesis 13.4 (Central perturbation map)**.** *There exists a finite, stable, physically derived central-transfer law that can convert the inherited two-pass state into an observationally acceptable, predominantly adiabatic, nearly scale-invariant third-leg perturbation spectrum.*

Hypothesis 13.5 (Observational recovery)**.** *The resulting third-leg background and perturbations reproduce the established observational successes of inflationary* ΛCDM*.*

Hypothesis 13.6 (Residual historical signature)**.** *At least one observable retains calculable information about the hidden first and second legs that is not naturally reproduced by unrestricted third-leg initial conditions.*

14 Claims Not Made

Paper II does not claim that:

1. the first outward leg is standard inflation;
2. inflationary calculations are observationally invalid;
3. a shared topological origin, added duration, or raw conformal reach automatically establishes a horizon candidate resolution;
4. all pre-third-leg sectors propagate at the speed of light;
5. the pre-singularity state is literally a flat two-dimensional surface;
6. near-flatness has already been derived;
7. the hidden legs automatically generate a scale-invariant spectrum;
8. the inherited web may be inserted as an arbitrary perturbation field;
9. the cosmic microwave background is dark energy;

10. the cosmic microwave background marks the outer boundary of the chasing sector;
11. the full Triple Bounce duration is fixed;
12. the three legs have exactly equal durations;
13. the 46.08-billion-light-year distance is an elapsed time;
14. the 10/3 ratio is an independent prediction;
15. the packet construction replaces the continuous expansion history; or
16. the incomplete-history hypothesis has already replaced inflation.

These restrictions protect the distinction between a research program and a completed result.

15 Scientific Success Conditions

The incomplete-history hypothesis succeeds only if the following conditions are met.

15.1 Defined classical history

The model must specify

$$\left\{a_j(t), H_j(t), R_E^{(j)}(t), c_{\text{eff}}^{(j)}(t)\right\} \tag{74}$$

over every classical leg and finite transition.

15.2 Regular matching

The metric and stress–energy must satisfy consistent matching conditions at

$$t_{\text{O}}, \qquad t_{\text{C}}. \tag{75}$$

15.3 Sufficient causal interval

The retained full-history causal domain must satisfy

$$2\chi_{\text{ret}}^{(123)}(t_*, k) \geq d_k(\chi_*, \theta) \tag{76}$$

throughout the observational domain for which common preparation is claimed.

15.4 Natural curvature suppression

The third-leg curvature must remain small over a finite basin of initial conditions rather than at one finely selected point.

15.5 Stable perturbations

The transition and third-leg perturbations must avoid:

- ghost instabilities;
- gradient instabilities;
- unbounded anisotropy;
- excessive isocurvature;
- unacceptable tensor power; and
- singular matching behavior.

15.6 Observational recovery

The model must reproduce the measured:

- cosmic microwave background acoustic peak positions;
- relative peak heights;
- polarization spectra;
- damping tail;
- large-scale matter spectrum;
- near-flat geometry; and
- late third-leg expansion.

15.7 Distinctiveness

The model must derive at least one observable for which

$$\mathcal{O}_{\mathrm{TB}} \neq \mathcal{O}_{\text{inflationary baseline}} \tag{77}$$

after both models are fitted fairly.

16 Primary Claim-Specific Failure Conditions

Paper II fails to support the incomplete-history hypothesis if any of the following conditions is unavoidable.

1. The first and second legs do not supply a well-defined raw propagation reach or common-state preparation mechanism.
2. The retained backward causal domains remain insufficient to account for the observed large-scale uniformity.
3. The hidden legs amplify curvature or anisotropy beyond observational limits.
4. The central transition cannot generate stable perturbations.
5. The inherited web necessarily produces excessive statistical anisotropy.
6. The model cannot reproduce the cosmic microwave background acoustic spectrum.
7. The required transition map violates conservation or covariance.
8. The model requires arbitrary functions for every observable.
9. Every proposed historical signature is exactly degenerate with freely chosen third-leg initial conditions.
10. The theory provides no predictive advantage over inflationary ΛCDM after accounting for its additional structure.

17 Notation and Reference Times

Table 1: Principal notation used in Paper II.

Symbol	**Meaning**
t_{res}	First resolved post-singularity spacetime boundary following the pre-singularity state and singular opening.
t_{O}	Center of the outer *Electro* reversal.
t_{C}	Center of the central *Electro* reversal.
t_*	Photon last-scattering time within the third leg.
t_0	Present third-leg cosmic time.
$a_j(t)$	Global scale factor during leg or transition interval j.
$H_j(t)$	Hubble rate during interval j.
$R_E(t)$	Effective physical position of the active *Electro* sector.
$\chi_E(t)$	Effective comoving *Electro* coordinate.

Symbol	Meaning
$c_{\text{eff}}^{(j)}$	Effective causal propagation speed of the relevant degree of freedom during interval j.
η	Conformal time.
$\chi_{\text{causal}}^{(3)}$	Causal comoving interval calculated from the third leg alone.
$\chi_{\text{raw}}^{(123)}$	Raw comoving propagation reach integrated across the resolved post-singularity three-leg history.
$\chi_{\text{ret}}^{(123)}$	Backward common-cause reach retained through the transitions and mapped into the third-leg state.
$\mathfrak{H}_{\text{raw}}$	Raw full-history extension ratio relative to the third-leg-only reach.
Ω_k	Curvature density parameter.
ϵ_{asym}	Pre-singularity asymmetry measure.
$\mathcal{W}_{12}$	Inherited first- and second-leg structural field.
$\mathcal{T}_{\text{O}}$	Outer-transition map.
$\mathcal{T}_{\text{C}}$	Central-transition map.
$\mathcal{R}_{\boldsymbol{k}}$	Third-leg curvature perturbation in Fourier space.
$S_{ij,\boldsymbol{k}}$	Relative-entropy or isocurvature perturbation.
$h_{\boldsymbol{k}}$	Tensor perturbation.
$\Delta_{\text{TB}}(k)$	Derived Triple Bounce modification to a baseline primordial spectrum.

18 Structure of the Analysis

Paper II proceeds through six connected analytical stages.

I. **Inflationary benchmark.** The paper identifies the causal, curvature, perturbative, and thermal functions that any proposed replacement or complement must reproduce.

II. **Full-history causal geometry.** The first leg, outer transition, second leg, central transition, and third leg are integrated as one ordered history, while raw propagation reach is kept distinct from retained common-cause information.

III. **Curvature and near-flatness.** Resolved boundary curvature is propagated through the first leg and both nonsingular reversals, and the model is tested against the standard curvature-transfer result.

IV. **Transition and perturbation transfer.** Finite-width matching laws connect the inherited two-pass state to a third-leg primordial covariance and a later radiation-rich thermal state.

V. **Observational recovery and historical signatures.** The derived initial state is propagated through the Einstein–Boltzmann system and compared with cosmic microwave background observables, large-scale structure, and candidate historical residues.

VI. **Joint comparison and falsification.** One frozen parameter ledger, shared background, complexity accounting, and claim-specific failure conditions determine whether the result is a replacement candidate, complement, partial success, or failure.

This organization is scientific rather than chronological: later sections may revisit the same physical interval at increasing levels of mathematical and observational specificity.

19 Controlling Distinctions

Four distinctions control every calculation in this paper.

1. The observable third-leg history is not assumed to be the complete cosmic history.
2. The first outward *Electro* leg is not inflation by definition; inflation is a condition on the metric expansion.
3. A longer history is not a horizon or flatness candidate resolution without explicit propagation, retention, and curvature calculations.
4. Observational equivalence does not establish physical identity or historical uniqueness.

Accordingly, Paper II must derive the retained causal domain, propagate curvature through the complete ordered history, obtain the third-leg primordial covariance from finite transition dynamics, and test the same model against the relevant observations and alternatives.

20 Standard Inflationary Benchmarks

The incomplete-history hypothesis must be compared with the actual functional achievements of inflation rather than with a simplified description of rapid expansion.

Inflation is successful because a period of accelerated background expansion can simultaneously alter causal geometry, suppress the relative contribution of spatial curvature, stretch microscopic fluctuations to cosmological scales, and prepare coherent perturbations for the later hot expanding universe.

The benchmark is therefore not merely

$$a(t_{\mathrm{end}}) > a(t_{\mathrm{begin}}). \tag{78}$$

Ordinary decelerating cosmology also expands.

The defining background property is accelerated expansion,

$$\ddot{a} > 0, \tag{79}$$

or equivalently,

$$\rho + \frac{3p}{c^2} < 0 \tag{80}$$

for the dominant effective component in a classical Friedmann–Lemaître–Robertson–Walker description.

The Hubble slow-roll parameter is

$$\epsilon_H = -\frac{\dot{H}}{H^2}. \tag{81}$$

Inflation occurs when

$$\epsilon_H < 1. \tag{82}$$

A complete alternative must reproduce the principal consequences of this condition even if it does not reproduce the same physical mechanism.

The Triple Bounce benchmark problem may be summarized as

$$\boxed{\begin{array}{l} \text{causal preparation} \\ +\,\text{curvature suppression} \\ +\,\text{acceptable primordial perturbations} \\ +\,\text{controlled hot-state transition} \\ +\,\text{observational agreement.} \end{array}} \tag{83}$$

These functions are developed separately below.

21 Accelerated Expansion and the Comoving Hubble Radius

21.1 Physical Hubble radius

The physical Hubble radius is

$$R_H = \frac{c}{H}. \tag{84}$$

The corresponding comoving Hubble radius is

$$\mathcal{R}_H = \frac{c}{aH}. \tag{85}$$

The comoving Hubble radius is not generally identical to the particle horizon.

It is nevertheless a central diagnostic because perturbation modes are often classified relative to the scale

$$\frac{kc}{aH}. \tag{86}$$

A mode is conventionally described as being well inside the Hubble scale when

$$\frac{kc}{aH} \gg 1, \tag{87}$$

and well outside it when

$$\frac{kc}{aH} \ll 1. \tag{88}$$

21.2 Shrinkage during inflation

Differentiating the comoving Hubble radius gives

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{c}{aH}\right) = -\frac{c(1-\epsilon_H)}{a}. \tag{89}$$

Therefore,

$$\epsilon_H < 1 \quad \Longrightarrow \quad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{c}{aH}\right) < 0. \tag{90}$$

A shrinking comoving Hubble radius permits modes that begin in a short-wavelength regime to evolve into a long-wavelength regime.

This behavior is opposite to ordinary radiation- and matter-dominated expansion, during which the comoving Hubble radius grows.

21.3 Mode exit and re-entry

A perturbation mode with comoving wavenumber k crosses the Hubble scale when

$$kc = aH. \tag{91}$$

During inflation, the decreasing comoving Hubble radius permits mode exit:

$$\frac{kc}{aH}: \quad \gg 1 \longrightarrow \ll 1. \tag{92}$$

During the later decelerating hot expansion, the comoving Hubble radius increases and the same mode may re-enter:

$$\frac{kc}{aH}: \quad \ll 1 \longrightarrow \gg 1. \tag{93}$$

This exit–re-entry sequence allows early microscopic fluctuations to influence later cosmological structure.

21.4 Triple Bounce comparison target

The Triple Bounce framework does not automatically inherit this mechanism from outward *Electro* motion.

The condition

$$\dot{R}_E > 0 \tag{94}$$

does not imply

$$\epsilon_H < 1. \tag{95}$$

Paper II must therefore determine whether the full three-leg history provides one of the following:

1. an interval with a shrinking comoving Hubble radius;
2. a different causal mechanism producing equivalent mode preparation;
3. a transition-generated perturbation spectrum that does not require conventional Hubble-scale exit; or
4. a combined mechanism involving both inherited structure and limited accelerated expansion.

Proposition 21.1 (Hubble-radius benchmark)**.** *The first outward Electro leg is functionally equivalent to inflation only if the complete model reproduces the observable consequences of inflationary mode preparation, whether or not $\mathcal{R}_H = c/(aH)$ decreases during that leg.*

22 The Inflationary Horizon Benchmark

22.1 The apparent causal problem

In a decelerating hot expansion extrapolated backward without an earlier causal mechanism, widely separated regions of the last-scattering surface may possess nonoverlapping past light cones.

Their nearly uniform temperature then requires explanation.

The relevant issue is not that every point in the observable universe must communicate at the time of recombination.

The issue is whether the matter and radiation occupying those regions descended from a sufficiently small common causal domain.

22.2 Inflationary causal preparation

Inflation permits a region initially contained within one causal domain to expand until it encompasses scales much larger than the later Hubble radius.

A simplified causal preparation sequence is

$$\boxed{\text{initial common patch} \longrightarrow \text{accelerated expansion} \longrightarrow \text{super-Hubble correlation} \longrightarrow \text{later re-entry.}} \tag{96}$$

The observed large-scale uniformity is then inherited from an earlier state in which causal interaction was possible.

Inflation does not require ordinary signals to travel across the entire post-inflationary last-scattering surface after the surface has become large.

It enlarges the physical scale corresponding to an initially correlated region.

22.3 Conformal-time representation

In conformal time,

$$d\eta = \frac{dt}{a(t)}. \tag{97}$$

A nearly de Sitter inflationary phase has approximately

$$a(\eta) \approx -\frac{1}{H\eta}, \qquad \eta < 0, \tag{98}$$

with η approaching zero from below as inflation proceeds.

The ability to extend the relevant modes to an earlier sub-Hubble regime depends on the conformal structure and the duration of inflation.

22.4 Causal benchmark for Triple Bounce

The Triple Bounce framework must demonstrate a corresponding common causal origin.

Its full-history causal domain may be represented as

$$\chi_{\mathrm{TB}} = \int_{t_{\mathrm{res}}}^{t_*} \frac{c_{\mathrm{eff}}(t)}{a(t)} \mathrm{d}t, \tag{99}$$

with piecewise definitions across the first leg, outer transition, second leg, central transition, and early third leg.

A successful alternative requires

$$\chi_{\mathrm{TB}} \geq \chi_{\mathrm{required}}. \tag{100}$$

The phrase *shared topological origin* is not by itself equivalent to this inequality.

The framework must determine:

- when classical causal propagation begins;
- which degrees of freedom transmit correlation;
- their effective propagation speeds;
- whether, and by what dynamics, the scaffold retains or transports the resulting correlation;
- whether both transitions preserve the correlation; and
- how the correlated state maps into the third-leg plasma.

23 The Inflationary Flatness Benchmark

23.1 Evolution of the curvature fraction

The curvature density parameter is

$$\Omega_k = -\frac{kc^2}{a^2H^2}. \tag{101}$$

Using the e-fold variable

$$N = \ln a, \tag{102}$$

its logarithmic evolution is

$$\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} = 2\,(\epsilon_H - 1)\,. \tag{103}$$

During inflation,

$$\epsilon_H < 1, \tag{104}$$

so

$$\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} < 0. \tag{105}$$

For approximately constant ϵ_H,

$$|\Omega_k|_{\mathrm{end}} \approx |\Omega_k|_{\mathrm{begin}} \exp\left[-2\,(1-\epsilon_H)\,N\right]. \tag{106}$$

A sufficiently long accelerated phase can therefore make the curvature fraction extremely small.

23.2 What inflation explains

Inflation does not necessarily force the fundamental spatial-curvature index k to vanish.

It suppresses the relative dynamical contribution

$$\frac{|k|c^2}{a^2H^2}. \tag{107}$$

The observable universe consequently appears close to spatially flat over the measured domain.

23.3 Triple Bounce target

The Triple Bounce framework proposes that near-flatness may be constrained at the first resolved post-singularity boundary by balanced pre-singularity ordering, then preserved or further suppressed through the prior legs and both nonsingular reversals.

To match inflation's achievement, it must derive a transfer relation

$$\Omega_k(t_0) = \mathcal{K}_{\mathrm{TB}}\left[\mathcal{B}_{\mathrm{res}}, \epsilon_{\mathrm{asym}}, \mathcal{U}_{k,1}, \mathcal{T}_{k,\mathrm{O}}, \mathcal{U}_{k,2}, \mathcal{T}_{k,\mathrm{C}}, \mathcal{U}_{k,3}\right], \tag{108}$$

such that

$$\left|\Omega_k^{(3)}\right| \ll 1 \tag{109}$$

for a finite and physically plausible region of initial-condition space.

A single specially selected initial state is insufficient.

The model must possess a stable near-flat basin.

Definition 23.1 (Near-flat basin)**.** A near-flat basin is a nonzero region of initial-condition and transition parameter space whose third-leg solutions satisfy the observational curvature bound without singular evolution or extreme parameter adjustment.

24 Relic Dilution and Initial Inhomogeneity

24.1 Inflationary dilution

Accelerated expansion dilutes the number density of nonrelativistic relics approximately as

$$n_X \propto a^{-3}. \tag{110}$$

If the scale factor grows by N e-folds,

$$n_{X,\mathrm{end}} = n_{X,\mathrm{begin}} e^{-3N} \tag{111}$$

for a conserved nonrelativistic population.

This can suppress unwanted relics such as monopole-like defects in high-energy theories.

Spatial gradients and anisotropies may also be strongly diluted, although their precise evolution depends on the matter content and background dynamics.

24.2 Triple Bounce alternatives

The Triple Bounce framework does not need to duplicate relic dilution if its pre-third-leg microphysics does not produce the same unwanted relic population.

It must instead satisfy the more general condition

$$\Omega_X^{(3)} < \Omega_{X,\max} \tag{112}$$

for every observationally constrained relic X.

This may occur through:

- absence of the relevant relic-generating transition;
- first-leg dilution;

- second-leg annihilation;
- central-transition conversion;
- decay before the observable hot state;
- confinement to an unobservable sector; or
- a limited accelerated phase retained within the larger framework.

The exact mechanism must be derived.

24.3 Anisotropy control

A general homogeneous but anisotropic background may contain a shear contribution satisfying approximately

$$\rho_\sigma \propto a^{-6}. \tag{113}$$

During ordinary contraction, this contribution can grow rapidly relative to matter or radiation.

The Triple Bounce second leg is not assumed to be global contraction, which may avoid the standard contraction-driven shear problem.

However, inward sector motion and multipass structure may generate their own anisotropies.

The framework must demonstrate

$$\frac{\rho_\sigma}{\rho_{\text{tot}}} \ll 1 \tag{114}$$

at the beginning of the observable third leg.

25 Scalar Perturbation Generation

25.1 Quantum origin in inflation

In standard inflationary theory, quantum fluctuations of the inflaton and metric are stretched to cosmological scales (Mukhanov and Chibisov, 1981; Mukhanov, 2005; Baumann, 2009).

For this subsection, natural units

$$c = \hbar = 1 \tag{115}$$

are used.

The quadratic action for the comoving curvature perturbation may be written as

$$S_{\mathcal{R}}^{(2)} = \frac{1}{2}\int \mathrm{d}\eta\, \mathrm{d}^3x\, z^2 \left[(\mathcal{R}')^2 - c_s^2\,(\nabla\mathcal{R})^2\right], \tag{116}$$

where:

- a prime denotes a derivative with respect to conformal time;
- c_s is the scalar sound speed; and
- z is a background-dependent normalization.

Defining the Mukhanov–Sasaki variable

$$v = z\mathcal{R}, \tag{117}$$

each Fourier mode satisfies

$$v_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_k = 0. \tag{118}$$

25.2 Short- and long-wavelength regimes

For

$$c_s^2 k^2 \gg \frac{z''}{z}, \tag{119}$$

the mode behaves approximately as an oscillatory vacuum fluctuation.

For

$$c_s^2 k^2 \ll \frac{z''}{z}, \tag{120}$$

the curvature perturbation develops a nearly conserved mode under adiabatic conditions.

Inflation therefore converts microscopic quantum fluctuations into classical stochastic perturbations that seed later structure.

25.3 Scalar power spectrum

The dimensionless curvature spectrum is defined by

$$\langle \mathcal{R}_{\boldsymbol{k}} \mathcal{R}_{\boldsymbol{k}'}^* \rangle = \frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(k)\delta_D\left(\boldsymbol{k} - \boldsymbol{k}'\right). \tag{121}$$

For a broad class of slowly varying single-field models,

$$\mathcal{P}_{\mathcal{R}}(k) \approx \frac{H_*^2}{8\pi^2 M_{\rm Pl}^2 \epsilon_{H*} c_{s*}}, \tag{122}$$

evaluated near sound-horizon crossing.

The spectrum is commonly parameterized as

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln(k/k_*) + \cdots}. \tag{123}$$

Here:

- A_s is the amplitude;
- n_s is the scalar spectral index; and
- α_s is its running.

25.4 Slow-variation tilt

Define

$$\eta_H = \frac{\dot{\epsilon}_H}{H\epsilon_H}, \tag{124}$$

and

$$s = \frac{\dot{c}_s}{Hc_s}. \tag{125}$$

To leading order in slow variation,

$$n_s - 1 \approx -2\epsilon_H - \eta_H - s. \tag{126}$$

The near scale invariance of the observed spectrum corresponds to a tilt close to, but not exactly equal to, zero.

25.5 Triple Bounce scalar target

The Triple Bounce framework must derive

$$\mathcal{P}_{\mathcal{R}}^{\rm TB}(k) \tag{127}$$

from the first-leg state, second-leg evolution, and central transition.

It must not insert the observed spectrum directly as third-leg initial data.

The central transition must determine or constrain:

$$\{A_s, n_s, \alpha_s, \text{phase}, \text{statistics}\}_{\text{TB}}\,. \tag{128}$$

26 Tensor, Vector, and Isocurvature Benchmarks

26.1 Tensor modes

Inflation also generates tensor perturbations.

For canonical slow-roll inflation, the tensor spectrum is approximately

$$\mathcal{P}_T(k) \approx \frac{2H_*^2}{\pi^2 M_{\text{Pl}}^2}. \tag{129}$$

The tensor-to-scalar ratio is

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}}. \tag{130}$$

For canonical single-field slow-roll inflation,

$$r \approx 16\epsilon_H, \tag{131}$$

while more general sound-speed models may yield

$$r \approx 16\epsilon_H c_s. \tag{132}$$

The Triple Bounce transitions may generate tensor modes through time-dependent geometry, anisotropic stress, or multipass dynamics.

Their amplitude must remain compatible with observational bounds (Planck Collaboration, 2020b; BICEP/Keck Collaboration, 2021).

26.2 Vector modes

Vector perturbations usually decay in simple inflationary and post-inflationary settings unless sustained by a source.

The inherited-web and dynamo-junction hypotheses may introduce vector structure.

A viable model must show that any primordial vector contribution is:

- sufficiently small;
- statistically compatible with isotropy;
- stable across the central transition; and
- consistent with cosmic microwave background polarization and large-scale structure.

26.3 Isocurvature modes

Multiple dynamical sectors can generate isocurvature perturbations.

For components i and j,

$$S_{ij} = 3\left(\zeta_i - \zeta_j\right), \tag{133}$$

where ζ_i is the curvature perturbation on hypersurfaces of uniform density for component i.

Single-clock inflation tends to produce predominantly adiabatic perturbations.

The Triple Bounce framework contains several effective sectors and may therefore generate nonzero isocurvature.

The central matching map must either suppress these modes or predict an observationally acceptable fraction (Planck Collaboration, 2020b).

27 Gaussianity, Statistical Isotropy, and Phase Coherence

27.1 Approximate Gaussianity

The simplest inflationary fluctuations are approximately Gaussian because they arise from weakly interacting linear quantum modes.

The leading non-Gaussian correction is often characterized through a bispectrum,

$$\langle \mathcal{R}_{\boldsymbol{k}_1} \mathcal{R}_{\boldsymbol{k}_2} \mathcal{R}_{\boldsymbol{k}_3} \rangle = (2\pi)^3 \delta_D\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3\right) B_{\mathcal{R}}\left(k_1, k_2, k_3\right). \tag{134}$$

A local-form parameterization may be written schematically as

$$\mathcal{R} = \mathcal{R}_g + \frac{3}{5} f_{\mathrm{NL}}^{\mathrm{local}} \left(\mathcal{R}_g^2 - \left\langle \mathcal{R}_g^2 \right\rangle\right), \tag{135}$$

where $\mathcal{R}_g$ is Gaussian.

The Triple Bounce central transition may be strongly nonequilibrium and could generate significant non-Gaussianity.

That contribution must be derived and constrained.

27.2 Statistical isotropy

For statistically homogeneous and isotropic scalar perturbations,

$$\langle \mathcal{R}_{\boldsymbol{k}} \mathcal{R}^*_{\boldsymbol{k}'} \rangle \propto \delta_D \left(\boldsymbol{k} - \boldsymbol{k}' \right) \mathcal{P}_{\mathcal{R}}(k), \tag{136}$$

with dependence only on the magnitude k.

An inherited web may introduce directional information.

The model must ensure that the largest-scale statistical anisotropy is small:

$$\mathcal{P}_{\mathcal{R}}(\boldsymbol{k}) = \mathcal{P}^{(0)}_{\mathcal{R}}(k) \left[1 + g_* \left(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}} \right)^2 + \cdots \right], \tag{137}$$

with

$$|g_*| \ll 1 \tag{138}$$

over observationally constrained scales.

27.3 Phase coherence

The acoustic peaks of the cosmic microwave background require coherent phases in the primordial perturbations (Hu and Dodelson, 2002; Planck Collaboration, 2020a).

Modes of the same wavenumber must begin the photon–baryon oscillation with correlated temporal phases.

A generic acoustic solution may be written as

$$\delta_\gamma(k, \eta) = A_k \cos\left[k r_s(\eta) \right] + B_k \sin\left[k r_s(\eta) \right]. \tag{139}$$

Inflationary adiabatic perturbations produce a highly coherent relation among A_k and B_k.

A purely stochastic active source operating continuously after horizon entry generally risks washing out the acoustic peak structure.

The Triple Bounce model must therefore show that its inherited and transition-generated perturbations become effectively passive and phase coherent before the observable acoustic evolution.

Proposition 27.1 (Acoustic-coherence benchmark)**.** *Any Triple Bounce perturbation mechanism that destroys the observed coherent acoustic phase structure fails even if it reproduces the overall scalar power amplitude.*

28 Duration and the E-Fold Benchmark

28.1 Inflationary e-folds

The duration of inflation is commonly measured by the number of e-folds,

$$N(t_a, t_b) = \int_{t_a}^{t_b} H(t)\mathrm{d}t = \ln\left[\frac{a(t_b)}{a(t_a)}\right]. \tag{140}$$

The number required for observable scales depends on:

- the inflationary energy scale;
- the evolution of H;
- the reheating equation of state;
- the reheating temperature;
- the post-inflationary thermal history; and
- the comoving scale being considered.

Values of order several tens of e-folds are commonly required, with a frequently discussed observational range near approximately fifty to sixty for many standard histories.

This is not a universal fixed number.

28.2 Triple Bounce replacement quantity

The Triple Bounce framework need not possess the same inflationary e-fold count.

It must instead produce sufficient causal and geometrical preparation.

Define an effective preparation functional

$$\mathfrak{P}_{\mathrm{TB}} = \mathfrak{P}\left[\Delta\eta_{123}, \Omega_k^{(3)}, \mathcal{P}_{\mathcal{R}}^{(3)}, \mathcal{R}_{13}, \mathcal{T}_{\mathrm{C}}\right]. \tag{141}$$

Functional equivalence requires

$$\mathfrak{P}_{\mathrm{TB}} \geq \mathfrak{P}_{\mathrm{required}}. \tag{142}$$

The model must not replace a defined inflationary e-fold requirement with an undefined statement that the prior history was simply "long enough."

29 The Hot-State Handoff and Reheating Benchmark

29.1 Inflationary reheating

Inflation must end and transfer energy into the particles and radiation of the hot expanding universe.

This transition is broadly called reheating.

A schematic sequence is

$$\begin{aligned}\text{inflationary energy} &\longrightarrow \text{particle production} \longrightarrow \text{thermalization} \\ &\longrightarrow \text{radiation-dominated expansion.}\end{aligned} \tag{143}$$

The effective background during reheating may be parameterized by an average equation of state

$$\bar{w}_{\rm reh} = \frac{\bar{p}_{\rm reh}}{\bar{\rho}_{\rm reh} c^2}. \tag{144}$$

The energy density evolves approximately as

$$\rho_{\rm reh} \propto a^{-3(1+\bar{w}_{\rm reh})}. \tag{145}$$

The details of reheating affect the mapping between observed scales and their inflationary time of origin.

29.2 Triple Bounce central handoff

The Triple Bounce history must eventually provide the functional handoff that inflationary cosmology assigns to reheating, though not necessarily through the same microphysics. Paper II does not assume that stable ordinary matter or a complete thermal bath exists at the instant of the central reversal.

A candidate handoff begins from

$$\mathcal{I}_{\rm C}^{-} = \left\{\rho_E, \rho_{\rm exc}, \rho_r, \mathcal{W}_{12}, \mathcal{M}_{ij}^{(12)}, g_{\mu\nu}, \dots\right\} \tag{146}$$

and must evolve into a viable early-third-leg hot state,

$$\mathcal{I}_{\rm C}^{+} = \{\rho_{\rm rad}, \rho_{\rm sc}, \rho_{\rm ch}, f_i, \mathcal{R}, \Phi, \Psi, \dots\}. \tag{147}$$

Here, $\rho_{\rm exc}$ denotes optional particle-like or pair-producing excitations; it is not a required pair-rich phase. The output notation summarizes an effective early-third-leg state from which standard thermal evolution and later matter organization must be recoverable.

The handoff must satisfy:

- energy and momentum conservation;
- finite temperature and density;
- acceptable particle abundances;
- sufficient thermalization;
- a predominantly radiation-rich early third leg;
- acceptable entropy production;
- stable perturbation transfer; and
- compatibility with nucleosynthesis.

29.3 Minimum thermal condition

The resulting hot state must reach a temperature sufficient for the standard thermal processes required before nucleosynthesis.

Denote the effective post-transition temperature by

$$T_{\mathrm{C}}^{+}. \tag{148}$$

A viable model requires

$$T_{\mathrm{C}}^{+} > T_{\min}, \tag{149}$$

where $T_{\min}$ is set by the particle and nuclear processes that must occur in the later thermal history.

Paper II does not determine the precise temperature.

It establishes the handoff as a required benchmark.

30 Inflationary Model Diversity

Inflation is not one unique potential or one unique field theory.

Representative classes include:

- single-field slow-roll inflation;
- plateau models;

- large-field models;
- small-field models;
- multifield inflation;
- noncanonical sound-speed models;
- modified-gravity inflation;
- warm inflation;
- hybrid inflation; and
- models with nontrivial reheating or spectator sectors.

These models may differ in:

$$\{n_s, r, \alpha_s, f_{\mathrm{NL}}, S_{ij}, T_{\mathrm{reh}}, N_*\}\,. \tag{150}$$

The Triple Bounce framework should therefore be compared against:

1. the general functional achievements shared by viable inflationary models; and
2. specific benchmark models when numerical predictions are calculated.

It would be misleading to disprove one inflationary potential and claim that inflation as a broad framework had been excluded.

Likewise, matching one inflationary prediction would not establish the Triple Bounce model as uniquely correct.

31 Inflationary Initial-Condition Questions

Inflation addresses several initial-condition problems, but it does not eliminate every question about initial conditions.

A complete inflationary model may still require explanation of:

- why an inflation-supporting region exists;
- why the relevant field begins in a suitable part of its potential;
- the measure assigned to possible initial states;
- the origin of the inflaton or equivalent degree of freedom;
- the transition from a more fundamental theory;
- the global past boundary; and

- the relationship between inflation and quantum gravity.

These open questions do not negate inflation's observational successes.

They establish that inflation may itself be part of a longer history.

This creates a possible complementarity with the Triple Bounce framework.

A hidden first and second leg could, in principle, prepare an inflationary third-leg boundary state.

The possible sequence would be

$$\text{Triple Bounce prehistory} \longrightarrow \text{inflation-compatible boundary state} \longrightarrow \text{hot third-leg expansion.} \tag{151}$$

This is the complement outcome defined in section 12.2.

Paper II therefore evaluates both:

$$\text{Triple Bounce instead of inflation} \tag{152}$$

and

$$\text{Triple Bounce before or around inflation.} \tag{153}$$

32 Exact Functional Targets for Triple Bounce

The inflationary benchmark can be summarized as a target vector,

$$\boldsymbol{\mathcal{B}}_{\rm inf} = \left(\mathcal{C}_{\rm hor}, \mathcal{F}_k, \mathcal{P}_{\mathcal{R}}, \mathcal{P}_T, \mathcal{I}_{\rm iso}, \mathcal{N}_{\rm G}, \mathcal{A}_{\rm phase}, \mathcal{H}_{\rm hot}, \mathcal{D}_{\rm relic}\right), \tag{154}$$

where:

- $\mathcal{C}_{\rm hor}$ measures causal sufficiency;
- $\mathcal{F}_k$ measures curvature suppression;
- $\mathcal{P}_{\mathcal{R}}$ is the scalar perturbation spectrum;
- $\mathcal{P}_T$ is the tensor spectrum;
- $\mathcal{I}_{\rm iso}$ measures isocurvature content;
- $\mathcal{N}_{\rm G}$ measures non-Gaussianity;
- $\mathcal{A}_{\rm phase}$ measures acoustic phase coherence;

- $\mathcal{H}_{\text{hot}}$ characterizes the hot-state handoff; and
- $\mathcal{D}_{\text{relic}}$ characterizes relic control.

The Triple Bounce model produces its own vector,

$$\boldsymbol{\mathcal{B}}_{\text{TB}} = \boldsymbol{\mathcal{B}}\left[\mathcal{S}_{\text{pre}}, \mathcal{U}_1, \mathcal{T}_{\text{O}}, \mathcal{U}_2, \mathcal{T}_{\text{C}}, \mathcal{U}_3\right]. \tag{155}$$

Functional recovery requires

$$\boldsymbol{\mathcal{B}}_{\text{TB}} \in \mathcal{V}_{\text{obs}}, \tag{156}$$

where $\mathcal{V}_{\text{obs}}$ is the observationally acceptable region.

Functional replacement requires the stronger condition

$$\boldsymbol{\mathcal{B}}_{\text{TB}} \in \mathcal{V}_{\text{obs}} \tag{157}$$

without an independently required inflationary stage.

Functional distinction additionally requires at least one derived observable

$$\mathcal{O}_{\text{res}} \tag{158}$$

satisfying

$$\mathcal{O}_{\text{res}}^{\text{TB}} \neq \mathcal{O}_{\text{res}}^{\text{inflation}} \tag{159}$$

at a measurable level.

33 Benchmark Matrix

Table 2: Inflationary functions and Triple Bounce requirements.

Problem	**Inflationary benchmark**	**Triple Bounce requirement**
Horizon	An initially causal region expands to encompass observable scales.	The full first–second–third-leg causal history must provide sufficient common causal preparation.
Flatness	A shrinking comoving Hubble radius exponentially suppresses $\lvert\Omega_k\rvert$.	Balanced initial ordering and transition dynamics must yield a stable near-flat third-leg basin.
Scalar spectrum	Quantum fluctuations generate a nearly scale-invariant, predominantly adiabatic spectrum.	The inherited state and central matching map must derive an acceptable $\mathcal{P}_{\mathcal{R}}(k)$.
Tensor modes	Background quantum fluctuations generate a model-dependent tensor spectrum.	The complete two-reversal history must yield tensor power within observational limits.
Isocurvature	Single-clock models naturally suppress relative-entropy modes.	The multisector Triple Bounce state must suppress or constrain isocurvature.
Gaussianity	Weakly interacting fluctuations are approximately Gaussian.	Central nonequilibrium must not generate excessive non-Gaussianity.
Phase coherence	Passive primordial perturbations generate coherent acoustic peaks.	Inherited and transition-generated perturbations must become coherent before acoustic evolution.
Relics	Accelerated expansion exponentially dilutes unwanted relics.	The framework must prevent, remove, decay, annihilate, or dilute any unacceptable relic population.
Hot-state handoff	Reheating transfers inflationary energy into a thermal particle bath.	The early third leg must reach a viable radiation-rich state through a derived handoff history.
Distinctiveness	Specific inflationary models predict relations among n_s, r, and non-Gaussianity.	Triple Bounce must derive at least one nondegenerate historical signature.

34 Benchmark Comparison and Claim-Specific Failure Conditions

The following results reject the corresponding Triple Bounce benchmark counterclaim or frozen realization. They do not automatically reject the triple directional history, and failure of a counterclaim does not prove that inflation is uniquely necessary:

1. an insufficient full-history causal interval;
2. a third-leg curvature fraction requiring extreme fine tuning;
3. a scalar spectrum with unacceptable amplitude or tilt;
4. loss of acoustic phase coherence;
5. excessive tensor power;
6. excessive vector or isocurvature modes;
7. large primordial statistical anisotropy;
8. non-Gaussianity beyond acceptable limits;
9. a singular or unstable central handoff;
10. a post-transition temperature incompatible with the hot thermal history;
11. unacceptable relic abundance;
12. failure to reproduce the cosmic microwave background acoustic spectrum; or
13. no distinctive output beyond freely selected initial conditions.

A longer history does not rescue a failed frozen mechanism. The failed mechanism remains rejected, while neighboring claim families retain their independent status.

35 Formal Summary of the Inflationary Benchmark

Inflation is characterized at the background level by

$$\boxed{\ddot{a} > 0 \quad \Longleftrightarrow \quad \epsilon_H < 1} \tag{160}$$

and therefore

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{c}{aH}\right) < 0.} \tag{161}$$

Its curvature effect is

$$\boxed{\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} = 2\,(\epsilon_H - 1) < 0.} \tag{162}$$

Its perturbative benchmark is

$$\boxed{\text{small} + \text{nearly scale invariant} + \text{adiabatic} + \text{nearly Gaussian} + \text{phase coherent.}} \tag{163}$$

Its hot-state requirement is

$$\boxed{\text{early driving sector} \longrightarrow \text{thermal radiation and particles.}} \tag{164}$$

The Triple Bounce framework is not required to reproduce the same microphysical steps.

It is required to reproduce the successful observable outputs.

The exact comparative question is

$$\boxed{\boldsymbol{\mathcal{B}}_{\mathrm{TB}} \overset{?}{\in} \mathcal{V}_{\mathrm{obs}}} \tag{165}$$

without hiding the required behavior inside arbitrary third-leg initial conditions.

36 Full-History Conformal Geometry

The horizon question is fundamentally a question about causal geometry.

A longer narrative does not automatically produce a larger causal domain.

The first and second legs contribute to the horizon calculation only if the relevant physical degrees of freedom can propagate, interact, or preserve correlations across a mathematically defined classical background.

Paper II therefore separates three concepts:

1. **Elapsed duration,** which measures the proper cosmic time assigned to an interval;
2. **sector trajectory length,** which measures the path followed by a particular dynamical sector; and
3. **causal comoving reach,** which measures how far physically relevant information can propagate through the expanding geometry.

These quantities need not be equal.

For example, a long first leg may contribute little causal distance if the scale factor grows too rapidly.

Conversely, a relatively short interval may contribute substantial comoving causal reach if the scale factor is small or if the relevant propagation speed is large.

The complete horizon test must therefore calculate

$$\chi_{\mathrm{raw}}^{(123)} = \sum_j \chi_j, \tag{166}$$

where each term is derived from the geometry and propagation law of one leg or transition interval.

Proposition 36.1 (Causal contribution criterion)**.** *A pre-third-leg interval contributes to the observable causal history only if a physically defined carrier of correlation propagates through that interval or if a previously established correlation is preserved through it by a specified dynamical memory mechanism.*

37 Radial Propagation in a Classical FLRW Regime

37.1 Conformal radial coordinate

For a spatially homogeneous and isotropic background, the line element may be written as

$$\mathrm{d}s^2 = a^2(\eta)\left[-c^2\mathrm{d}\eta^2 + \mathrm{d}\Sigma_k^2\right], \tag{167}$$

where

$$\mathrm{d}\eta = \frac{\mathrm{d}t}{a(t)} \tag{168}$$

and $\mathrm{d}\Sigma_k^2$ is the metric of the constant-curvature spatial hypersurface.

For radial propagation, one may use a comoving geodesic coordinate χ such that

$$\mathrm{d}s^2 = a^2(\eta)\left[-c^2\mathrm{d}\eta^2 + \mathrm{d}\chi^2\right] \tag{169}$$

along a radial path.

For an ordinary radial null signal,

$$\mathrm{d}s^2 = 0 \tag{170}$$

implies

$$\frac{\mathrm{d}\chi}{\mathrm{d}\eta} = \pm c. \tag{171}$$

Equivalently,

$$\frac{\mathrm{d}\chi}{\mathrm{d}t} = \pm \frac{c}{a(t)}. \tag{172}$$

The comoving distance traveled between t_a and t_b is therefore

$$\Delta\chi_\gamma(t_a, t_b) = \int_{t_a}^{t_b} \frac{c\,\mathrm{d}t}{a(t)}. \tag{173}$$

37.2 Physical distance

The corresponding physical separation at time t is

$$D_{\mathrm{phys}}(t) = a(t)\Delta\chi. \tag{174}$$

The comoving distance and physical distance therefore answer different questions.

The horizon calculation is most naturally expressed in comoving coordinates because the present separation of distant regions may be compared with the accumulated comoving reach of their past causal domains.

38 Effective Pre-Electromagnetic Propagation

38.1 Generalized characteristic speed

Ordinary electromagnetic signals propagate locally at speed c in vacuum during the established third-leg regime.

The earlier legs may involve degrees of freedom whose characteristic propagation law has not yet been derived.

Paper II therefore introduces

$$c_{\mathrm{eff}}^{(j)} = c_{\mathrm{eff}}^{(j)}\left(t, k, \mathcal{S}_j\right), \tag{175}$$

where:

- j identifies the leg or transition interval;

- t is cosmic time;
- k is comoving wavenumber when the propagation is dispersive; and
- $\mathcal{S}_j$ represents the local background state.

The effective comoving propagation law is

$$\frac{\mathrm{d}\chi_{\mathrm{sig}}}{\mathrm{d}t} = \pm \frac{c_{\mathrm{eff}}^{(j)}(t,k)}{a_j(t)}. \tag{176}$$

The causal contribution of interval j is then

$$\chi_j(k) = \int_{\mathcal{I}_j} \frac{c_{\mathrm{eff}}^{(j)}(t,k)}{a_j(t)} \mathrm{d}t. \tag{177}$$

38.2 No universal speed assumption

The framework does not assume

$$c_{\mathrm{eff}}^{(1)} = c_{\mathrm{eff}}^{(2)} = c_{\mathrm{eff}}^{(3)} = c. \tag{178}$$

Such an equality may emerge from a later theory, but it is not inserted here.

A physically acceptable completion must nevertheless satisfy:

1. a well-defined characteristic cone;
2. stable propagation;
3. no uncontrolled acausal behavior;
4. compatibility with the effective background geometry;
5. a consistent low-energy limit; and
6. recovery of ordinary relativistic propagation during the third leg.

38.3 Frequency-dependent propagation

If the early medium is dispersive, the relevant speed may depend on wavenumber.

The full causal reach then becomes

$$\chi_{\mathrm{causal}}(k) = \sum_j \int_{\mathcal{I}_j} \frac{c_{\mathrm{eff}}^{(j)}(t,k)}{a_j(t)} \mathrm{d}t. \tag{179}$$

This scale dependence could leave observable signatures.

It could also prevent one early causal process from preparing all cosmological scales equally.

The model must therefore determine whether the relevant long-wavelength modes experience a common effective causal history.

39 Piecewise Decomposition of the Full Raw Causal Reach

The complete raw comoving propagation reach from the first resolved post-singularity boundary to a third-leg event at time t is

$$\chi_{\text{raw}}^{(123)}(t,k) = \chi_1(k) + \chi_{\text{O}}(k) + \chi_2(k) + \chi_{\text{C}}(k) + \chi_3(t,k), \tag{180}$$

where

$$\chi_1(k) = \int_{t_{\text{res}}}^{t_{\text{O}}^-} \frac{c_{\text{eff}}^{(1)}(t,k)}{a_1(t)}\text{d}t, \tag{181}$$

$$\chi_{\text{O}}(k) = \int_{t_{\text{O}}^-}^{t_{\text{O}}^+} \frac{c_{\text{eff}}^{(\text{O})}(t,k)}{a_{\text{O}}(t)}\text{d}t, \tag{182}$$

$$\chi_2(k) = \int_{t_{\text{O}}^+}^{t_{\text{C}}^-} \frac{c_{\text{eff}}^{(2)}(t,k)}{a_2(t)}\text{d}t, \tag{183}$$

$$\chi_{\text{C}}(k) = \int_{t_{\text{C}}^-}^{t_{\text{C}}^+} \frac{c_{\text{eff}}^{(\text{C})}(t,k)}{a_{\text{C}}(t)}\text{d}t, \tag{184}$$

$$\chi_3(t,k) = \int_{t_{\text{C}}^+}^{t} \frac{c_{\text{eff}}^{(3)}(t',k)}{a_3(t')}\text{d}t'. \tag{185}$$

For ordinary photons after electromagnetic emergence,

$$c_{\text{eff}}^{(3)} = c. \tag{186}$$

At last scattering,

$$\chi_{\text{raw}}^{(123)}(t_*) = \chi_1 + \chi_{\text{O}} + \chi_2 + \chi_{\text{C}} + \chi_3(t_*). \tag{187}$$

The third-leg-only comparison quantity is

$$\chi_{\text{causal}}^{(3)}(t_*) = \chi_3(t_*). \tag{188}$$

40 Contribution Fractions and History Extension

40.1 Interval fractions

Define the fractional contribution of interval j by

$$f_j(k) = \frac{\chi_j(k)}{\chi_{\rm raw}^{(123)}(t_*, k)}. \tag{189}$$

The fractions satisfy

$$f_1 + f_{\rm O} + f_2 + f_{\rm C} + f_3 = 1. \tag{190}$$

This decomposition reveals which part of the history performs the causal work.

A model in which

$$f_1 + f_{\rm O} + f_2 + f_{\rm C} \ll 1 \tag{191}$$

does not gain substantial horizon explanatory power from its hidden history.

40.2 History-extension ratio

The raw history-extension ratio is

$$\mathfrak{H}_{\rm raw}(k) = \frac{\chi_{\rm raw}^{(123)}(t_*, k)}{\chi_3(t_*, k)}. \tag{192}$$

Using the piecewise decomposition,

$$\mathfrak{H}_{\rm raw}(k) = 1 + \frac{\chi_1 + \chi_{\rm O} + \chi_2 + \chi_{\rm C}}{\chi_3(t_*)}. \tag{193}$$

A meaningful extension requires

$$\mathfrak{H}_{\rm raw} > 1. \tag{194}$$

A horizon candidate resolution requires the stronger condition

$$\mathfrak{H}_{\rm ret} \geq \mathfrak{H}_{\rm required}, \tag{195}$$

where the required ratio depends on the baseline third-leg geometry and the separation of the regions being compared.

40.3 Causal gain

Define the absolute causal gain as

$$\Delta\chi_{\mathrm{pre}} = \chi_1 + \chi_{\mathrm{O}} + \chi_2 + \chi_{\mathrm{C}}. \tag{196}$$

The hidden history is causally significant when

$$\Delta\chi_{\mathrm{pre}} \sim \chi_{\mathrm{required}} \tag{197}$$

or larger.

The raw ratio $\mathfrak{H}_{\mathrm{raw}}$, the retained ratio $\mathfrak{H}_{\mathrm{ret}}$, and the absolute gain $\Delta\chi_{\mathrm{pre}}$ must both be reported because a large ratio can be misleading when the denominator is very small.

41 Conformal Matching Across Finite Transitions

41.1 Finite-width transitions

The preferred foundational treatment assigns finite widths to both reversals.

Let

$$\Delta t_{\mathrm{O}} = t_{\mathrm{O}}^{+} - t_{\mathrm{O}}^{-}, \tag{198}$$

$$\Delta t_{\mathrm{C}} = t_{\mathrm{C}}^{+} - t_{\mathrm{C}}^{-}. \tag{199}$$

The conformal durations are

$$\Delta\eta_{\mathrm{O}} = \int_{t_{\mathrm{O}}^{-}}^{t_{\mathrm{O}}^{+}} \frac{\mathrm{d}t}{a_{\mathrm{O}}(t)}, \tag{200}$$

$$\Delta\eta_{\mathrm{C}} = \int_{t_{\mathrm{C}}^{-}}^{t_{\mathrm{C}}^{+}} \frac{\mathrm{d}t}{a_{\mathrm{C}}(t)}. \tag{201}$$

Finite transitions contribute finite causal distance when

$$0 < \int \frac{c_{\mathrm{eff}}(t)}{a(t)} \mathrm{d}t < \infty. \tag{202}$$

41.2 Metric continuity

For a smooth classical transition, the scale factor should satisfy

$$a(t_j^-) = a(t_j^+). \tag{203}$$

The Hubble rate should also remain finite:

$$|H(t_j)| < \infty. \tag{204}$$

The foundational continuously expanding version additionally requires

$$H(t_j) > 0. \tag{205}$$

A continuous scale factor ensures that the conformal coordinate does not jump discontinuously because of an arbitrary normalization change.

41.3 Signal matching

A signal or correlation carrier crossing a transition may undergo a change in propagation law.

Its incoming and outgoing perturbation variables are related through a transition matrix,

$$\boldsymbol{X}_j^+ = \mathsf{M}_j(k)\boldsymbol{X}_j^-, \tag{206}$$

where $\boldsymbol{X}_j$ may contain:

$$\boldsymbol{X}_j = (\delta\psi, \Pi_\psi, \mathcal{R}, \Phi, \Psi, \delta\rho_i, \theta_i)^{\mathsf{T}}. \tag{207}$$

A causal interval preceding the transition remains observationally relevant only if the transition does not erase the correlation.

This introduces a transfer-efficiency factor

$$0 \le \mathcal{E}_j(k) \le 1. \tag{208}$$

The effective retained contribution is then

$$\chi_j^{\mathrm{ret}}(k) = \mathcal{E}_j(k)\chi_j(k). \tag{209}$$

The full *retained* causal history may therefore be smaller than the raw integrated causal distance.

42 First-Leg Causal Contribution

The first outward leg begins at the first resolved post-singularity boundary,

$$t_{\mathrm{res}} \leq t \leq t_{\mathrm{O}}^{-}. \tag{210}$$

Its causal contribution is

$$\chi_1(k) = \int_{t_{\mathrm{res}}}^{t_{\mathrm{O}}^{-}} \frac{c_{\mathrm{eff}}^{(1)}(t,k)}{a_1(t)} \mathrm{d}t. \tag{211}$$

The magnitude of χ_1 depends on three independent properties:

1. the duration of the first leg;
2. the scale-factor history $a_1(t)$; and
3. the characteristic propagation speed $c_{\mathrm{eff}}^{(1)}$.

A large physical *Electro* excursion does not by itself guarantee a large value of χ_1.

The first leg contributes substantially when the ratio

$$\frac{c_{\mathrm{eff}}^{(1)}}{a_1} \tag{212}$$

remains large over a sufficient interval.

42.1 First-leg correlation production

The first leg may prepare large-scale correlation through:

- direct signal propagation;
- common interaction with the released *Electro* field;
- coherent scaffold formation;
- a shared boundary-state mode; or
- a combination of these processes.

The resulting two-point correlation may be written as

$$C_1(\boldsymbol{x}, \boldsymbol{y}) = \langle \delta X_1(\boldsymbol{x}) \delta X_1(\boldsymbol{y}) \rangle . \tag{213}$$

The first-leg causal mechanism must explain why this correlation remains significant for separations later visible across the third-leg last-scattering surface.

43 Second-Leg Causal Contribution

The second leg occurs during

$$t_{\mathrm{O}}^{+} \le t \le t_{\mathrm{C}}^{-}. \tag{214}$$

Its causal contribution is

$$\chi_2(k) = \int_{t_{\mathrm{O}}^{+}}^{t_{\mathrm{C}}^{-}} \frac{c_{\mathrm{eff}}^{(2)}(t,k)}{a_2(t)} \mathrm{d}t. \tag{215}$$

The active *Electro* trajectory satisfies

$$\dot{R}_E^{(2)} < 0, \tag{216}$$

while the background may satisfy

$$H_2 > 0. \tag{217}$$

The sign of $\dot{R}_E$ does not change the sign of elapsed conformal time.

The second leg adds positive causal duration whenever

$$c_{\mathrm{eff}}^{(2)} > 0 \tag{218}$$

and the interval is nonzero.

43.1 Return and repeated contact

The inward trajectory may allow *Electro* to revisit regions traversed during the first leg.

This creates the possibility of repeated interaction.

Let

$$\mathcal{D}_{12} = \mathcal{D}_1 \cap \mathcal{D}_2 \tag{219}$$

represent the spatial domain traversed during both passes.

A repeated-contact measure may be defined as

$$\mathcal{C}_{12} = \frac{\mathrm{Vol}\,(\mathcal{D}_{12})}{\mathrm{Vol}\,(\mathcal{D}_1 \cup \mathcal{D}_2)}. \tag{220}$$

This quantity measures pathway overlap.

It does not directly measure causal reach.

Repeated traversal may strengthen correlations within an already connected domain, but it cannot replace the requirement that the domain was causally prepared.

43.2 Second-leg memory preservation

A first-leg correlation transported through the second leg may decay.

Let

$$C_{12}(k) = \mathcal{E}_{12}(k) C_1(k), \tag{221}$$

where

$$0 \leq \mathcal{E}_{12}(k) \leq 1. \tag{222}$$

A strong inherited-history model requires

$$\mathcal{E}_{12}(k) \approx 1 \tag{223}$$

on the large scales relevant to the horizon problem.

44 Electro Trajectory Length Is Not the Causal Horizon

The effective comoving *Electro* coordinate is

$$\chi_E(t) = \frac{R_E(t)}{a(t)}. \tag{224}$$

Its comoving trajectory length during leg j is

$$L_E^{(j)} = \int_{\mathcal{I}_j} |\dot{\chi}_E(t)| \, \mathrm{d}t. \tag{225}$$

The physical trajectory length is

$$L_{E,\mathrm{phys}}^{(j)} = \int_{\mathcal{I}_j} \left| \dot{R}_E(t) \right| \mathrm{d}t. \tag{226}$$

Neither quantity is automatically equal to

$$\chi_j = \int_{\mathcal{I}_j} \frac{c_{\text{eff}}^{(j)}}{a_j} \mathrm{d}t. \tag{227}$$

Equality occurs only under a defined propagation relation.

For example, if *Electro* itself carries the relevant correlation and

$$c_E(t) = a(t) \left| \dot{\chi}_E(t) \right|, \tag{228}$$

then

$$\int \frac{c_E(t)}{a(t)} \mathrm{d}t = \int \left| \dot{\chi}_E(t) \right| \mathrm{d}t = L_E. \tag{229}$$

This equality must be derived from the *Electro* field equations.

It is not assumed merely from the geometric picture of an outward and inward path.

Limitation 44.1*.* The radial extent or retraced length of the *Electro* path cannot be used as the cosmological particle horizon unless *Electro* is shown to be a physical carrier of the relevant correlations with a defined characteristic propagation law.

45 Analytical Toy Backgrounds

Toy backgrounds do not establish the physical Triple Bounce solution.

They clarify which combinations of duration, expansion, and propagation speed enlarge the causal interval.

45.1 Constant scale factor

If

$$a(t) = a_0 \tag{230}$$

and

$$c_{\text{eff}} = c_X, \tag{231}$$

then

$$\Delta\chi = \frac{c_X}{a_0} \Delta t. \tag{232}$$

The comoving causal reach grows linearly with duration.

45.2 Power-law expansion

Let

$$a(t) = a_r \left(\frac{t}{t_r}\right)^p, \tag{233}$$

where p is constant and $t > 0$.

For constant $c_{\text{eff}} = c_X$ and $p \neq 1$,

$$\Delta\chi = \frac{c_X t_r^p}{a_r(1-p)} \left[t_b^{1-p} - t_a^{1-p}\right]. \tag{234}$$

For

$$p = 1, \tag{235}$$

the result is

$$\Delta\chi = \frac{c_X t_r}{a_r} \ln\left(\frac{t_b}{t_a}\right). \tag{236}$$

For decelerating expansion with

$$0 < p < 1, \tag{237}$$

the accumulated comoving causal distance grows as a positive power of time.

45.3 Exponential expansion

Let

$$a(t) = a_i \exp\left[H_X(t - t_i)\right], \tag{238}$$

with constant $H_X > 0$.

Then

$$\Delta\chi = \frac{c_X}{a_i H_X} \left[1 - \exp\left(-H_X \Delta t\right)\right]. \tag{239}$$

As Δt becomes large,

$$\Delta\chi \longrightarrow \frac{c_X}{a_i H_X}. \tag{240}$$

This illustrates why elapsed duration alone does not determine causal reach.

Rapid expansion may cause the future-directed comoving distance accumulated during a finite starting epoch to approach a limiting value.

Inflation nevertheless prepares large observable correlations because the physical scale associated with an initially causal region is stretched dramatically.

45.4 Piecewise power laws

A basic Triple Bounce toy background may assign

$$a_j(t) = A_j \left(\frac{t - t_{j,0}}{\tau_j}\right)^{p_j} \tag{241}$$

within each classical interval.

The total causal distance is then

$$\chi_{\text{toy}} = \sum_j \frac{c_j \tau_j^{p_j}}{A_j(1 - p_j)} \left[u_{j,b}^{1-p_j} - u_{j,a}^{1-p_j}\right], \tag{242}$$

where

$$u_j = t - t_{j,0}, \tag{243}$$

for $p_j \neq 1$.

The constants must be chosen so that $a(t)$ remains continuous across the transitions.

46 Expansion-Normalization Invariance

The numerical value of a comoving coordinate depends on the normalization of the scale factor.

If

$$a(t) \longrightarrow \lambda a(t), \tag{244}$$

then comoving coordinates transform as

$$\chi \longrightarrow \frac{\chi}{\lambda} \tag{245}$$

so that physical distance remains unchanged:

$$a\chi \longrightarrow \lambda a \frac{\chi}{\lambda} = a\chi. \tag{246}$$

Therefore, causal comparisons must use a common scale-factor normalization.

A convenient convention is

$$a(t_0) = 1. \tag{247}$$

All first-, second-, and third-leg scale factors must then be connected to that same normalization through the transition matching.

One may not normalize each leg independently and add the resulting comoving distances without applying the appropriate conversion factors.

Proposition 46.1 (Common-normalization requirement)**.** *Every interval entering the full-history conformal sum must be expressed using one globally consistent normalization of the scale factor.*

47 Common-Cause Domains

47.1 Two-point overlap condition

Consider two comoving points A and B on the last-scattering hypersurface separated by comoving geodesic distance

$$\Delta\chi_{AB}. \tag{248}$$

Let the backward causal reach of each point to an earlier preparation hypersurface be

$$\chi_{\text{back}}(t_{\text{prep}}, t_*). \tag{249}$$

In a symmetric flat-background approximation, their backward causal domains overlap when

$$2\chi_{\text{back}} \geq \Delta\chi_{AB}. \tag{250}$$

More generally, the condition is that the intersection

$$J^-(A) \cap J^-(B) \tag{251}$$

contains a physically connected preparation region before the correlations are fixed.

47.2 Whole-surface preparation

To explain large-angle cosmic microwave background uniformity, the condition must hold for the largest relevant separations on the last-scattering surface.

Define

$$\Delta\chi_{\max} = \max\{\Delta\chi_{AB}\} \tag{252}$$

over the observational domain requiring common preparation.

A sufficient symmetric condition is

$$2\chi_{\mathrm{ret}}^{(123)}(t_*) \geq \Delta\chi_{\max}. \tag{253}$$

The precise calculation must account for:

- spatial curvature;
- the last-scattering geometry;
- the observer's past light cone;
- the selected preparation hypersurface;
- transition transfer efficiencies; and
- the characteristic cone of the relevant early degree of freedom.

47.3 Common cause without repeated late communication

Once a common preparation process establishes correlated initial data, the separated regions do not need to remain in continuous causal contact throughout the entire third leg.

The causal sequence may be

$$\boxed{\begin{aligned} &\text{early connected preparation} \longrightarrow \text{separation} \\ &\qquad \longrightarrow \text{preserved correlation} \longrightarrow \text{last-scattering observation.} \end{aligned}} \tag{254}$$

The scientific burden therefore contains two parts:

1. establish the early causal preparation; and
2. demonstrate preservation of the prepared state.

48 Raw Causal Reach Versus Retained Causal Information

The raw integrated causal distance is

$$\chi_{\text{raw}} = \sum_j \chi_j. \tag{255}$$

The retained causal information must also include transition and retention efficiencies. The word *memory* is used only for a specified coarse-grained variable that demonstrably persists; it does not imply exact storage of a microscopic state.

Define the cumulative transfer factor from interval j to the third-leg hot state as

$$\mathcal{E}_{j\to 3}(k) = \prod_{m\in\mathcal{P}_{j\to 3}} \mathcal{E}_m(k), \tag{256}$$

where $\mathcal{P}_{j\to 3}$ is the ordered set of transitions, propagation intervals, and any explicitly modeled retention stages between interval j and the third leg.

The retained causal contribution is

$$\chi_j^{\text{ret}}(k) = \mathcal{E}_{j\to 3}(k)\chi_j(k). \tag{257}$$

The effective retained causal history is

$$\chi_{\text{ret}}^{(123)}(k) = \sum_j \chi_j^{\text{ret}}(k). \tag{258}$$

A strong horizon candidate resolution requires both

$$\chi_{\text{raw}} \geq \chi_{\text{required}} \tag{259}$$

and

$$\chi_{\text{ret}}^{(123)} \geq \chi_{\text{required}}^{\text{eff}}. \tag{260}$$

The second inequality prevents the model from counting correlations that are later erased.

49 Causal-Diagram Requirements

A complete analysis should include a conformal causal diagram.

The diagram must distinguish:

- the first resolved post-singularity spacetime boundary;
- the first outward leg;
- the outer transition;
- the second inward *Electro* leg;
- the central transition;
- the third-leg hot state;
- the last-scattering hypersurface;
- the present observer;
- ordinary photon null paths;
- the effective early characteristic cones; and
- the inherited scaffold trajectory.

The diagram must not depict the inward *Electro* path as a contraction of the entire spacetime unless the background solution actually contracts.

The global conformal geometry is determined by

$$a(t), \qquad H(t), \qquad k, \tag{261}$$

while the *Electro* history is represented by

$$\chi_E(t). \tag{262}$$

These should appear as separate objects.

A complete causal-diagram presentation should contain at least two panels:

1. a global conformal diagram showing causal cones and transition hypersurfaces; and
2. a sector-trajectory diagram showing the outward–inward–outward *Electro* path inside the expanding background.

Combining both into one unlabeled diagram could falsely imply that the metric itself follows the *Electro* reversal.

50 Horizon-Ready Background Parameterization

The later horizon calculation requires a finite parameterization of the background.

A minimum parameter set is

$$\boldsymbol{\theta}_{\mathrm{hor}} = \left\{t_{\mathrm{res}}, t_{\mathrm{O}}^{\pm}, t_{\mathrm{C}}^{\pm}, a_j(t), H_j(t), c_{\mathrm{eff}}^{(j)}(t,k), \mathcal{E}_j(k)\right\}. \tag{263}$$

The following quantities must be computed from this set:

$$\chi_1(k), \qquad \chi_{\mathrm{O}}(k), \qquad \chi_2(k), \qquad \chi_{\mathrm{C}}(k), \qquad \chi_3(t_*, k), \tag{264}$$

together with

$$\chi_{\mathrm{raw}}^{(123)}(k), \qquad \chi_{\mathrm{ret}}^{(123)}(k), \qquad \mathfrak{H}_{\mathrm{raw}}(k), \qquad \mathfrak{H}_{\mathrm{ret}}(k), \qquad f_j(k). \tag{265}$$

The observational geometry supplies

$$\Delta\chi_{\mathrm{max}}, \tag{266}$$

or its curved-space generalization.

The primary horizon statistic may then be defined as

$$\mathcal{Q}_{\mathrm{hor}}(k) = \frac{2\chi_{\mathrm{ret}}^{(123)}(t_*, k)}{\Delta\chi_{\mathrm{max}}}. \tag{267}$$

The causal criterion is

$$\mathcal{Q}_{\mathrm{hor}} \geq 1. \tag{268}$$

A value

$$\mathcal{Q}_{\mathrm{hor}} < 1 \tag{269}$$

indicates insufficient retained causal preparation under the adopted background and transfer model.

51 Minimal Toy Evaluation Protocol

Before constructing a complete field theory, the causal proposal may be tested through a controlled toy-model sequence.

51.1 Step 1: choose one common normalization

Set

$$a(t_0) = 1. \tag{270}$$

Propagate that normalization backward continuously through all intervals.

51.2 Step 2: assign explicit background functions

Choose analytic functions for

$$a_1(t), \qquad a_{\mathrm{O}}(t), \qquad a_2(t), \qquad a_{\mathrm{C}}(t), \qquad a_3(t). \tag{271}$$

The functions must satisfy

$$a(t) > 0 \tag{272}$$

and, for the foundational expanding-background version,

$$H(t) > 0. \tag{273}$$

51.3 Step 3: assign propagation laws

Specify

$$c_{\mathrm{eff}}^{(1)}, \qquad c_{\mathrm{eff}}^{(\mathrm{O})}, \qquad c_{\mathrm{eff}}^{(2)}, \qquad c_{\mathrm{eff}}^{(\mathrm{C})}, \qquad c. \tag{274}$$

Each value or function must be physically interpreted.

51.4 Step 4: integrate the intervals

Calculate every term in

$$\chi_{\rm raw}^{(123)} = \chi_1 + \chi_{\rm O} + \chi_2 + \chi_{\rm C} + \chi_3. \tag{275}$$

51.5 Step 5: apply retention factors

Assign transparent trial values to

$$\mathcal{E}_{1\to 3}(k), \qquad \mathcal{E}_{2\to 3}(k), \qquad \mathcal{E}_{\rm C\to 3}(k). \tag{276}$$

The first calculation should include both:

$$\mathcal{E} = 1 \tag{277}$$

as the optimistic upper bound, and reduced values to test sensitivity.

51.6 Step 6: compare with the required domain

Compute

$$\mathcal{Q}_{\rm hor} = \frac{2\chi_{\rm ret}^{(123)}}{\Delta\chi_{\rm max}}. \tag{278}$$

The calculation should report which interval contributes most strongly and how sensitive the result is to each assumption.

52 Claim-Specific Failure Conditions for Full-History Conformal Geometry

The conformal-history proposal fails or requires revision if any of the following conditions is unavoidable.

1. No classical geometric interval can be defined before the third leg.
2. The relevant pre-third-leg degree of freedom has no stable characteristic propagation law.
3. The first and second legs contribute negligible comoving causal reach.
4. The transition intervals erase the correlations established earlier.

5. The full causal sum depends on inconsistent scale-factor normalizations.
6. The proposed horizon length is actually only the radial *Electro* trajectory length.
7. The effective propagation law becomes uncontrolled or acausal.
8. The calculated common-cause domains fail to overlap for the required last-scattering separations.
9. The result succeeds only by assigning an arbitrary superluminal speed without a consistent field theory.
10. The raw causal interval is large, but the retained correlation after the central transition is too small.

A longer elapsed time is not sufficient to avoid these failure conditions.

53 Formal Summary of Full-History Conformal Geometry

The classical conformal interval is defined by

$$\boxed{\mathrm{d}\eta = \frac{\mathrm{d}t}{a(t)}.} \tag{279}$$

The causal contribution of interval j is

$$\boxed{\chi_j(k) = \int_{\mathcal{I}_j} \frac{c_{\mathrm{eff}}^{(j)}(t,k)}{a_j(t)} \mathrm{d}t.} \tag{280}$$

The complete raw history is

$$\boxed{\chi_{\mathrm{raw}}^{(123)} = \chi_1 + \chi_{\mathrm{O}} + \chi_2 + \chi_{\mathrm{C}} + \chi_3.} \tag{281}$$

The history-extension ratio is

$$\boxed{\mathfrak{H}_{\mathrm{raw}} = 1 + \frac{\chi_1 + \chi_{\mathrm{O}} + \chi_2 + \chi_{\mathrm{C}}}{\chi_3}.} \tag{282}$$

The retained causal history is

$$\boxed{\chi_{\mathrm{ret}}^{(123)}(k) = \sum_j \mathcal{E}_{j\to 3}(k)\chi_j(k).} \tag{283}$$

The two-point common-cause condition is

$$\boxed{2\chi_{\rm ret}^{(123)} \geq \Delta\chi_{AB}.} \tag{284}$$

The whole-domain horizon statistic is

$$\boxed{\mathcal{Q}_{\rm hor} = \frac{2\chi_{\rm ret}^{(123)}}{\Delta\chi_{\max}}.} \tag{285}$$

Causal sufficiency requires

$$\boxed{\mathcal{Q}_{\rm hor} \geq 1.} \tag{286}$$

The decisive distinction is

$$\boxed{\textit{Electro}\text{ trajectory length} \neq \text{causal horizon}} \tag{287}$$

unless the field equations explicitly connect them.

The incomplete-history hypothesis therefore advances from a narrative claim to a calculable question:

$$\boxed{\text{Does the retained full-history causal domain encompass the observed last-scattering domain?}} \tag{288}$$

54 The Dedicated Horizon Test

The preceding conformal construction defines the raw propagation reach. This section turns that construction into a direct horizon test by adding last-scattering geometry and transition-retention requirements.

The central question is not merely whether the first and second legs add positive conformal duration.

It is whether the retained causal domain established before last scattering is large enough to account for correlations observed across the cosmic microwave background sky.

The test requires four quantities:

1. the comoving radius of the last-scattering surface;
2. the comoving separation of the regions being compared;
3. the backward causal reach of each region to an earlier preparation hypersurface; and

4. the fraction of the earlier correlation that survives both reversals and enters the third-leg plasma.

The horizon candidate resolution remains viable only if the relevant backward causal domains overlap before the observed perturbations are fixed. Paper II defines this criterion; numerical satisfaction awaits an explicit background and transfer model.

Define the scale- and angle-dependent horizon statistic

$$\mathcal{Q}_{\mathrm{hor}}\left(k,\theta\right)=\frac{2\chi_{\mathrm{ret}}^{(123)}(k)}{d_k\left(\chi_*,\theta\right)}, \tag{289}$$

where:

- χ_* is the present comoving radial distance to last scattering;
- θ is the observed angular separation;
- $d_k(\chi_*,\theta)$ is the spatial geodesic separation of the two last-scattering points; and
- $\chi_{\mathrm{ret}}^{(123)}$ is the retained backward causal reach from last scattering into the full three-leg history.

Causal overlap requires

$$\boxed{\mathcal{Q}_{\mathrm{hor}}\left(k,\theta\right)\geq 1.} \tag{290}$$

The all-sky horizon problem concerns the largest relevant angular separations.

55 Geometry of the Last-Scattering Surface

55.1 Radial distance to last scattering

The comoving radial distance from the present observer to the last-scattering surface is

$$\chi_*=\int_{t_*}^{t_0}\frac{c\,\mathrm{d}t}{a_3(t)}. \tag{291}$$

Equivalently, in redshift space,

$$\chi_*=c\int_0^{z_*}\frac{\mathrm{d}z}{H_3(z)}. \tag{292}$$

The observer is placed at the coordinate origin for convenience.

This does not imply that the observer occupies a preferred physical center of the universe.

55.2 Two points on the last-scattering surface

Consider two points A and B on the last-scattering surface observed at angular separation

$$0 \leq \theta \leq \pi. \tag{293}$$

Their comoving spatial separation is

$$\Delta\chi_{AB} = d_k\left(\chi_*, \theta\right), \tag{294}$$

where d_k is the geodesic distance on the relevant constant-curvature spatial hypersurface.

In the spatially flat limit,

$$d_0\left(\chi_*, \theta\right) = 2\chi_* \sin\left(\frac{\theta}{2}\right). \tag{295}$$

For small angular separation,

$$d_0\left(\chi_*, \theta\right) \approx \chi_*\theta. \tag{296}$$

For antipodal points,

$$\theta = \pi, \tag{297}$$

and therefore, in the flat limit,

$$d_0\left(\chi_*, \pi\right) = 2\chi_*. \tag{298}$$

The antipodal comparison provides the strongest simple all-sky causal test.

56 Backward Causal Domains

56.1 Preparation hypersurface

Let

$$\Sigma_{\text{prep}} \tag{299}$$

denote a hypersurface on which the relevant common state is established.

This hypersurface may lie:

- during the first outward leg;
- near the outer transition;
- during the second inward leg;
- near the central transition; or
- during the earliest third-leg plasma stage.

The preparation time is denoted

$$t_{\mathrm{prep}} < t_*. \tag{300}$$

56.2 Backward reach

The backward causal reach from last scattering to the preparation hypersurface is

$$\chi_{\mathrm{back}}\left(t_{\mathrm{prep}}, t_*; k\right) = \int_{t_{\mathrm{prep}}}^{t_*} \frac{c_{\mathrm{eff}}(t,k)}{a(t)} \mathrm{d}t, \tag{301}$$

with the integral evaluated piecewise across all intervening legs and transitions.

The backward causal domain of point A on Σ_{prep} is

$$\mathcal{B}_A = \left\{\boldsymbol{x} \in \Sigma_{\mathrm{prep}} : d_k\left(\boldsymbol{x}, \boldsymbol{x}_A\right) \leq \chi_{\mathrm{back}}\right\}. \tag{302}$$

Similarly,

$$\mathcal{B}_B = \left\{\boldsymbol{x} \in \Sigma_{\mathrm{prep}} : d_k\left(\boldsymbol{x}, \boldsymbol{x}_B\right) \leq \chi_{\mathrm{back}}\right\}. \tag{303}$$

A common causal preparation region exists when

$$\mathcal{B}_A \cap \mathcal{B}_B \neq \varnothing. \tag{304}$$

For equal causal radii in a symmetric background, this reduces to

$$2\chi_{\mathrm{back}} \geq d_k\left(\chi_*, \theta\right). \tag{305}$$

57 The Standard Decelerating Horizon Deficit

57.1 Finite conformal beginning

Consider a decelerating power-law background,

$$a(t) = At^p, \qquad 0 < p < 1. \tag{306}$$

The conformal time from a finite classical beginning at $t = 0$ is

$$\eta(t) = \int_0^t \frac{\mathrm{d}t'}{At'^p} = \frac{t^{1-p}}{A(1-p)}. \tag{307}$$

The particle horizon at time t_* is therefore finite:

$$\chi_\mathrm{p}(t_*) = c\eta(t_*). \tag{308}$$

For a third-leg-only hot history, the causal reach available before last scattering is

$$\chi_3(t_*) = \int_{t_\mathrm{C}^+}^{t_*} \frac{c\,\mathrm{d}t}{a_3(t)}. \tag{309}$$

The radial distance from the observer to last scattering is instead

$$\chi_* = \int_{t_*}^{t_0} \frac{c\,\mathrm{d}t}{a_3(t)}. \tag{310}$$

In a conventional noninflationary reconstruction,

$$\chi_3(t_*) < \chi_*. \tag{311}$$

Thus, the backward causal domains of antipodal last-scattering points do not overlap:

$$2\chi_3(t_*) < 2\chi_*. \tag{312}$$

57.2 Deficit factor

Define the standard third-leg horizon deficit factor as

$$\mathcal{D}_\mathrm{hor}^{(3)} = \frac{d_k\left(\chi_*, \theta\right)}{2\chi_3(t_*)}. \tag{313}$$

For the antipodal flat-space comparison,

$$\mathcal{D}_{\rm hor}^{(3)}(\pi) = \frac{\chi_*}{\chi_3(t_*)}. \tag{314}$$

The third-leg-only history is causally insufficient when

$$\mathcal{D}_{\rm hor}^{(3)} > 1. \tag{315}$$

The Triple Bounce prehistory must supply the missing causal reach rather than merely add a nonzero correction.

58 Required Triple Bounce Causal Extension

58.1 Minimum retained prehistory

For angular separation θ, the minimum additional retained causal reach required beyond the third leg is

$$\chi_{\rm pre,req}(\theta) = \max\left[0, \frac{1}{2}d_k(\chi_*, \theta) - \chi_3(t_*)\right]. \tag{316}$$

For antipodal points in the flat limit,

$$\chi_{\rm pre,req}(\pi) = \max\left[0, \chi_* - \chi_3(t_*)\right]. \tag{317}$$

The retained first- and second-leg contribution must satisfy

$$\chi_{\rm pre,ret} = \chi_1^{\rm ret} + \chi_{\rm O}^{\rm ret} + \chi_2^{\rm ret} + \chi_{\rm C}^{\rm ret} \geq \chi_{\rm pre,req}. \tag{318}$$

58.2 Efficiency-corrected requirement

Using transfer efficiencies,

$$\chi_{\rm pre,ret}(k) = \sum_{j\in\{1,{\rm O},2,{\rm C}\}} \mathcal{E}_{j\to3}(k)\chi_j(k). \tag{319}$$

The complete causal criterion is

$$\boxed{\chi_3(t_*, k) + \sum_{j\in\{1,{\rm O},2,{\rm C}\}} \mathcal{E}_{j\to3}(k)\chi_j(k) \geq \frac{1}{2}d_k(\chi_*, \theta).} \tag{320}$$

For all-sky antipodal correlation in the flat limit,

$$\boxed{\chi_{\rm ret}^{(123)}(t_*, k) \geq \chi_*.} \tag{321}$$

58.3 Required history-extension ratio

The required extension ratio is

$$\mathfrak{H}_{\rm required}(\theta) = \frac{d_k(\chi_*, \theta)}{2\chi_3(t_*)}. \tag{322}$$

A successful model must satisfy

$$\mathfrak{H}_{\rm ret}(k) \geq \mathfrak{H}_{\rm required}(\theta), \tag{323}$$

where

$$\mathfrak{H}_{\rm ret}(k) = \frac{\chi_{\rm ret}^{(123)}(k)}{\chi_3(t_*, k)}. \tag{324}$$

59 Angular Causal Scale

59.1 Flat-background expression

In the flat approximation, the maximum angular separation whose backward causal domains overlap satisfies

$$2\chi_{\rm ret}^{(123)} = 2\chi_* \sin\left(\frac{\theta_{\rm causal}}{2}\right). \tag{325}$$

Therefore,

$$\theta_{\rm causal} = 2\arcsin\left[\min\left(1, \frac{\chi_{\rm ret}^{(123)}}{\chi_*}\right)\right]. \tag{326}$$

When

$$\chi_{\rm ret}^{(123)} \ll \chi_*, \tag{327}$$

the small-angle approximation gives

$$\theta_{\rm causal} \approx 2\frac{\chi_{\rm ret}^{(123)}}{\chi_*}. \tag{328}$$

Whole-sky antipodal overlap requires

$$\theta_{\text{causal}} = \pi, \tag{329}$$

which occurs when

$$\chi_{\text{ret}}^{(123)} \geq \chi_*. \tag{330}$$

59.2 Curvature correction

When spatial curvature is nonzero, the relationship between θ_{causal} and the radial causal reach must be calculated using the spatial geodesic function

$$d_k\left(\chi_*, \theta\right). \tag{331}$$

The horizon test and the flatness test are therefore related.

A curvature model that changes the last-scattering geometry also changes the required causal overlap distance.

60 Direct Transport and Common-State Preparation

The Triple Bounce framework permits two broad causal mechanisms.

60.1 Direct transport

A physical carrier may propagate information between different regions during the first or second leg.

Let X denote that carrier.

Its correlation field may satisfy an effective hyperbolic equation,

$$\mathcal{D}_X \delta X = \mathcal{J}_X, \tag{332}$$

with characteristic speed

$$c_X(t, k). \tag{333}$$

Direct transport contributes through

$$\chi_X = \int \frac{c_X(t,k)}{a(t)} \mathrm{d}t. \tag{334}$$

60.2 Common-state preparation

Alternatively, a connected region may be prepared in a common state before its descendants become widely separated.

Let

$$\Xi_{\mathrm{prep}}(\boldsymbol{x}) \tag{335}$$

represent the prepared field.

Its two-point correlation is

$$C_\Xi(\boldsymbol{x},\boldsymbol{y}) = \langle \Xi_{\mathrm{prep}}(\boldsymbol{x}) \Xi_{\mathrm{prep}}(\boldsymbol{y}) \rangle . \tag{336}$$

The preparation correlation length is

$$\xi_{\mathrm{prep}}. \tag{337}$$

A successful common-state mechanism requires

$$\xi_{\mathrm{prep}} \geq d_k(\chi_*, \theta) \tag{338}$$

after conversion to the common comoving normalization and application of the transition maps.

60.3 No causality exemption

A common initial state does not remove the need to explain how that state was generated.

The model must show either:

- that the preparation domain was causally connected;
- that a fundamental boundary condition imposed the correlation;
- that a pre-singularity theory supplies an alternative causal structure; or
- that a later dynamical attractor erased earlier differences.

Each possibility has a different physical and evidential burden.

61 Causal Contact Versus Equilibration

Causal contact permits interaction.

It does not guarantee thermal or dynamical equilibration.

Let

$$\Gamma_X(t) \tag{339}$$

denote the interaction rate of the relevant preparation process.

The number of effective interactions during an interval is

$$\mathcal{N}_{\rm int} = \int \Gamma_X(t)\mathrm{d}t. \tag{340}$$

Significant equilibration generally requires

$$\mathcal{N}_{\rm int} \gtrsim 1, \tag{341}$$

while strong equilibration may require

$$\mathcal{N}_{\rm int} \gg 1. \tag{342}$$

An expansion-timescale comparison is

$$\frac{\Gamma_X}{H}. \tag{343}$$

Local equilibration is efficient when

$$\Gamma_X \gg H. \tag{344}$$

The horizon explanation must therefore answer two separate questions:

1. Could the relevant regions influence one another?
2. Did the interaction dynamics actually drive them toward a sufficiently uniform prepared state?

A large causal domain without an effective homogenizing process does not fully explain the observed uniformity.

62 Survival Through the Outer Reversal

Let the first-leg prepared correlation spectrum be

$$P_{\Xi}^{(1)}(k). \tag{345}$$

The outer transition maps it into

$$P_{\Xi}^{(\mathrm{O}+)}(k) = |T_{\mathrm{O}}(k)|^2 \, P_{\Xi}^{(1)}(k) + N_{\mathrm{O}}(k), \tag{346}$$

where:

- $T_{\mathrm{O}}(k)$ is the outer-transition transfer function; and
- $N_{\mathrm{O}}(k)$ is transition-generated stochastic power.

Large-scale survival requires

$$|T_{\mathrm{O}}(k)| \approx 1 \tag{347}$$

over the modes relevant to the horizon problem.

The outer transition must not produce excessive decorrelation,

$$N_{\mathrm{O}}(k) \ll P_{\Xi}^{(1)}(k) \tag{348}$$

on those scales.

If the first-leg correlation is erased at the outer reversal, the first leg cannot perform the required horizon work.

63 Survival Through the Second Leg and Central Reversal

63.1 Second-leg transfer

The second-leg state may be written as

$$P_{\Xi}^{(2)}(k,t) = \left|T_2(k;t,t_{\mathrm{O}}^{+})\right|^2 P_{\Xi}^{(\mathrm{O}+)}(k) + N_2(k,t). \tag{349}$$

Large-scale preservation requires

$$\left|T_2(k;t_{\mathrm{C}}^{-},t_{\mathrm{O}}^{+})\right| \not\ll 1. \tag{350}$$

The second leg may strengthen, weaken, or reorganize the correlation.

It need not preserve every microscopic detail.

63.2 Central conversion

Represent the candidate conversion of the inherited correlation into third-leg perturbation variables by

$$\begin{pmatrix} \mathcal{R}_{\boldsymbol{k}} \\ S_{\boldsymbol{k}} \\ h_{\boldsymbol{k}} \\ \delta_i(\boldsymbol{k}) \end{pmatrix}_{t_{\mathrm{C}}^{+}} = \mathsf{M}_{\mathrm{C}}(k) \begin{pmatrix} \Xi_{\boldsymbol{k}} \\ \mathcal{W}_{12,\boldsymbol{k}} \\ \mathcal{P}_{12,\boldsymbol{k}} \\ \mathcal{M}_{12,\boldsymbol{k}} \end{pmatrix}_{t_{\mathrm{C}}^{-}} + \boldsymbol{N}_{\mathrm{C}}(k). \tag{351}$$

A successful horizon candidate requires a substantial projection of the retained common-state component into the visible adiabatic curvature mode. In a linearized description this requires, over the relevant scales,

$$\left| T^{\mathrm{C}}_{\mathcal{R}\Xi}(k) \right|^2 \mathcal{P}_{\Xi_{\mathrm{common}}}(k) \gg \mathcal{P}^{(\Xi)}_{S,\mathrm{vis}}(k). \tag{352}$$

The transition need not preserve every microscopic detail. If the retained common-state power enters mainly an isocurvature, vector, or unstable visible mode, the model may preserve historical correlation while still failing observationally.

64 Scale Dependence of the Horizon Test

The effective propagation speeds and transition efficiencies may depend on wavenumber.

The horizon statistic is therefore

$$\mathcal{Q}_{\mathrm{hor}} = \mathcal{Q}_{\mathrm{hor}}\left(k, \theta\right). \tag{353}$$

A scale-dependent causal reach may produce:

- full correlation on large scales but not smaller scales;
- selective damping across transition bands;
- features in the primordial power spectrum;
- phase shifts;
- scale-dependent statistical anisotropy; or
- different scalar and tensor preparation lengths.

For an angular multipole ℓ, a useful approximate correspondence is

$$k_\ell \approx \frac{\ell + \frac{1}{2}}{D_M(z_*)}, \tag{354}$$

where $D_M(z_*)$ is the transverse comoving distance to last scattering.

The horizon condition should therefore be evaluated over the range

$$k_{\min} \leq k \leq k_{\max} \tag{355}$$

corresponding to the angular and physical scales whose common preparation is being claimed.

A model that succeeds only at one isolated wavenumber does not explain the broad observed uniformity.

65 Inherited Scaffold as a Candidate Correlation Carrier

The dark-matter-associated scaffold may act as one carrier of coarse-grained correlation established during the first leg. This is a candidate retention mechanism, not a claim that the scaffold stores a complete microscopic record.

Represent the relevant scaffold-retention field by

$$\mathcal{W}_{12}(\boldsymbol{x}). \tag{356}$$

Its correlation function is

$$C_{\mathcal{W}}(r) = \langle \mathcal{W}_{12}(\boldsymbol{x}) \mathcal{W}_{12}(\boldsymbol{x} + \boldsymbol{r}) \rangle . \tag{357}$$

Define the scaffold correlation length by

$$\xi_{\mathcal{W}} = \text{characteristic scale over which } C_{\mathcal{W}}(r) \text{ remains significant.} \tag{358}$$

For the scaffold alone to support a flat-space antipodal common-state explanation, its third-leg correlation scale or an equivalent transfer range must satisfy

$$\xi_{\mathcal{W}}^{(3)} \gtrsim d_0(\chi_*, \pi) = 2\chi_*. \tag{359}$$

This does not require the later visible matter distribution to be uniform over that scale.

It requires the relevant preparation variable to possess a common large-scale ancestry and to couple through the central transfer into an observable adiabatic mode. A long correlation length by itself is not a horizon candidate resolution.

The scaffold must also avoid imprinting excessive visible anisotropy.

Thus, the model must satisfy both

$$\xi_{\mathcal{W}}^{(3)} \text{ sufficiently large,} \tag{360}$$

and

$$\epsilon_{\mathcal{W},\text{anis}} \ll 1 \tag{361}$$

on the largest observed scales.

66 Observer Independence and Homogeneity

The horizon construction must not rely on the present observer occupying a privileged position.

The coordinate origin used to define χ_* is observational.

It is not a physical center of the Triple Bounce geometry.

For statistically homogeneous initial conditions,

$$C(\boldsymbol{x}, \boldsymbol{y}) = C(|\boldsymbol{x} - \boldsymbol{y}|) \tag{362}$$

in the flat limit.

For statistical isotropy,

$$C(\boldsymbol{x}, \boldsymbol{y}) \tag{363}$$

depends only on geodesic separation rather than absolute direction.

A viable Triple Bounce preparation mechanism must generate a causal domain and correlation structure that can be centered on any typical comoving observer.

If the model explains the observed sky only when

$$\boldsymbol{x}_{\text{obs}} = \boldsymbol{x}_{\text{special}}, \tag{364}$$

it violates the intended homogeneous cosmological interpretation.

67 Horizon Test Cases

The first numerical analysis should evaluate four nested cases.

67.1 Case A: third leg only

Set

$$\chi_1 = \chi_{\mathrm{O}} = \chi_2 = \chi_{\mathrm{C}} = 0. \tag{365}$$

Then

$$\mathcal{Q}_{\mathrm{hor}}^{A} = \frac{2\chi_3(t_*)}{d_k(\chi_*, \theta)}. \tag{366}$$

This reproduces the baseline noninflationary horizon deficit.

67.2 Case B: raw full history

Set all retention efficiencies to unity:

$$\mathcal{E}_{j\to 3} = 1. \tag{367}$$

Then

$$\mathcal{Q}_{\mathrm{hor}}^{B} = \frac{2\,(\chi_1 + \chi_{\mathrm{O}} + \chi_2 + \chi_{\mathrm{C}} + \chi_3)}{d_k(\chi_*, \theta)}. \tag{368}$$

This is the optimistic geometric upper bound.

67.3 Case C: efficiency-corrected history

Use scale-dependent transfer efficiencies:

$$\mathcal{Q}_{\mathrm{hor}}^{C} = \frac{2\sum_j \mathcal{E}_{j\to 3}(k)\chi_j(k)}{d_k(\chi_*, \theta)}. \tag{369}$$

This is the physically relevant retained-history test.

67.4 Case D: preparation and equilibration

Require both causal overlap and effective preparation:

$$\mathcal{Q}_{\rm hor}^{C} \geq 1, \tag{370}$$

and

$$\mathcal{N}_{\rm int} \gtrsim 1. \tag{371}$$

The model passes the complete horizon test only if both conditions are met while the resulting perturbations remain observationally acceptable.

68 Explicit Horizon-Claim Rejection Criteria

The Triple Bounce horizon explanation fails if any of the following is unavoidable.

1. The full classical background cannot be defined early enough to calculate a pre-third-leg causal interval.
2. The relevant correlation carrier has no stable characteristic propagation law.
3. The raw full-history result satisfies

$$\mathcal{Q}_{\rm hor}^{B} < 1. \tag{372}$$

4. The raw history is sufficient, but transition losses give

$$\mathcal{Q}_{\rm hor}^{C} < 1. \tag{373}$$

5. The backward causal domains overlap only for small angular separations and not across the large-angle cosmic microwave background domain being explained.
6. The causal reach is obtained only by assigning an arbitrary superluminal speed without a consistent effective theory.
7. The first- and second-leg correlation is erased at either reversal.
8. The derived common-state transfer places most visible power in unacceptable isocurvature, vector, tensor, or unstable modes.
9. The preparation domain is causally connected but lacks interactions capable of producing the required uniformity.

10. The inherited scaffold provides long-range correlation only by introducing excessive statistical anisotropy.
11. The model requires the observer to occupy a privileged location.
12. The horizon candidate resolution depends on incompatible scale-factor normalizations across the three legs.
13. The quantity used as causal reach is merely the geometric *Electro* trajectory rather than a derived characteristic domain.
14. The calculated horizon success occurs only at an isolated wavenumber and not across the observationally relevant scale range.
15. The model reproduces large-scale uniformity only by inserting the uniform third-leg state directly as an unexplained boundary condition.

69 Formal Summary of the Horizon Test

The radial distance to last scattering is

$$\boxed{\chi_* = \int_{t_*}^{t_0} \frac{c\,\mathrm{d}t}{a_3(t)}.} \tag{374}$$

The separation of two flat-space last-scattering points is

$$\boxed{\Delta\chi_{AB} = 2\chi_* \sin\left(\frac{\theta}{2}\right).} \tag{375}$$

The full retained backward reach is

$$\boxed{\chi_{\mathrm{ret}}^{(123)}(k) = \chi_3(t_*, k) + \sum_{j\in\{1,\mathrm{O},2,\mathrm{C}\}} \mathcal{E}_{j\to 3}(k)\chi_j(k).} \tag{376}$$

The angular horizon statistic is

$$\boxed{\mathcal{Q}_{\mathrm{hor}}(k,\theta) = \frac{2\chi_{\mathrm{ret}}^{(123)}(k)}{d_k(\chi_*,\theta)}.} \tag{377}$$

The causal-overlap condition is

$$\boxed{\mathcal{Q}_{\mathrm{hor}}(k,\theta) \geq 1.} \tag{378}$$

For antipodal points in the flat limit,

$$\boxed{\chi_{\text{ret}}^{(123)}(k) \geq \chi_*.} \tag{379}$$

The minimum required retained prehistory is

$$\boxed{\chi_{\text{pre,req}}(\theta) = \max\left[0, \frac{1}{2}d_k\left(\chi_*, \theta\right) - \chi_3(t_*)\right].} \tag{380}$$

Causal contact must be accompanied by sufficient preparation dynamics:

$$\boxed{\mathcal{Q}_{\text{hor}} \geq 1 \qquad \text{and} \qquad \mathcal{N}_{\text{int}} \gtrsim 1.} \tag{381}$$

The horizon claim is therefore reduced to a direct calculation:

$$\boxed{\begin{array}{l}\text{Does the efficiency-corrected full-history causal domain overlap across the required}\\ \text{last-scattering separations and survive as an acceptable third-leg perturbation state?}\end{array}} \tag{382}$$

70 Resolved Boundary Curvature and First-Leg Evolution

The flatness question begins before the observable third leg.

If the third-leg curvature parameter is inserted directly as an arbitrarily small initial value,

$$\left|\Omega_k^{(3)}\right| \ll 1, \tag{383}$$

then the Triple Bounce framework has not explained near-flatness.

It has only relocated the standard initial-condition problem to the central transition.

The proposed alternative is that small observable curvature descends from a sequence of distinct requirements:

1. balanced pre-singularity ordering that constrains, but is not identified with, classical spatial curvature;
2. a singular-opening boundary prescription producing the first resolved post-singularity curvature data;
3. controlled first-leg curvature evolution;
4. regular transfer through the nonsingular outer reversal;
5. controlled second-leg curvature evolution;

6. regular transfer through the nonsingular central reversal; and
7. third-leg evolution consistent with the present observational bound.

This part of Paper II develops the resolved boundary and first-leg components before constructing the two reversal maps and the complete present-day curvature budget.

Its endpoint is the state immediately before the outer reversal,

$$t = t_{\mathrm{O}}^{-}. \tag{384}$$

The resolved-boundary and first-leg curvature sequence is written as

$$\mathcal{S}_{\mathrm{pre}} \xrightarrow{\mathcal{B}_{\mathrm{res}}} \Omega_k(t_{\mathrm{res}}^{+}) \xrightarrow{\mathcal{U}_{k,1}} \Omega_k(t_{\mathrm{O}}^{-}), \tag{385}$$

where:

- $\mathcal{B}_{\mathrm{res}}$ is a provisional singular-opening boundary prescription, not a classical evolution operator through the singularity; and
- $\mathcal{U}_{k,1}$ is the first-leg curvature-evolution operator.

The first-leg flatness problem is therefore divided into two questions:

$$\boxed{\begin{array}{l}\text{How small is the resolved boundary curvature?}\\ \text{Does the first leg preserve, suppress, or amplify it?}\end{array}} \tag{386}$$

71 Classical Spatial Curvature

71.1 Curvature index and curvature scalar

In a homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker regime, the spatial-curvature index is

$$k \in \{-1, 0, +1\} \tag{387}$$

after conventional normalization.

The intrinsic scalar curvature of a constant-time spatial hypersurface is

$${}^{(3)}R = \frac{6k}{a^2}. \tag{388}$$

The curvature density parameter is

$$\Omega_k = -\frac{kc^2}{a^2 H^2}. \tag{389}$$

The sign convention implies:

$$k = -1 \quad \Longrightarrow \quad \Omega_k > 0, \tag{390}$$

$$k = 0 \quad \Longrightarrow \quad \Omega_k = 0, \tag{391}$$

$$k = +1 \quad \Longrightarrow \quad \Omega_k < 0. \tag{392}$$

Near-flatness refers to the small magnitude

$$|\Omega_k| \ll 1, \tag{393}$$

not necessarily to the exact condition

$$k = 0. \tag{394}$$

71.2 Geometric curvature versus dynamical relevance

The intrinsic curvature scalar scales as

$$^{(3)}R \propto a^{-2}. \tag{395}$$

Its dynamical importance relative to the Hubble expansion is measured by

$$|\Omega_k| = \frac{|k|c^2}{a^2 H^2}. \tag{396}$$

Thus, the absolute spatial-curvature scalar may decrease while the relative curvature fraction increases.

This distinction is essential.

A universe may become geometrically less curved in dimensional units while becoming less flat relative to its total expansion dynamics.

72 The Singular-Opening Boundary Prescription

Paper II does not assign an ordinary Friedmann–Lemaître–Robertson–Walker curvature parameter to the pre-singularity state, and it does not claim to evolve classical curvature continuously through the singularity. Instead, it introduces a provisional boundary prescription from pre-singularity ordering data to the earliest resolved post-singularity geometric data.

The pre-singularity state is therefore not assumed to possess a classical spatial-curvature index.

Therefore,

$$k_{\rm pre} \tag{397}$$

is not treated as a conventional Friedmann–Lemaître–Robertson–Walker variable.

Instead, the pre-singularity state contains ordering data,

$$\mathcal{S}_{\rm pre} = \{\Psi_0, \delta\Psi, \mathcal{A}_{ij}, \mathcal{C}_{\rm pre}, \mathcal{Q}_{\rm pre}, \ldots\}, \tag{398}$$

where:

- Ψ_0 is the dominant balanced mode;
- $\delta\Psi$ represents departures from that mode;
- $\mathcal{A}_{ij}$ represents directional asymmetry;
- $\mathcal{C}_{\rm pre}$ represents pre-singularity correlation structure; and
- $\mathcal{Q}_{\rm pre}$ represents additional conserved or approximately conserved ordering information.

The first resolved post-singularity hypersurface is provisionally related to the pre-singularity ordering data through

$$\mathcal{B}_{\rm res} : \mathcal{S}_{\rm pre} \longrightarrow \left\{a_{\rm res}, H_{\rm res}, {}^{(3)}R_{\rm g}, \sigma_{ij,{\rm res}}, \rho_{i,{\rm res}}, p_{i,{\rm res}}\right\}. \tag{399}$$

Here,

$$a_{\rm res} = a(t^+_{\rm res}), \qquad H_{\rm res} = H(t^+_{\rm res}). \tag{400}$$

The resolved boundary curvature parameter is

$$\Omega_{k,{\rm res}} = -\frac{k_{\rm res}c^2}{a^2_{\rm res}H^2_{\rm res}}. \tag{401}$$

A successful flatness account must derive or statistically constrain $\Omega_{k,\mathrm{res}}$ from an explicit boundary law or statistical boundary prescription tied to the pre-singularity ordering variables. Merely assigning a small value at t^+_{res} does not explain near-flatness.

Limitation 72.1. Classical spatial curvature is used only after a resolved post-singularity spatial metric is available. The balanced pre-singularity state may constrain the resolved boundary curvature, but it is not identified with an ordinary flat Friedmann–Lemaître–Robertson–Walker hypersurface. The boundary prescription $\mathcal{B}_{\mathrm{res}}$ is therefore an effective parametrization pending a derived boundary law; it is not evidence that curvature has been classically propagated through the singularity.

73 Balanced-Mode Asymmetry

73.1 Dominant balanced mode

Paper I proposed that the pre-singularity state is dominated by a balanced internal mode.

Write the state schematically as

$$\Psi_{\mathrm{pre}} = A_0\Psi_0 + \delta\Psi, \tag{402}$$

where

$$\|\delta\Psi\| \ll \|A_0\Psi_0\|. \tag{403}$$

The dimensionless asymmetry measure is

$$\epsilon_{\mathrm{asym}} = \frac{\|\delta\Psi\|}{\|A_0\Psi_0\|}. \tag{404}$$

The limit

$$\epsilon_{\mathrm{asym}} \to 0 \tag{405}$$

represents exact dominance of the balanced mode.

The framework does not require the physical universe to occupy this exact limit.

It requires only that the resolved boundary curvature and anisotropy remain small for a finite range

$$0 \leq \epsilon_{\mathrm{asym}} < \epsilon_{\mathrm{max}}. \tag{406}$$

73.2 Curvature response expansion

Near the balanced state, the resolved boundary curvature may be expanded as

$$\Omega_{k,\mathrm{res}} = K_0 + K_1\epsilon_{\mathrm{asym}} + K_2\epsilon_{\mathrm{asym}}^2 + \mathcal{O}\left(\epsilon_{\mathrm{asym}}^3\right). \tag{407}$$

A symmetry-protected balanced state may satisfy

$$K_0 = 0, \qquad K_1 = 0, \tag{408}$$

so that

$$\Omega_{k,\mathrm{res}} \approx K_2\epsilon_{\mathrm{asym}}^2. \tag{409}$$

This quadratic response would suppress curvature more strongly than a generic linear response. However, the vanishing of K_0 and K_1 must follow from a defined symmetry or boundary law. It cannot be imposed solely because it produces near-flatness.

73.3 Anisotropy response

The resolved boundary shear may be expanded similarly:

$$\frac{\sigma_{\mathrm{res}}}{H_{\mathrm{res}}} = S_0 + S_1\epsilon_{\mathrm{asym}} + S_2\epsilon_{\mathrm{asym}}^2 + \cdots. \tag{410}$$

A balanced isotropic boundary should satisfy

$$S_0 = 0. \tag{411}$$

The model must determine whether the leading correction is linear or quadratic.

Hypothesis 73.1 (Balanced-boundary suppression)**.** *A singular-opening boundary law constrained by a dominant balanced pre-singularity mode may suppress the leading resolved curvature and shear responses, yielding a near-flat, nearly isotropic first resolved post-singularity hypersurface without requiring exact geometric flatness.*

74 Statistical Distribution of Resolved Curvature

The singular-opening boundary prescription may generate a distribution of resolved classical initial conditions rather than one deterministic geometry.

Let

$$P_{\mathrm{pre}}\left(\mathcal{S}_{\mathrm{pre}}\right) \tag{412}$$

be the distribution over candidate pre-singularity states.

The induced distribution of resolved boundary curvature is

$$P\left(\Omega_{k,\mathrm{res}}\right)=\int \mathcal{D}\mathcal{S}_{\mathrm{pre}}\, P_{\mathrm{pre}}\left(\mathcal{S}_{\mathrm{pre}}\right)\delta_D\left[\Omega_{k,\mathrm{res}}-\mathcal{K}_{\mathrm{res}}\left(\mathcal{S}_{\mathrm{pre}}\right)\right]. \tag{413}$$

The mean resolved boundary curvature is

$$\left\langle\Omega_{k,\mathrm{res}}\right\rangle=\int \Omega_{k,\mathrm{res}} P\left(\Omega_{k,\mathrm{res}}\right)\mathrm{d}\Omega_{k,\mathrm{res}}. \tag{414}$$

The variance is

$$\sigma_k^2=\left\langle\Omega_{k,\mathrm{res}}^2\right\rangle-\left\langle\Omega_{k,\mathrm{res}}\right\rangle^2. \tag{415}$$

A balanced ensemble may satisfy

$$\left\langle\Omega_{k,\mathrm{res}}\right\rangle\approx 0 \tag{416}$$

while retaining a nonzero variance.

Near-flatness then requires

$$\sigma_k\ll 1. \tag{417}$$

A small ensemble mean alone is insufficient.

Large positive and negative curvatures could cancel statistically while individual realizations remain strongly curved.

Proposition 74.1 (Curvature concentration requirement)**.** *A statistical balanced-state explanation of near-flatness requires the probability distribution of resolved boundary curvature to be concentrated near zero, not merely symmetric about zero.*

75 First-Leg Curvature Evolution

75.1 Exact logarithmic evolution

During a classical Friedmann–Lemaître–Robertson–Walker interval with constant normalized curvature index k,

$$\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}t} = -2H - 2\frac{\dot{H}}{H}. \tag{418}$$

Using

$$\epsilon_H = -\frac{\dot{H}}{H^2}, \tag{419}$$

and

$$\mathrm{d}N = H\mathrm{d}t, \tag{420}$$

one obtains

$$\boxed{\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} = 2\left(\epsilon_H - 1\right).} \tag{421}$$

The first-leg transfer is therefore

$$\ln\left[\frac{|\Omega_k(t_{\mathrm{O}}^-)|}{|\Omega_k(t_{\mathrm{res}}^+)|}\right] = 2\int_{N_{\mathrm{res}}}^{N_{\mathrm{O}}^-}\left[\epsilon_H^{(1)}(N) - 1\right]\mathrm{d}N. \tag{422}$$

Define the first-leg curvature transfer factor

$$T_k^{(1)} = \frac{|\Omega_k(t_{\mathrm{O}}^-)|}{|\Omega_k(t_{\mathrm{res}}^+)|}. \tag{423}$$

Then

$$\boxed{T_k^{(1)} = \exp\left\{2\int_{N_{\mathrm{res}}}^{N_{\mathrm{O}}^-}\left[\epsilon_H^{(1)}(N) - 1\right]\mathrm{d}N\right\}.} \tag{424}$$

75.2 Suppression, preservation, and amplification

The first leg suppresses curvature when

$$T_k^{(1)} < 1. \tag{425}$$

It preserves the curvature fraction when

$$T_k^{(1)} \approx 1. \tag{426}$$

It amplifies the curvature fraction when

$$T_k^{(1)} > 1. \tag{427}$$

Using the first-leg average

$$\left\langle \epsilon_H^{(1)} \right\rangle_N = \frac{1}{\Delta N_1} \int_{N_{\rm res}}^{N_{\rm O}^-} \epsilon_H^{(1)}(N)\mathrm{d}N, \tag{428}$$

the transfer factor becomes

$$T_k^{(1)} = \exp\left[2\Delta N_1 \left(\left\langle \epsilon_H^{(1)} \right\rangle_N - 1\right)\right]. \tag{429}$$

Thus:

$$\left\langle \epsilon_H^{(1)} \right\rangle_N < 1 \qquad \Longrightarrow T_k^{(1)} < 1, \tag{430}$$

$$\left\langle \epsilon_H^{(1)} \right\rangle_N = 1 \qquad \Longrightarrow T_k^{(1)} = 1, \tag{431}$$

$$\left\langle \epsilon_H^{(1)} \right\rangle_N > 1 \qquad \Longrightarrow T_k^{(1)} > 1. \tag{432}$$

76 Effective Equation-of-State Representation

When the first-leg background can be represented by a dominant effective fluid with equation of state

$$w_1 = \frac{p_1}{\rho_1 c^2}, \tag{433}$$

the Hubble slow-roll parameter is approximately

$$\epsilon_H = \frac{3}{2}(1 + w_1) \tag{434}$$

when curvature and additional corrections are subdominant.

Then

$$\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} = 1 + 3w_1. \tag{435}$$

For nearly constant w_1,

$$|\Omega_k| \propto a^{1+3w_1}. \tag{436}$$

The curvature fraction therefore:

- decreases when

$$w_1 < -\frac{1}{3}; \tag{437}$$

- remains approximately constant when

$$w_1 = -\frac{1}{3}; \tag{438}$$

  and

- increases when

$$w_1 > -\frac{1}{3}. \tag{439}$$

This creates a direct constraint on a noninflationary first leg.

If

$$w_1 > -\frac{1}{3} \tag{440}$$

throughout a long expanding interval, then the first leg amplifies the relative curvature fraction.

Near-flatness must then originate from an even smaller boundary value or from a later suppression mechanism.

77 Power-Law First-Leg Toy Models

Let the first-leg scale factor be

$$a_1(t) = A_1 \left(\frac{t}{t_1}\right)^p, \qquad p > 0. \tag{441}$$

The Hubble rate is

$$H_1(t) = \frac{p}{t}. \tag{442}$$

The slow-roll parameter is

$$\epsilon_H = \frac{1}{p}. \tag{443}$$

The curvature fraction evolves as

$$|\Omega_k| \propto a^{2(1/p-1)}. \tag{444}$$

Equivalently,

$$|\Omega_k| \propto t^{2-2p}. \tag{445}$$

77.1 Radiation-like expansion

For

$$p = \frac{1}{2}, \tag{446}$$

one obtains

$$|\Omega_k| \propto a^2. \tag{447}$$

77.2 Matter-like expansion

For

$$p = \frac{2}{3}, \tag{448}$$

one obtains

$$|\Omega_k| \propto a. \tag{449}$$

77.3 Coasting expansion

For

$$p = 1, \tag{450}$$

one obtains

$$|\Omega_k| = \text{constant} \tag{451}$$

within the toy approximation.

77.4 Accelerated power-law expansion

For

$$p > 1, \tag{452}$$

the curvature fraction decreases:

$$|\Omega_k| \propto a^{-2(1-1/p)}. \tag{453}$$

This regime is inflationary in the metric sense because

$$\ddot{a} > 0. \tag{454}$$

Calling the active sector *Electro* does not change that mathematical classification.

78 First-Leg Expansion Is Not Automatically a Flatness Mechanism

The first outward leg contains two distinct outward properties:

$$\dot{a} > 0, \qquad \dot{R}_E > 0. \tag{455}$$

Neither condition alone suppresses the relative curvature fraction.

Curvature suppression requires

$$\left\langle \epsilon_H^{(1)} \right\rangle_N < 1, \tag{456}$$

or a modified gravitational or transition mechanism that changes the standard curvature evolution.

Therefore,

$$\boxed{\text{outward } \textit{Electro} \text{ propagation} \neq \text{curvature suppression.}} \tag{457}$$

The foundational Triple Bounce interpretation intentionally avoids identifying the first leg with inflation.

Consequently, near-flatness cannot be attributed to the first leg merely because the active sector moves outward.

The framework must instead choose among four logically distinct possibilities:

I. **Boundary-dominated near-flatness:**

The singular-opening boundary prescription produces an extremely small $\Omega_{k,\mathrm{res}}$, and the first leg does not amplify it beyond the allowed range.

II. **First-leg preservation:**

The effective first-leg background remains close to

$$\left\langle \epsilon_H^{(1)} \right\rangle_N \approx 1, \tag{458}$$

so the curvature fraction changes little.

III. **Limited metric acceleration:**

A portion of the first leg satisfies

$$\epsilon_H < 1 \tag{459}$$

and suppresses curvature, even though the entire *Electro* leg is not identified with standard inflation.

IV. **Nonstandard curvature transfer:**

Additional fields or modified geometric dynamics alter

$$\frac{\mathrm{d} \ln |\Omega_k|}{\mathrm{d}N} = 2\left(\epsilon_H - 1\right) + \mathcal{S}_k, \tag{460}$$

where $\mathcal{S}_k$ is a derived curvature-transfer source.

The fourth possibility requires a consistent extension of the gravitational equations.

79 Curvature Budget Across the First Leg

Define the maximum curvature allowed immediately before the outer transition as

$$\Omega_{k,\mathrm{O}}^{\max}. \tag{461}$$

The resolved boundary curvature must satisfy

$$|\Omega_{k,\mathrm{res}}| \leq \frac{\Omega_{k,\mathrm{O}}^{\max}}{T_k^{(1)}}. \tag{462}$$

If

$$T_k^{(1)} \gg 1, \tag{463}$$

then the required boundary curvature becomes correspondingly smaller.

Define the first-leg curvature budget in logarithmic form:

$$\mathcal{B}_k^{(1)} = \ln\left[\frac{\Omega_{k,\mathrm{O}}^{\max}}{|\Omega_{k,\mathrm{res}}|}\right]. \tag{464}$$

The first-leg evolution consumes

$$\Delta_k^{(1)} = \ln T_k^{(1)}. \tag{465}$$

The first-leg state remains within the curvature budget when

$$\Delta_k^{(1)} \leq \mathcal{B}_k^{(1)}. \tag{466}$$

This formulation makes the fine-tuning burden explicit.

A model with a large positive $\Delta_k^{(1)}$ must explain why the resolved boundary curvature was sufficiently tiny.

80 Shear and Anisotropy During the First Leg

80.1 Shear variable

A nearly homogeneous but weakly anisotropic geometry may be described by a shear tensor

$$\sigma_{ij}. \tag{467}$$

Define

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}. \tag{468}$$

A normalized shear parameter is

$$\Omega_\sigma = \frac{\sigma^2}{3H^2}. \tag{469}$$

A generalized background constraint may be written schematically as

$$1 = \Omega_\rho + \Omega_k + \Omega_\sigma, \tag{470}$$

with the precise form depending on the selected homogeneous geometry and normalization conventions.

80.2 Shear dilution in expansion

In a source-free classical regime,

$$\rho_\sigma \propto a^{-6}. \tag{471}$$

For an effective fluid,

$$\rho \propto a^{-3(1+w)}. \tag{472}$$

Their ratio evolves as

$$\frac{\rho_\sigma}{\rho} \propto a^{-3(1-w)}. \tag{473}$$

For

$$w < 1, \tag{474}$$

the shear contribution decays relative to the dominant fluid during expansion.

Thus, an expanding noninflationary first leg may suppress shear even while the relative curvature fraction grows.

This distinction is important:

$$\boxed{\text{anisotropy dilution} \neq \text{curvature suppression.}} \tag{475}$$

80.3 Anisotropic stress source

If the first-leg *Electro* or scaffold sectors carry anisotropic stress, the shear evolution may be sourced:

$$\dot{\sigma}_{ij} + 3H\sigma_{ij} = \frac{8\pi G}{c^4}\pi_{ij}^{\text{aniso}}, \tag{476}$$

where π_{ij}^{aniso} is the anisotropic stress.

The first-leg model must therefore derive or constrain

$$\pi_{ij}^{\text{aniso}}. \tag{477}$$

A rapidly expanding background does not guarantee isotropization if the source remains large.

81 Curvature and Shear Phase Space

Define the first-leg geometric state vector

$$\boldsymbol{Y}_1 = \left(\Omega_k, \Omega_\sigma, \epsilon_H, \epsilon_{\text{asym}}, \ldots\right)^{\mathsf{T}}. \tag{478}$$

Its e-fold evolution is

$$\frac{\mathrm{d}\boldsymbol{Y}_1}{\mathrm{d}N} = \boldsymbol{F}_1\left(\boldsymbol{Y}_1\right). \tag{479}$$

Let

$$\boldsymbol{Y}_{1*} = \left(0, 0, \epsilon_{H*}, 0, \ldots\right) \tag{480}$$

represent a candidate near-flat, nearly isotropic trajectory or fixed point.

Perturbations around it are

$$\delta \boldsymbol{Y}_1 = \boldsymbol{Y}_1 - \boldsymbol{Y}_{1*}. \tag{481}$$

Linearization gives

$$\frac{\mathrm{d}\delta \boldsymbol{Y}_1}{\mathrm{d}N} = \mathsf{J}_1 \delta \boldsymbol{Y}_1, \tag{482}$$

where

$$\mathsf{J}_1 = \left.\frac{\partial \boldsymbol{F}_1}{\partial \boldsymbol{Y}_1}\right|_{\boldsymbol{Y}_{1*}}. \tag{483}$$

The candidate state is locally attracting when

$$\mathrm{Re}\,(\lambda_a) < 0 \tag{484}$$

for all relevant eigenvalues λ_a of J_1.

If the curvature eigenvalue is positive,

$$\mathrm{Re}\,(\lambda_k) > 0, \tag{485}$$

near-flatness is unstable during the first leg.

Definition 81.1 (First-leg near-flat attractor)**.** A first-leg near-flat attractor is a stable trajectory or fixed-point region toward which a finite set of initially small curvature and shear perturbations evolve before the outer reversal.

The foundational framework does not yet establish that such an attractor exists.

82 Comparison with Inflationary Curvature Suppression

Inflation suppresses curvature through

$$T_{k,\mathrm{inf}} = \exp\left[2 \int_{\mathrm{inf}} (\epsilon_H - 1)\,\mathrm{d}N\right]. \tag{486}$$

For approximately constant small ϵ_H,

$$T_{k,\mathrm{inf}} \approx e^{-2N_{\mathrm{inf}}}. \tag{487}$$

This provides strong dynamical suppression over sufficient e-folds.

The Triple Bounce first-leg alternatives may be classified as follows.

82.1 Equivalent metric suppression

If the first-leg background satisfies

$$\int_{\mathcal{I}_1} (1 - \epsilon_H) \, \mathrm{d}N \gg 1, \tag{488}$$

then it suppresses curvature through the same geometric mechanism as inflation.

Even if *Electro* has additional sector dynamics, the background portion is inflationary in the mathematical sense.

82.2 Boundary suppression with later preservation

If

$$T_k^{(1)} \gtrsim 1, \tag{489}$$

near-flatness must be supplied primarily by

$$|\Omega_{k,\mathrm{res}}| \ll 1. \tag{490}$$

This approach differs physically from inflation.

Its scientific credibility depends on deriving the concentration of the boundary curvature distribution near zero.

82.3 Modified geometric suppression

If new dynamics produce

$$\mathcal{S}_k < -2 \left(\epsilon_H - 1 \right), \tag{491}$$

then curvature may decrease even when

$$\epsilon_H > 1. \tag{492}$$

Such a mechanism would be distinct from standard inflation.

It would also require a new covariant field theory and stability analysis.

82.4 No semantic substitution

The framework should not call a period noninflationary merely because it belongs to the first *Electro* leg.

If

$$\ddot{a} > 0 \tag{493}$$

for a sufficient interval, then the background undergoes accelerated expansion.

The distinction should be stated as:

$$\boxed{\begin{array}{l}\text{the first } \textit{Electro} \text{ leg is not identical to inflation,}\\ \text{but it may contain an inflationary metric subinterval.}\end{array}} \tag{494}$$

83 First-Leg Curvature Transfer Function

A general scale-dependent first-leg curvature transfer may be written as

$$\widetilde{\Omega}_k\left(\boldsymbol{q}, t_{\mathrm{O}}^-\right) = T_\Omega^{(1)}\left(q\right)\widetilde{\Omega}_k\left(\boldsymbol{q}, t_{\mathrm{res}}^+\right) + N_\Omega^{(1)}\left(\boldsymbol{q}\right), \tag{495}$$

where $\boldsymbol{q}$ is used here as a perturbation wavenumber to distinguish it from the background curvature index k.

The transfer function contains:

$$T_\Omega^{(1)} = T_{\mathrm{bg}}^{(1)} T_{\mathrm{int}}^{(1)} T_{\mathrm{mem}}^{(1)}, \tag{496}$$

where:

- $T_{\mathrm{bg}}^{(1)}$ is the homogeneous background transfer;
- $T_{\mathrm{int}}^{(1)}$ is the effect of interactions among sectors; and
- $T_{\mathrm{mem}}^{(1)}$ is the scaffold-memory contribution.

The stochastic term

$$N_\Omega^{(1)} \tag{497}$$

represents curvature or anisotropy generated during first-leg evolution.

Large-scale near-flatness requires

$$\left|T_{\Omega}^{(1)}(q)\widetilde{\Omega}_k\left(\boldsymbol{q}, t_{\mathrm{res}}^{+}\right) + N_{\Omega}^{(1)}(\boldsymbol{q})\right| \ll 1 \tag{498}$$

for the long-wavelength modes relevant to the observable background.

The framework must distinguish homogeneous curvature from long-wavelength curvature perturbations.

They are related but not identical variables.

84 Outer-Reversal Input State

Immediately before the outer reversal, the first-leg geometric state is

$$\mathcal{I}_{\mathrm{O}}^{-} = \left\{a_{\mathrm{O}}^{-}, H_{\mathrm{O}}^{-}, \Omega_{k,\mathrm{O}}^{-}, \sigma_{ij,\mathrm{O}}^{-}, \rho_{i,\mathrm{O}}^{-}, p_{i,\mathrm{O}}^{-}, \mathcal{W}_1, \boldsymbol{\mathcal{P}}_1, \delta_i, \Phi, \Psi\right\}. \tag{499}$$

The first-leg flatness analysis must supply finite values satisfying

$$a_{\mathrm{O}}^{-} > 0, \tag{500}$$

$$H_{\mathrm{O}}^{-} > 0, \tag{501}$$

$$\left|\Omega_{k,\mathrm{O}}^{-}\right| < \infty, \tag{502}$$

and

$$\frac{\sigma_{\mathrm{O}}^{-}}{H_{\mathrm{O}}^{-}} < \infty. \tag{503}$$

For a near-flat transfer into the second leg, the stronger conditions are

$$\left|\Omega_{k,\mathrm{O}}^{-}\right| \ll 1, \tag{504}$$

and

$$\Omega_{\sigma,\mathrm{O}}^{-} \ll 1. \tag{505}$$

The outer transition cannot repair an arbitrarily pathological first-leg state unless a specific smoothing mechanism is derived.

85 First-Leg Flatness Test Cases

The first numerical curvature analysis should compare four cases.

85.1 Case A: exact balanced boundary

Assume

$$\epsilon_{\mathrm{asym}} = 0, \qquad \Omega_{k,\mathrm{res}} = 0, \qquad \Omega_{\sigma,\mathrm{res}} = 0. \tag{506}$$

This is an ideal limiting solution.

It tests whether the first-leg equations preserve exact flatness and isotropy.

It does not test naturalness.

85.2 Case B: small perturbed boundary

Assign

$$0 < |\Omega_{k,\mathrm{res}}| \ll 1, \qquad 0 < \Omega_{\sigma,\mathrm{res}} \ll 1. \tag{507}$$

Evolve a neighborhood of initial conditions to determine whether the balanced trajectory is stable.

85.3 Case C: decelerating first leg

Require

$$\epsilon_H^{(1)} > 1 \tag{508}$$

throughout the first leg.

This measures the boundary fine-tuning required when curvature grows monotonically.

85.4 Case D: mixed first-leg background

Permit

$$\epsilon_H^{(1)} = \epsilon_H^{(1)}(N) \tag{509}$$

to cross unity.

The total transfer remains

$$T_k^{(1)} = \exp\left[2\int \left(\epsilon_H^{(1)} - 1\right) \mathrm{d}N\right]. \tag{510}$$

This case tests whether limited accelerated subintervals can compensate for decelerating portions without identifying the entire first leg as standard inflation.

86 First-Leg Flatness-Mechanism Rejection Criteria

The first-leg near-flatness proposal fails or requires revision if any of the following is unavoidable.

1. No mathematically defined boundary map connects pre-singularity ordering to classical curvature and shear.
2. The balanced mode suppresses only the ensemble mean curvature while leaving a broad distribution of strongly curved realizations.
3. Small curvature perturbations grow without bound during the first leg.
4. The first-leg transfer factor satisfies

$$T_k^{(1)} \gg 1 \tag{511}$$

   and the required boundary value is unexplained or excessively tuned.
5. The first-leg *Electro* sector generates large anisotropic stress that prevents isotropization.
6. The near-flat trajectory is a repeller rather than an attractor or stable neutral trajectory.
7. Curvature suppression is claimed from outward *Electro* motion without a corresponding metric or modified-gravity derivation.
8. The framework describes a long interval with $\epsilon_H < 1$ but denies that the background is inflationary solely through terminology.
9. The first-leg state reaches the outer transition with divergent curvature, shear, density, or pressure.
10. Near-flatness occurs only at one measure-zero initial state rather than within a finite basin.
11. A nonstandard curvature source $\mathcal{S}_k$ is introduced without covariance, conservation, and stability.
12. The first-leg flatness candidate resolution is incompatible with the causal history required by the horizon test.

The last condition is especially important.

A background chosen to suppress curvature may reduce or alter the available conformal interval.

The horizon and flatness calculations must therefore use the same first-leg solution.

87 Formal Summary of First-Leg Curvature Evolution

The provisional singular-opening boundary prescription is

$$\boxed{\mathcal{B}_{\mathrm{res}} : \mathcal{S}_{\mathrm{pre}} \longrightarrow \{\Omega_{k,\mathrm{res}}, \Omega_{\sigma,\mathrm{res}}, a_{\mathrm{res}}, H_{\mathrm{res}}\}\,.} \tag{512}$$

This notation does not assert a classical continuation through the singularity; it defines the boundary problem that a completed theory must solve.

The balanced-state asymmetry is

$$\boxed{\epsilon_{\mathrm{asym}} = \frac{\|\delta\Psi\|}{\|A_0\Psi_0\|}.} \tag{513}$$

A symmetry-suppressed curvature response may take the form

$$\boxed{\Omega_{k,\mathrm{res}} \approx K_2\epsilon_{\mathrm{asym}}^2,} \tag{514}$$

but this relation must be derived.

The exact classical curvature evolution is

$$\boxed{\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} = 2\,(\epsilon_H - 1)\,.} \tag{515}$$

The first-leg transfer factor is

$$\boxed{T_k^{(1)} = \exp\left\{2\int_{N_{\mathrm{res}}}^{N_{\mathrm{O}}^-}\left[\epsilon_H^{(1)}(N) - 1\right]\mathrm{d}N\right\}.} \tag{516}$$

Near-flatness at the outer transition requires

$$\boxed{\left|\Omega_{k,\mathrm{O}}^-\right| = T_k^{(1)}\,|\Omega_{k,\mathrm{res}}| \ll 1.} \tag{517}$$

The shear distinction is

$$\boxed{\rho_\sigma \propto a^{-6} \quad \text{does not imply} \quad |\Omega_k| \text{ decreases.}} \tag{518}$$

The central first-leg flatness question is therefore

$$\boxed{\begin{array}{l}\text{Does balanced boundary suppression, together with first-leg evolution,}\\ \text{deliver a finite and naturally near-flat state to the outer reversal?}\end{array}} \tag{519}$$

88 Curvature Matching Across the Two Reversals

The first-leg analysis delivered a geometric state immediately before the outer reversal,

$$\mathcal{I}_{\mathrm{O}}^{-}. \tag{520}$$

That state must pass through:

$$\mathcal{I}_{\mathrm{O}}^{-} \xrightarrow{\mathcal{T}_{\mathrm{O}}} \mathcal{I}_{\mathrm{O}}^{+} \xrightarrow{\mathcal{U}_2} \mathcal{I}_{\mathrm{C}}^{-} \xrightarrow{\mathcal{T}_{\mathrm{C}}} \mathcal{I}_{\mathrm{C}}^{+}, \tag{521}$$

where:

- $\mathcal{T}_{\mathrm{O}}$ is the outer-reversal transition map;
- $\mathcal{U}_2$ is the second-leg evolution operator; and
- $\mathcal{T}_{\mathrm{C}}$ is the central-reversal transition map.

The final state

$$\mathcal{I}_{\mathrm{C}}^{+} \tag{522}$$

provides the initial background and perturbation data for the observable third leg.

The matching problem has two distinct levels:

1. **Background matching,** which governs the scale factor, Hubble rate, energy density, pressure, curvature, and shear; and
2. **Perturbation matching,** which governs scalar, vector, tensor, entropy, and inherited-memory modes.

The *Electro* direction changes at both transitions.

The global expansion is not required to reverse.

Thus, the fundamental sign changes are

$$\dot{R}_E : + \longrightarrow - \qquad \text{at the outer reversal,} \tag{523}$$

$$\dot{R}_E : - \longrightarrow + \qquad \text{at the central reversal,} \tag{524}$$

while the continuously expanding version requires

$$H_{\mathrm{O}} > 0, \qquad H_{\mathrm{C}} > 0. \tag{525}$$

The two transition maps must therefore be constructed independently. Neither reversal is a singularity, and Paper II leaves the physical cause of each reversal unresolved. The maps cannot be assumed to possess identical durations, energies, transfer matrices, or underlying causes.

88.1 Curvature transfer across the two nonsingular reversals

At the homogeneous level, define the curvature transfer maps

$$\Omega_k(t_{\mathrm{O}}^+) = \mathcal{T}_{k,\mathrm{O}}\left[\Omega_k(t_{\mathrm{O}}^-), \mathcal{I}_{\mathrm{O}}^-, \theta_{\mathrm{O}}\right], \tag{526}$$

$$\Omega_k(t_{\mathrm{C}}^+) = \mathcal{T}_{k,\mathrm{C}}\left[\Omega_k(t_{\mathrm{C}}^-), \mathcal{I}_{\mathrm{C}}^-, \theta_{\mathrm{C}}\right]. \tag{527}$$

Here θ_{O} and θ_{C} denote the distinct transition parameters. Regular transfer does not require exact conservation of Ω_k, because $\Omega_k \propto (aH)^{-2}$ can change when H or the effective stress-energy content changes. It does require that any change follow from a specified finite-width evolution or a controlled thin-limit matching law rather than an arbitrary reset.

For a common normalized curvature index k and continuous a and H, the homogeneous curvature fraction is continuous. A discontinuity in Ω_k therefore requires a derived change in the effective spatial geometry, a controlled jump in the extrinsic curvature, or an explicit surface source. These possibilities are tests, not assumptions.

89 Finite-Width Transition Geometry

89.1 Transition intervals

The outer and central transitions occupy finite intervals,

$$\mathcal{I}_{\mathrm{O}} = \left[t_{\mathrm{O}}^{-}, t_{\mathrm{O}}^{+}\right], \tag{528}$$

$$\mathcal{I}_{\mathrm{C}} = \left[t_{\mathrm{C}}^{-}, t_{\mathrm{C}}^{+}\right]. \tag{529}$$

Their proper-time widths are

$$\Delta t_{\mathrm{O}} = t_{\mathrm{O}}^{+} - t_{\mathrm{O}}^{-}, \tag{530}$$

$$\Delta t_{\mathrm{C}} = t_{\mathrm{C}}^{+} - t_{\mathrm{C}}^{-}. \tag{531}$$

No equality is assumed:

$$\Delta t_{\mathrm{O}} \neq \Delta t_{\mathrm{C}} \tag{532}$$

in the general model.

89.2 Smooth reversal profiles

A useful kinematic profile for reversal j is

$$\dot{R}_E^{(j)}(t) = \frac{v_j^- + v_j^+}{2} + \frac{v_j^+ - v_j^-}{2} \tanh\left(\frac{t - t_j}{\tau_j}\right), \tag{533}$$

where $j \in \{\mathrm{O}, \mathrm{C}\}$.

For the outer reversal,

$$v_{\mathrm{O}}^- > 0, \qquad v_{\mathrm{O}}^+ < 0. \tag{534}$$

For the central reversal,

$$v_{\mathrm{C}}^- < 0, \qquad v_{\mathrm{C}}^+ > 0. \tag{535}$$

The parameters

$$\left\{v_{\mathrm{O}}^{-}, v_{\mathrm{O}}^{+}, \tau_{\mathrm{O}}, v_{\mathrm{C}}^{-}, v_{\mathrm{C}}^{+}, \tau_{\mathrm{C}}\right\} \tag{536}$$

remain distinct.

The tanh profile provides a regular interpolation.

It does not supply the physical force producing either reversal.

89.3 Smooth background requirements

Within a finite classical transition, the scale factor must satisfy

$$a_j(t) > 0. \tag{537}$$

The continuously expanding version requires

$$H_j(t) > 0. \tag{538}$$

The background variables should remain finite:

$$\left\{a, H, \dot{H}, \rho_{\mathrm{tot}}, p_{\mathrm{tot}}, \Omega_k, \sigma^2\right\} < \infty. \tag{539}$$

The transition may contain rapid variation.

It may not contain an unexplained divergence.

90 Thin-Hypersurface Limit

Although finite-width transitions are preferred, the thin-hypersurface limit provides a useful consistency check.

Let

$$\Sigma_j \tag{540}$$

denote the limiting transition hypersurface.

For any quantity X, define the jump

$$[X]_j = X(t_j^+) - X(t_j^-). \tag{541}$$

The thin limit is obtained schematically as

$$\Delta t_j \longrightarrow 0 \tag{542}$$

while holding finite any physically meaningful integrated transition source.

A thin transition may contain a surface stress–energy tensor,

$$S_{ab}^{(j)}. \tag{543}$$

If no physical surface layer is present, then

$$S_{ab}^{(j)} = 0. \tag{544}$$

The thin-limit equations must reproduce the finite-width solution as the transition duration is reduced.

A model that gives unrelated answers in the two descriptions lacks a controlled transition limit.

91 Induced Metric and Extrinsic Curvature

91.1 Induced metric

Let n^μ be the unit normal to the transition hypersurface.

The induced metric is

$$h_{\mu\nu} = g_{\mu\nu} - \epsilon_n n_\mu n_\nu, \tag{545}$$

where $\epsilon_n = n_\mu n^\mu$ depends on the signature and whether the hypersurface is spacelike or timelike.

For a standard constant-time cosmological transition, the induced spatial metric is

$$h_{ij} = a^2(t)\gamma_{ij}, \tag{546}$$

where γ_{ij} is the normalized constant-curvature spatial metric.

Continuity of the induced metric requires

$$[h_{ij}]_j = 0. \tag{547}$$

For a common spatial geometry, this implies

$$[a]_j = 0. \tag{548}$$

91.2 Extrinsic curvature

The extrinsic curvature is

$$K_{\mu\nu} = h_\mu{}^\alpha h_\nu{}^\beta \nabla_\alpha n_\beta. \tag{549}$$

For a homogeneous constant-time Friedmann–Lemaître–Robertson–Walker hypersurface,

$$K_{ij} \propto H h_{ij}, \tag{550}$$

up to the orientation and sign convention for n^μ.

Thus, continuity of K_{ij} implies continuity of the Hubble rate:

$$[H]_j = 0 \tag{551}$$

when no surface layer is present and the same time orientation is used on both sides.

This is consistent with sectoral reversal:

$$[H]_j = 0, \qquad \operatorname{sgn}\left(\dot{R}_E^+\right) = -\operatorname{sgn}\left(\dot{R}_E^-\right). \tag{552}$$

The *Electro* velocity may reverse while the metric expansion remains continuous.

92 Surface Stress and the No-Shell Condition

In the thin-hypersurface description, the jump in extrinsic curvature is related to the surface stress–energy tensor.

Using one common sign convention (Israel, 1966),

$$[K_{ab} - h_{ab}K]_j = -\frac{8\pi G}{c^4} S_{ab}^{(j)}. \tag{553}$$

The overall sign depends on the normal-vector convention.

The physical content is that a discontinuity in extrinsic curvature requires a localized surface source.

When

$$S_{ab}^{(j)} = 0, \tag{554}$$

the no-shell matching conditions are

$$[h_{ab}]_j = 0, \qquad [K_{ab}]_j = 0. \tag{555}$$

For the homogeneous background, this gives

$$[a]_j = 0, \qquad [H]_j = 0. \tag{556}$$

A discontinuity such as

$$[H]_j \neq 0 \tag{557}$$

must therefore be supported by a specified surface layer or by a departure from the assumed classical junction structure.

The Triple Bounce framework does not presently require such a layer.

The preferred baseline is smooth finite-width evolution with no distributional surface stress.

93 Background Energy Transfer Through the Transitions

The total stress–energy tensor satisfies

$$\nabla_\mu T^{\mu\nu}_{\text{tot}} = 0. \tag{558}$$

Individual sectors may exchange energy and momentum:

$$\nabla_\mu T^{\mu\nu}_i = Q^\nu_i. \tag{559}$$

Total conservation requires

$$\sum_i Q^\nu_i = 0. \tag{560}$$

At the homogeneous level,

$$\dot{\rho}_i + 3H\left(\rho_i + \frac{p_i}{c^2}\right) = Q_i, \tag{561}$$

with

$$\sum_i Q_i = 0. \tag{562}$$

The outer transition may redistribute energy among:

$$\{\textit{Electro}, \text{scaffold}, \text{chasing sector}, \text{precursor sectors}\}. \tag{563}$$

The central transition may additionally populate or source:

$$\left\{\begin{matrix} \text{effective radiation}, & \text{optional particle-like excitations}, \\ \text{candidate electromagnetic modes}, & \text{third-leg matter precursors} \end{matrix}\right\}. \tag{564}$$

This schematic set does not require a pair-rich phase or stable ordinary matter at the instant of reversal. Those outcomes must emerge, if at all, from the subsequent third-leg dynamics.

The total energy density should remain finite and continuous in a smooth transition:

$$[\rho_{\text{tot}}]_j = 0 \tag{565}$$

in the no-shell thin limit.

Individual sector densities need not remain separately constant.

94 Background Curvature Through a Smooth Transition

In an ordinary Friedmann–Lemaître–Robertson–Walker geometry with fixed normalized spatial-curvature index k,

$$\Omega_k = -\frac{kc^2}{a^2H^2}. \tag{566}$$

If a and H are continuous, then

$$[\Omega_k]_j = 0. \tag{567}$$

Thus, a smooth reversal does not permit an arbitrary instantaneous jump in the homogeneous curvature fraction.

Curvature may still evolve rapidly across a finite transition because

$$\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} = 2\left(\epsilon_H - 1\right) \tag{568}$$

continues to apply wherever the standard Friedmann–Lemaître–Robertson–Walker equations remain valid.

Define the finite-width transition transfer factor

$$T_k^{(j)} = \exp\left\{2\int_{\mathcal{I}_j} [\epsilon_H(t) - 1]\, \mathrm{d}N\right\}. \tag{569}$$

For a regular thin transition,

$$\Delta N_j \longrightarrow 0 \quad \Longrightarrow \quad T_k^{(j)} \longrightarrow 1. \tag{570}$$

Any proposal in which

$$[\Omega_k]_j \neq 0 \tag{571}$$

must identify one of the following:

- a surface stress layer;
- a discontinuity in H;
- a change in the spatial-curvature index or topology;
- modified gravitational field equations; or
- a breakdown of the classical Friedmann–Lemaître–Robertson–Walker description.

The baseline Triple Bounce model assumes none of these without explicit derivation.

95 Perturbation Variables and Gauge-Invariant Matching

For the perturbation-matching discussion in this section, natural units

$$c = \hbar = 1 \tag{572}$$

are used.

95.1 Scalar metric perturbations

A general scalar-perturbed metric may be written as

$$\mathrm{d}s^2 = a^2(\eta)\left\{-(1+2A)\,\mathrm{d}\eta^2 + 2\partial_i B\,\mathrm{d}x^i\mathrm{d}\eta + [(1+2\psi)\,\delta_{ij} + 2\partial_i\partial_j E]\,\mathrm{d}x^i\mathrm{d}x^j\right\}. \tag{573}$$

Gauge-invariant combinations include the Bardeen potentials (Bardeen, 1980). Standard cosmological applications are developed by Ma and Bertschinger (1995) and reviewed by Malik and Wands (2009).

$$\Phi, \qquad \Psi, \tag{574}$$

the curvature perturbation on uniform-density hypersurfaces,

$$\zeta = -\psi - H\frac{\delta\rho}{\dot{\rho}}, \tag{575}$$

and the comoving curvature perturbation,

$$\mathcal{R}. \tag{576}$$

The matching conditions should be stated in gauge-invariant variables or derived directly from the perturbed induced metric and extrinsic curvature.

95.2 Perturbation state vector

Define a scalar-sector state vector

$$\boldsymbol{X}_{\mathrm{s}} = \left(\mathcal{R}, \Pi_{\mathcal{R}}, S_1, \Pi_{S_1}, \ldots, S_n, \Pi_{S_n}, \Phi, \Psi\right)^{\mathsf{T}}, \tag{577}$$

where:

- $\Pi_{\mathcal{R}}$ is the canonical or effective momentum associated with $\mathcal{R}$; and
- S_A are independent entropy or isocurvature modes.

The tensor state may be written as

$$\boldsymbol{X}_{\mathrm{T}} = \left(h_+, \Pi_+, h_\times, \Pi_\times\right)^{\mathsf{T}}. \tag{578}$$

Vector variables and a specified coarse-grained scaffold-retention variable may be added when their dynamics are defined.

96 Finite-Width Perturbation Evolution

For a finite transition, the perturbations should be evolved through the transition rather than matched by an arbitrary jump rule.

A general linear system is

$$\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} = \mathsf{A}_j\left(t, k\right)\boldsymbol{X} + \boldsymbol{J}_j\left(t, k\right), \tag{579}$$

where:

- A_j is the transition evolution matrix; and
- $\boldsymbol{J}_j$ contains transition-generated sources.

The formal solution is

$$\boldsymbol{X}(t_j^+) = \mathsf{U}_j\left(t_j^+, t_j^-; k\right)\boldsymbol{X}(t_j^-) + \int_{t_j^-}^{t_j^+} \mathsf{U}_j\left(t_j^+, t; k\right)\boldsymbol{J}_j(t, k)\mathrm{d}t. \tag{580}$$

Define

$$\mathsf{M}_j(k) = \mathsf{U}_j\left(t_j^+, t_j^-; k\right), \tag{581}$$

and

$$\boldsymbol{N}_j(k) = \int_{t_j^-}^{t_j^+} \mathsf{U}_j\left(t_j^+, t; k\right)\boldsymbol{J}_j(t, k)\mathrm{d}t. \tag{582}$$

Then

$$\boxed{\boldsymbol{X}_j^+ = \mathsf{M}_j(k)\boldsymbol{X}_j^- + \boldsymbol{N}_j(k).} \tag{583}$$

The transfer matrix must be derived from the transition dynamics.

It may not be selected independently at every wavenumber.

97 Thin-Limit Perturbation Matching

In the no-shell thin limit, the perturbations of the induced metric and extrinsic curvature should remain continuous on the selected transition hypersurface.

Schematically,

$$[\delta h_{ij}]_j = 0, \tag{584}$$

and

$$[\delta K_{ij}]_j = 0 \tag{585}$$

when no perturbed surface stress is present.

For a transition defined by a uniform-total-density hypersurface, these conditions commonly imply continuity of

$$[\zeta]_j = 0 \tag{586}$$

on sufficiently large scales under regular adiabatic conditions.

The Newtonian potential need not remain separately constant in every transition model.

Its matching depends on the transition hypersurface, anisotropic stress, and the equation-of-state change.

The framework should therefore avoid assuming

$$[\Phi]_j = 0 \tag{587}$$

without deriving the appropriate hypersurface conditions.

Limitation 97.1. Perturbation matching is not unique until the physical transition hypersurface is specified. Matching on constant coordinate time, constant total density, constant field value, or constant transition order parameter may produce different intermediate expressions.

98 Large-Scale Curvature Evolution and Entropy Sources

On scales much larger than the relevant sound horizon, the comoving curvature perturbation evolves schematically as

$$\dot{\mathcal{R}} = -\frac{H}{\rho + p}\delta p_{\text{nad}} + \mathcal{S}_{\text{int}} + \mathcal{O}\left(\frac{k^2}{a^2 H^2}\right), \tag{588}$$

where:

- δp_{nad} is the nonadiabatic pressure perturbation; and
- $\mathcal{S}_{\text{int}}$ represents interaction and modified-sector sources.

The nonadiabatic pressure is

$$\delta p_{\text{nad}} = \delta p - c_a^2 \delta\rho, \tag{589}$$

with

$$c_a^2 = \frac{\dot{p}}{\dot{\rho}}. \tag{590}$$

For an adiabatic single-clock transition,

$$\delta p_{\mathrm{nad}} \approx 0 \quad \Longrightarrow \quad \dot{\mathcal{R}} \approx 0 \tag{591}$$

on large scales.

The Triple Bounce transitions are multisector and may generate entropy perturbations.

Thus,

$$\dot{\mathcal{R}} \neq 0 \tag{592}$$

is allowed during the reversals.

The resulting curvature must nevertheless remain finite and observationally acceptable.

99 Outer-Reversal Perturbation Map

The outer transition changes the first outward *Electro* state into the second inward state. Its cause remains unresolved, so the transfer law below is a general candidate parameterization rather than a derived mechanism.

Its scalar transfer may be written as

$$\begin{pmatrix} \mathcal{R} \\ S_E \\ S_{\mathrm{sc}} \\ S_{\mathrm{ch}} \end{pmatrix}^{+}_{\mathrm{O}} = \mathsf{M}^{(\mathrm{s})}_{\mathrm{O}}(k) \begin{pmatrix} \mathcal{R} \\ S_E \\ S_{\mathrm{sc}} \\ S_{\mathrm{ch}} \end{pmatrix}^{-}_{\mathrm{O}} + \boldsymbol{N}^{(\mathrm{s})}_{\mathrm{O}}(k), \tag{593}$$

where:

- S_E is an *Electro* entropy mode;
- S_{sc} is a scaffold-relative entropy mode; and
- S_{ch} is a chasing-sector entropy mode.

A useful block form is

$$\mathsf{M}^{(\mathrm{s})}_{\mathrm{O}} = \begin{pmatrix} T^{\mathrm{O}}_{\mathcal{R}\mathcal{R}} & \boldsymbol{T}^{\mathrm{O}}_{\mathcal{R}S} \\ \boldsymbol{T}^{\mathrm{O}}_{S\mathcal{R}} & \mathsf{T}^{\mathrm{O}}_{SS} \end{pmatrix}. \tag{594}$$

The outgoing curvature perturbation is

$$\mathcal{R}^{+}_{\mathrm{O}} = T^{\mathrm{O}}_{\mathcal{R}\mathcal{R}} \mathcal{R}^{-}_{\mathrm{O}} + \boldsymbol{T}^{\mathrm{O}}_{\mathcal{R}S} \boldsymbol{S}^{-}_{\mathrm{O}} + N^{\mathrm{O}}_{\mathcal{R}}. \tag{595}$$

Regularity requires the complete scalar transfer matrix to remain finite,

$$\left\| \mathsf{M}_{\mathrm{O}}^{(\mathrm{s})}(k) \right\| < \infty. \tag{596}$$

Strong *direct* curvature preservation on large scales corresponds to

$$T_{\mathcal{R}\mathcal{R}}^{\mathrm{O}} \approx 1. \tag{597}$$

This is a benchmark, not a universal requirement. Common-state information may temporarily occupy entropy or scaffold-correlated channels and later return to the curvature sector. The transition may therefore convert entropy into curvature through

$$\boldsymbol{T}_{\mathcal{R}S}^{\mathrm{O}} \neq 0. \tag{598}$$

That conversion must not generate excessive scale dependence, non-Gaussianity, or anisotropy.

100 Second-Leg Propagation of the Matched State

The outer output becomes the initial condition for second-leg propagation:

$$\boldsymbol{X}_2(t_{\mathrm{O}}^{+}) = \boldsymbol{X}_{\mathrm{O}}^{+}. \tag{599}$$

The linear second-leg solution is

$$\boldsymbol{X}_{\mathrm{C}}^{-} = \mathsf{U}_2\left(t_{\mathrm{C}}^{-}, t_{\mathrm{O}}^{+}; k\right) \boldsymbol{X}_{\mathrm{O}}^{+} + \boldsymbol{N}_2(k). \tag{600}$$

Combining the outer reversal and second leg gives

$$\boldsymbol{X}_{\mathrm{C}}^{-} = \mathsf{U}_2 \mathsf{M}_{\mathrm{O}} \boldsymbol{X}_{\mathrm{O}}^{-} + \mathsf{U}_2 \boldsymbol{N}_{\mathrm{O}} + \boldsymbol{N}_2. \tag{601}$$

The second leg must preserve the regularity of:

$$\left\{\mathcal{R}, S_A, h_\lambda, \Phi, \Psi, \mathcal{W}_{12}, \mathcal{M}_{ij}^{(12)}\right\}. \tag{602}$$

The inward direction of *Electro* does not reverse the sign of cosmic time or conformal time.

The perturbation equations remain forward initial-value equations from

$$t_{\mathrm{O}}^{+} \longrightarrow t_{\mathrm{C}}^{-}. \tag{603}$$

101 Central-Reversal Perturbation Map

The central reversal has a broader observational burden than the outer reversal because it initiates the third outward leg and candidate *Electro*–magnetism coupling. Its physical cause and microphysics remain unresolved.

A viable central-transfer theory must connect the inherited two-pass state to a regular early-third-leg perturbation state containing:

- scalar, tensor, vector, and entropy perturbations;
- effective radiation, scaffold, and chasing-sector fluctuations;
- any candidate electromagnetic-sector perturbations generated by coupling; and
- an inherited-web occupancy variable or distribution function.

This list does not require stable ordinary matter or a pair-rich phase to exist at the instant of reversal. Stable matter organization may occur later during the third leg.

Define the central input vector

$$\boldsymbol{X}_{\mathrm{C}}^{-} = \left(\mathcal{R}^{-}, \boldsymbol{S}^{-}, \boldsymbol{h}^{-}, \boldsymbol{V}^{-}, \mathcal{W}_{12}, \boldsymbol{\mathcal{P}}_{12}, \mathcal{M}_{ij}^{(12)}, \delta f_i^{-}\right)^{\mathsf{T}}. \tag{604}$$

The central output vector is

$$\boldsymbol{X}_{\mathrm{C}}^{+} = \left(\mathcal{R}^{+}, \boldsymbol{S}^{+}, \boldsymbol{h}^{+}, \boldsymbol{V}^{+}, \delta_{\mathrm{rad}}, \delta_{\mathrm{sc}}, \delta_{\mathrm{ch}}, \delta_{\mathrm{em}}, \delta f_i^{+}\right)^{\mathsf{T}}. \tag{605}$$

A candidate linearized transfer law is

$$\boxed{\boldsymbol{X}_{\mathrm{C}}^{+} = \mathsf{M}_{\mathrm{C}}(k)\boldsymbol{X}_{\mathrm{C}}^{-} + \boldsymbol{N}_{\mathrm{C}}(k).} \tag{606}$$

The central matrix is not assumed to equal the outer matrix:

$$\mathsf{M}_{\mathrm{C}} \not\equiv \mathsf{M}_{\mathrm{O}} \tag{607}$$

in the general formulation. The physical environments and required outputs differ, although special limits may share individual transfer coefficients. Every coefficient must be derived from one finite transition theory rather than fitted independently.

102 Entropy-to-Curvature Conversion at the Central Reversal

The third-leg curvature perturbation may be decomposed as

$$\mathcal{R}_{\mathrm{C}}^{+} = T_{\mathcal{R}\mathcal{R}}^{\mathrm{C}} \mathcal{R}_{\mathrm{C}}^{-} + \boldsymbol{T}_{\mathcal{R}S}^{\mathrm{C}} \boldsymbol{S}_{\mathrm{C}}^{-} + T_{\mathcal{R}\mathcal{W}}^{\mathrm{C}} \mathcal{W}_{12} + N_{\mathcal{R}}^{\mathrm{C}}. \tag{608}$$

The four terms represent:

1. direct transfer of precentral curvature;
2. conversion of entropy modes into curvature;
3. candidate transfer of scaffold-correlated retention into curvature; and
4. a transition-sourced curvature contribution.

The final scalar power spectrum is therefore

$$\mathcal{P}_{\mathcal{R}}^{+}(k) = \left\langle \left| \mathcal{R}_{\mathrm{C}}^{+}(k) \right|^2 \right\rangle. \tag{609}$$

Expanding schematically,

$$\begin{aligned} \mathcal{P}_{\mathcal{R}}^{+} = {} & \left| T_{\mathcal{R}\mathcal{R}}^{\mathrm{C}} \right|^2 \mathcal{P}_{\mathcal{R}}^{-} + \boldsymbol{T}_{\mathcal{R}S}^{\mathrm{C}} \mathsf{P}_{SS}^{-} \boldsymbol{T}_{\mathcal{R}S}^{\mathrm{C}\dagger} \\ & + \left| T_{\mathcal{R}\mathcal{W}}^{\mathrm{C}} \right|^2 \mathcal{P}_{\mathcal{W}} + \mathcal{P}_{N_{\mathcal{R}}} + \text{cross terms}. \end{aligned} \tag{610}$$

The cross terms encode correlations among the incoming modes.

They cannot be omitted if the first and second legs create a correlated two-pass state.

A viable central transfer must yield:

$$\boxed{\begin{aligned} &\text{small amplitude} + \text{near scale invariance} + \text{predominant adiabaticity} \\ &+ \text{phase coherence} + \text{acceptable statistics.} \end{aligned}} \tag{611}$$

103 Adiabatic and Isocurvature Output

For each third-leg component i, define

$$\zeta_i = -\psi - H\frac{\delta\rho_i}{\dot{\rho}_i}. \tag{612}$$

Relative-entropy perturbations are

$$S_{ij} = 3\left(\zeta_i - \zeta_j\right). \tag{613}$$

Purely adiabatic initial conditions satisfy

$$S_{ij} = 0 \tag{614}$$

for all relevant pairs.

The Triple Bounce transition need not produce exactly zero isocurvature. For visible modes, however, the derived transfer must yield an observationally allowed fraction.

Define

$$\beta_{\text{iso}}(k) = \frac{\mathcal{P}_S(k)}{\mathcal{P}_{\mathcal{R}}(k) + \mathcal{P}_S(k)}. \tag{615}$$

The required condition is

$$\beta_{\text{iso}}(k) < \beta_{\text{iso,max}}(k), \tag{616}$$

where the bound depends on the isocurvature type and correlation with the adiabatic mode.

For visible modes, the derived central transfer must therefore place most of the observable power in

$$\mathcal{R}_{\text{adiabatic}} \tag{617}$$

while keeping correlated and uncorrelated isocurvature components below the applicable bounds. Power assigned to unobserved sectors remains a physical claim and cannot be used as an unconstrained sink.

104 Shear and Anisotropy Matching

The shear evolution is

$$\dot{\sigma}_{ij} + 3H\sigma_{ij} = \frac{8\pi G}{c^4}\pi_{ij}^{\mathrm{aniso}}, \tag{618}$$

where $\pi_{ij}^{\mathrm{aniso}}$ is the anisotropic stress.

Integrating across a finite transition gives

$$\sigma_{ij}^{+} - \sigma_{ij}^{-} = \int_{t_j^-}^{t_j^+} \left[-3H\sigma_{ij} + \frac{8\pi G}{c^4}\pi_{ij}^{\mathrm{aniso}}\right] \mathrm{d}t. \tag{619}$$

For a regular thin transition without surface anisotropic stress,

$$[\sigma_{ij}]_j = 0. \tag{620}$$

If a surface anisotropic stress exists, then the shear may acquire a finite jump proportional to its integrated strength.

Define a normalized transition anisotropy impulse,

$$\mathcal{J}_\sigma^{(j)} = \frac{8\pi G}{c^4 H_j}\int_{t_j^-}^{t_j^+} \pi^{\mathrm{aniso}}(t)\mathrm{d}t. \tag{621}$$

A controlled transition requires

$$\left|\mathcal{J}_\sigma^{(j)}\right| \ll 1 \tag{622}$$

on the homogeneous and largest observable scales.

The multipass nature of the Triple Bounce may generate preferred directions.

Those directions must average or transfer in a manner consistent with

$$\Omega_{\sigma,\mathrm{C}}^{+} \ll 1. \tag{623}$$

105 Tensor and Vector Transfer

The tensor modes satisfy a transition equation of the form

$$h_\lambda'' + 2\mathcal{H}h_\lambda' + \left(c_T^2 k^2 + a^2 m_T^2\right) h_\lambda = \Pi_\lambda^{\mathrm{T}}, \tag{624}$$

where:

- $\lambda \in \{+, \times\}$ labels polarization;
- $\mathcal{H} = a'/a$ is the conformal Hubble rate;
- c_T is the tensor propagation speed;
- m_T is an effective tensor mass if present; and
- Π_λ^{T} is transverse-traceless anisotropic stress.

The tensor transfer is

$$\boldsymbol{X}_{\mathrm{T},j}^{+} = \mathsf{M}_{\mathrm{T},j}(k)\boldsymbol{X}_{\mathrm{T},j}^{-} + \boldsymbol{N}_{\mathrm{T},j}(k). \tag{625}$$

The two reversals may source tensor power.

The combined output must satisfy

$$\mathcal{P}_T^{(3)}(k) < \mathcal{P}_{T,\max}(k). \tag{626}$$

Vector perturbations may be written schematically as

$$\boldsymbol{V}_j^{+} = \mathsf{M}_{\mathrm{V},j}(k)\boldsymbol{V}_j^{-} + \boldsymbol{N}_{\mathrm{V},j}(k). \tag{627}$$

Because multipass pathways and dynamo junctions may generate rotational structure, the vector source need not vanish.

It must nevertheless remain compatible with the observed near-isotropy of the early third leg.

106 Transition Stability Conditions

A regular background solution is insufficient if the perturbations are unstable.

For a general quadratic perturbation action,

$$S^{(2)} = \frac{1}{2} \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \left[\dot{\boldsymbol{q}}^{\mathsf{T}} \mathsf{K} \dot{\boldsymbol{q}} - \frac{1}{a^2} (\nabla \boldsymbol{q})^{\mathsf{T}}\, \mathsf{G}\, (\nabla \boldsymbol{q}) - \boldsymbol{q}^{\mathsf{T}} \mathsf{M}^2 \boldsymbol{q} \right], \tag{628}$$

where $\boldsymbol{q}$ contains the independent perturbation modes.

106.1 No-ghost condition

The kinetic matrix must be positive definite:

$$\lambda_a (\mathsf{K}) > 0 \tag{629}$$

for every physical degree of freedom.

106.2 No gradient instability

The eigenvalues of

$$\mathsf{C}_s^2 = \mathsf{K}^{-1} \mathsf{G} \tag{630}$$

must satisfy

$$c_{s,a}^2 \geq 0. \tag{631}$$

106.3 Controlled tachyonic behavior

A negative effective mass squared may be temporarily allowed if the growth time exceeds the transition duration or if the instability is the controlled mechanism ending a metastable state.

It must not produce uncontrolled amplification:

$$\|\mathsf{M}_j(k)\| < \infty. \tag{632}$$

106.4 Invertibility

A deterministic linear transition should possess a finite-rank transfer map over the retained physical modes.

A useful diagnostic is

$$\det \mathsf{M}_j(k). \tag{633}$$

A vanishing determinant may represent physical mode loss, dissipation, or projection into untracked sectors.

It must be explained rather than ignored.

A large condition number,

$$\kappa\left(\mathsf{M}_j\right) \gg 1, \tag{634}$$

indicates extreme sensitivity to initial data and numerical error.

107 Combined Transfer Across Both Reversals

The complete linear transfer from the first-leg outer input to the third-leg central output is

$$\boldsymbol{X}_{\mathrm{C}}^{+} = \mathsf{M}_{\mathrm{C}}\mathsf{U}_2\mathsf{M}_{\mathrm{O}}\boldsymbol{X}_{\mathrm{O}}^{-} + \boldsymbol{N}_{\mathrm{tot}}, \tag{635}$$

where

$$\boldsymbol{N}_{\mathrm{tot}} = \mathsf{M}_{\mathrm{C}}\mathsf{U}_2\boldsymbol{N}_{\mathrm{O}} + \mathsf{M}_{\mathrm{C}}\boldsymbol{N}_2 + \boldsymbol{N}_{\mathrm{C}}. \tag{636}$$

Define the total transfer matrix

$$\mathsf{M}_{123} = \mathsf{M}_{\mathrm{C}}\mathsf{U}_2\mathsf{M}_{\mathrm{O}}. \tag{637}$$

Then

$$\boxed{\boldsymbol{X}_{\mathrm{C}}^{+} = \mathsf{M}_{123}(k)\boldsymbol{X}_{\mathrm{O}}^{-} + \boldsymbol{N}_{\mathrm{tot}}(k).} \tag{638}$$

The observable output depends on the ordered product.

In general,

$$\mathsf{M}_{\mathrm{C}}\mathsf{U}_2\mathsf{M}_{\mathrm{O}} \neq \mathsf{M}_{\mathrm{O}}\mathsf{U}_2\mathsf{M}_{\mathrm{C}}. \tag{639}$$

The sequence of the reversals is physically meaningful.

108 Complete Background Curvature Budget

For standard classical background evolution with fixed k, the complete curvature transfer from the first resolved post-singularity boundary to the start of the third leg is

$$T_k^{(\mathrm{res}\to 3)} = \exp\left\{2\int_{N_{\mathrm{res}}}^{N_{\mathrm{C}}^{+}} \left[\epsilon_H(N) - 1\right] \mathrm{d}N\right\}. \tag{640}$$

This may be decomposed as

$$T_k^{(\mathrm{res}\to 3)} = T_k^{(1)} T_k^{(\mathrm{O})} T_k^{(2)} T_k^{(\mathrm{C})}. \tag{641}$$

Therefore,

$$\boxed{\left|\Omega_{k,\mathrm{C}}^{+}\right| = T_k^{(1)} T_k^{(\mathrm{O})} T_k^{(2)} T_k^{(\mathrm{C})} \left|\Omega_{k,\mathrm{res}}\right|.} \tag{642}$$

In a smooth thin-transition limit,

$$T_k^{(\mathrm{O})} \approx 1, \qquad T_k^{(\mathrm{C})} \approx 1, \tag{643}$$

so the dominant background change occurs during the extended legs.

The final curvature condition is

$$\left|\Omega_{k,\mathrm{C}}^{+}\right| < \Omega_{k,\max}^{(3)}. \tag{644}$$

This condition must be satisfied by the same background used in the horizon calculation.

109 Matching Toy Models

The first computational matching study should contain four increasingly complete models.

109.1 Model A: continuous adiabatic matching

Assume:

$$[a]_j = 0, \qquad [H]_j = 0, \qquad [\mathcal{R}]_j = 0, \qquad \boldsymbol{S}_j = 0. \tag{645}$$

This provides the simplest no-shell benchmark.

109.2 Model B: entropy conversion

Permit

$$\boldsymbol{S}_j \neq 0, \qquad \boldsymbol{T}^{(j)}_{\mathcal{R}S} \neq 0. \tag{646}$$

This tests whether the transitions can generate an acceptable curvature spectrum from inherited sector differences.

109.3 Model C: inherited scaffold conversion

Include

$$T^{\mathrm{C}}_{\mathcal{R}\mathcal{W}} \neq 0. \tag{647}$$

This tests whether a coarse-grained scaffold-correlated retention field can seed third-leg curvature without excessive anisotropy.

109.4 Model D: full finite-width multisector transition

Solve the coupled background and perturbation equations across both finite intervals.

The model should include:

- time-dependent sector energy transfer;
- time-dependent kinetic and gradient matrices;
- nonadiabatic pressure;
- anisotropic stress;

- radiation and matter production;
- scaffold-memory transfer; and
- scalar, vector, and tensor outputs.

Model D is the first level capable of supporting a complete candidate realization.

110 Central Boundary and Thermal Initial States

The central reversal supplies the earliest resolved third-leg boundary state,

$$\mathcal{I}_{\mathrm{C}}^{+}. \tag{648}$$

This state contains the background, sector, and perturbation data required to continue the theory beyond the reversal. A schematic minimum is

$$\begin{aligned}\mathcal{I}_{\mathrm{C}}^{+} = \{&a_{\mathrm{C}}^{+}, H_{\mathrm{C}}^{+}, \dot{H}_{\mathrm{C}}^{+}, \Omega_{k,\mathrm{C}}^{+}, \sigma_{ij,\mathrm{C}}^{+},\\ &\rho_E, \rho_{\mathrm{DM}}, \rho_{\mathrm{ch}}, \rho_{\mathrm{other}},\\ &\mathcal{R}_{\boldsymbol{k}}, S_{A,\boldsymbol{k}}, h_{\lambda,\boldsymbol{k}}, V_{A,\boldsymbol{k}},\\ &\Phi_{\boldsymbol{k}}, \Psi_{\boldsymbol{k}}, \mathcal{W}_{12}, \boldsymbol{\mathcal{P}}_{12}, \mathcal{M}_{ij}^{(12)}\}.\end{aligned} \tag{649}$$

The central boundary state need not yet contain a conventional photon, baryon, neutrino, or stable-matter inventory. Those variables enter the earliest standard thermal state only after the derived handoff,

$$\mathcal{I}_{\mathrm{th}} = \mathcal{H}_{\mathrm{th}}\left[\mathcal{I}_{\mathrm{C}}^{+}\right]. \tag{650}$$

A minimum thermal state includes

$$\begin{aligned}\mathcal{I}_{\mathrm{th}} = \{&a_{\mathrm{th}}, H_{\mathrm{th}}, \Omega_{k,\mathrm{th}}, \sigma_{ij,\mathrm{th}},\\ &\rho_\gamma, \rho_b, \rho_{\mathrm{DM}}, \rho_{\mathrm{ch}}, \rho_{\mathrm{other}},\\ &p_\gamma, p_b, p_{\mathrm{DM}}, p_{\mathrm{ch}}, p_{\mathrm{other}},\\ &\delta_i(\boldsymbol{k}), \theta_i(\boldsymbol{k}), f_i(\boldsymbol{x}, \boldsymbol{p})\}.\end{aligned} \tag{651}$$

The central boundary must satisfy the appropriate background constraint, for example

$$H_{\mathrm{C}}^{+\,2} = \frac{8\pi G}{3}\rho_{\mathrm{tot,C}}^{+} - \frac{kc^2}{a_{\mathrm{C}}^{+\,2}}, \tag{652}$$

along with the perturbative constraints of the derived sector theory. The thermal state must separately satisfy the standard species constraints at the first integration time for which the Einstein–Boltzmann description is valid.

The required qualitative conditions are therefore staged:

$$\boxed{\begin{aligned} &H^+_{\mathrm{C}} > 0, \qquad \left|\Omega^+_{k,\mathrm{C}}\right| < \infty, \qquad \Omega^+_{\sigma,\mathrm{C}} < \infty, \\ &H_{\mathrm{th}} > 0, \qquad \rho_{\gamma,\mathrm{th}} > 0, \qquad \beta_{\mathrm{iso}}(k) < \beta_{\mathrm{iso,max}}(k), \\ &\mathcal{P}_{\mathcal{R}}(k) \text{ lies in the observationally viable region.} \end{aligned}} \tag{653}$$

Neither state is free input. The central boundary is the output of the first two legs and both reversals; the thermal state is the output of a physical handoff that Paper II requires but does not yet derive.

111 Matching-Realization Rejection Criteria

The tested matching realization is rejected or requires a new version if any of the following is unavoidable. Architecture-level rejection requires showing that the conflict follows from the defining transition postulates rather than from the selected matching law.

1. The induced metric becomes discontinuous without a defined surface geometry.
2. The Hubble rate jumps without a corresponding surface stress or modified junction law.
3. The homogeneous curvature fraction is assigned an arbitrary instantaneous jump despite continuous a, H, and fixed k.
4. The total stress–energy tensor is not conserved across either transition.
5. The outer and central reversals are represented by one identical unconstrained transfer matrix.
6. The perturbation matching depends on an unspecified transition hypersurface.
7. The transfer matrix diverges for observable modes.
8. The kinetic matrix develops a ghost.
9. The gradient matrix produces an uncontrolled instability.
10. The transition generates excessive tensor, vector, or anisotropic power.
11. The derived central transfer places the inherited common state primarily in isocurvature rather than the observed adiabatic mode.
12. The output scalar spectrum requires a separately chosen transfer function at every wavenumber.
13. The correlation prepared during the first leg is erased at the outer or central transition.

14. The second-leg propagator produces singular growth.
15. The third-leg initial state violates the Friedmann or perturbative constraint equations.
16. The background used for curvature matching differs from the background used for the horizon calculation.
17. Near-flatness is obtained only by introducing an unmotivated surface layer.
18. No derived thermal handoff produces a viable radiation-rich hot state.

112 Formal Summary of Reversal Matching

The directional reversals are

$$\boxed{\dot{R}_E : + \to - \quad \text{and} \quad - \to +,} \tag{654}$$

while the baseline metric condition is

$$\boxed{H > 0 \text{ through both transitions.}} \tag{655}$$

For a no-shell thin transition,

$$\boxed{[h_{ab}]_j = 0, \qquad [K_{ab}]_j = 0.} \tag{656}$$

For the homogeneous background, this implies

$$\boxed{[a]_j = 0, \qquad [H]_j = 0.} \tag{657}$$

With fixed k,

$$\boxed{[\Omega_k]_j = 0} \tag{658}$$

in the regular thin limit.

The finite-width perturbation map is

$$\boxed{\boldsymbol{X}_j^+ = \mathsf{M}_j(k)\boldsymbol{X}_j^- + \boldsymbol{N}_j(k).} \tag{659}$$

The full two-reversal transfer is

$$\boxed{\boldsymbol{X}_\mathrm{C}^+ = \mathsf{M}_\mathrm{C}\mathsf{U}_2\mathsf{M}_\mathrm{O}\boldsymbol{X}_\mathrm{O}^- + \boldsymbol{N}_\mathrm{tot}.} \tag{660}$$

The complete background curvature transfer is

$$\boxed{\left|\Omega^{+}_{k,\mathrm{C}}\right| = T^{(1)}_{k} T^{(\mathrm{O})}_{k} T^{(2)}_{k} T^{(\mathrm{C})}_{k} \left|\Omega_{k,\mathrm{res}}\right|.} \tag{661}$$

The third-leg scalar curvature output is

$$\boxed{\mathcal{R}^{+}_{\mathrm{C}} = T^{\mathrm{C}}_{\mathcal{R}\mathcal{R}} \mathcal{R}^{-}_{\mathrm{C}} + \boldsymbol{T}^{\mathrm{C}}_{\mathcal{R}S} \boldsymbol{S}^{-}_{\mathrm{C}} + T^{\mathrm{C}}_{\mathcal{R}\mathcal{W}} \mathcal{W}_{12} + N^{\mathrm{C}}_{\mathcal{R}}.} \tag{662}$$

The required endpoint is

$$\boxed{\begin{aligned} &\text{regular background} + \text{near-flat geometry} + \text{small shear} \\ &+ \text{stable perturbations} + \text{radiation-rich third-leg state.} \end{aligned}} \tag{663}$$

The decisive transition question is therefore

$$\boxed{\begin{aligned} &\text{Can both sectoral reversals preserve geometric regularity while converting the} \\ &\text{inherited two-pass state into acceptable third-leg background and perturbation data?} \end{aligned}} \tag{664}$$

113 Near-Flatness Without Identifying the First Leg as Inflation

The purpose of this section is to determine whether the Triple Bounce history can account for observed near-flatness without simply renaming the first outward *Electro* leg as inflation.

The distinction is physical and mathematical.

The first leg is defined by

$$\dot{R}^{(1)}_{E} > 0. \tag{665}$$

Inflation is defined by accelerated metric expansion,

$$\ddot{a} > 0, \tag{666}$$

or equivalently,

$$\epsilon_H < 1. \tag{667}$$

These conditions are not equivalent:

$$\boxed{\dot{R}_E^{(1)} > 0 \quad \not\Rightarrow \quad \epsilon_H^{(1)} < 1.} \tag{668}$$

The Triple Bounce flatness problem must therefore be addressed through one or more of the following:

I. balanced singular-opening boundary conditions;

II. curvature preservation across the hidden history;

III. limited intervals of accelerated metric expansion;

IV. a derived nonstandard curvature-transfer mechanism;

V. a transition mechanism that changes the effective geometric state; or

VI. a combination of these effects.

The model may not claim curvature suppression solely from the directional motion of *Electro*.

The full question is

$$\boxed{\begin{array}{l}\text{Can a naturally generated near-flat state survive the complete three-leg history}\\ \text{and remain compatible with the background used for the horizon candidate resolution?}\end{array}} \tag{669}$$

114 Complete Curvature Transfer to the Present

For a classical Friedmann–Lemaître–Robertson–Walker background with fixed spatial-curvature index k, the curvature fraction is

$$\Omega_k = -\frac{kc^2}{a^2H^2}. \tag{670}$$

Its logarithmic evolution is

$$\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} = 2\left(\epsilon_H - 1\right), \tag{671}$$

where

$$\epsilon_H = -\frac{\dot{H}}{H^2}. \tag{672}$$

The transfer from the first resolved post-singularity spacetime boundary to the present is

$$|\Omega_{k,0}| = T_k^{(\mathrm{res}\to 0)} |\Omega_{k,\mathrm{res}}|, \tag{673}$$

with

$$T_k^{(\mathrm{res}\to 0)} = \exp\left\{ 2\int_{N_{\mathrm{res}}}^{N_0} [\epsilon_H(N) - 1]\,\mathrm{d}N \right\}. \tag{674}$$

The transfer may be decomposed as

$$T_k^{(\mathrm{res}\to 0)} = T_k^{(1)} T_k^{(\mathrm{O})} T_k^{(2)} T_k^{(\mathrm{C})} T_k^{(3)}. \tag{675}$$

Thus,

$$\boxed{|\Omega_{k,0}| = T_k^{(1)} T_k^{(\mathrm{O})} T_k^{(2)} T_k^{(\mathrm{C})} T_k^{(3)} |\Omega_{k,\mathrm{res}}|.} \tag{676}$$

The present observational requirement is represented by

$$|\Omega_{k,0}| < \Omega_{k,\mathrm{obs}}^{\max}. \tag{677}$$

Therefore, the resolved boundary curvature must satisfy

$$\boxed{|\Omega_{k,\mathrm{res}}| < \frac{\Omega_{k,\mathrm{obs}}^{\max}}{T_k^{(1)} T_k^{(\mathrm{O})} T_k^{(2)} T_k^{(\mathrm{C})} T_k^{(3)}}.} \tag{678}$$

This equation exposes the total flatness burden.

If the total transfer factor is large, the boundary curvature must be correspondingly smaller.

115 A Conditional No-Suppression Result

A useful constraint follows directly from the standard curvature evolution equation.

Proposition 115.1 (No suppression under everywhere-decelerating expansion)**.** *Assume that the classical Triple Bounce background satisfies:*

(i) $H > 0$ *throughout the relevant interval;*

(ii) the spatial-curvature index k *remains fixed;*

(iii) the standard Friedmann–Lemaître–Robertson–Walker curvature relation remains valid; and

(iv) $\epsilon_H \geq 1$ *throughout the interval.*

Then the magnitude of the curvature fraction cannot decrease during that interval.

Proof. From

$$\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} = 2\left(\epsilon_H - 1\right), \tag{679}$$

and

$$\epsilon_H - 1 \geq 0, \tag{680}$$

it follows that

$$\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} \geq 0. \tag{681}$$

Because $H > 0$, the e-fold variable $N = \ln a$ increases with time. Therefore,

$$|\Omega_k(t_b)| \geq |\Omega_k(t_a)| \tag{682}$$

for $t_b > t_a$ within the specified interval. □

The proposition has an immediate consequence.

If the first leg, outer transition, second leg, and central transition are all globally expanding and everywhere satisfy

$$\epsilon_H \geq 1, \tag{683}$$

then the hidden history does not dynamically suppress the standard curvature fraction.

Near-flatness must then come primarily from:

- a sharply concentrated singular-opening boundary distribution;
- a later accelerated interval;
- modified geometric evolution; or
- a transition that changes the assumptions of the proposition.

The inward direction of the second-leg *Electro* sector does not alter this result.

116 Balanced-Boundary Naturalness

The balanced-boundary proposal states that the pre-singularity state preferentially maps into a nearly flat classical geometry.

Let

$$\boldsymbol{\lambda}_{\rm pre} = (\epsilon_{\rm asym}, \alpha_1, \alpha_2, \ldots, \alpha_n) \tag{684}$$

denote the parameters characterizing the pre-singularity state.

The boundary map is

$$\Omega_{k,\rm res} = \mathcal{K}_{\rm res}\left(\boldsymbol{\lambda}_{\rm pre}\right). \tag{685}$$

Define the successful pre-singularity set

$$\mathcal{B}_{\rm flat}^{\rm pre} = \left\{\boldsymbol{\lambda}_{\rm pre} : \left|\Omega_{k,0}\left(\boldsymbol{\lambda}_{\rm pre}\right)\right| < \Omega_{k,\rm obs}^{\max}\right\}. \tag{686}$$

The total allowed pre-singularity parameter domain is

$$\mathcal{B}_{\rm pre}. \tag{687}$$

Given a measure $\mu_{\rm pre}$, define the near-flat basin fraction

$$f_{\rm flat} = \frac{\mu_{\rm pre}\left(\mathcal{B}_{\rm flat}^{\rm pre}\right)}{\mu_{\rm pre}\left(\mathcal{B}_{\rm pre}\right)}. \tag{688}$$

A broad natural basin corresponds to

$$f_{\rm flat} \not\ll 1. \tag{689}$$

A sharply tuned basin corresponds to

$$f_{\rm flat} \ll 1. \tag{690}$$

A logarithmic tuning diagnostic is

$$\mathfrak{F}_{\rm flat} = -\ln f_{\rm flat}. \tag{691}$$

Larger values indicate a smaller successful basin.

Limitation 116.1. The numerical value of a naturalness fraction depends on the measure chosen over pre-singularity states. The framework must therefore report the measure and test whether its conclusions remain stable under reasonable alternatives.

117 Symmetry Protection and Curvature Concentration

Suppose the balanced state is invariant under a transformation

$$\mathcal{G} : \Psi_0 \longrightarrow \Psi_0. \tag{692}$$

Let the asymmetry variable transform as

$$\epsilon_{\text{asym}} \longrightarrow -\epsilon_{\text{asym}}. \tag{693}$$

If the resolved boundary curvature is invariant under this transformation, then

$$\Omega_{k,\text{res}}\left(\epsilon_{\text{asym}}\right) = \Omega_{k,\text{res}}\left(-\epsilon_{\text{asym}}\right). \tag{694}$$

The leading odd response vanishes:

$$K_1 = 0. \tag{695}$$

The expansion becomes

$$\Omega_{k,\text{res}} = K_0 + K_2\epsilon_{\text{asym}}^2 + K_4\epsilon_{\text{asym}}^4 + \cdots . \tag{696}$$

If the exact balanced mode additionally satisfies

$$K_0 = 0, \tag{697}$$

then

$$\Omega_{k,\text{res}} \approx K_2\epsilon_{\text{asym}}^2. \tag{698}$$

For an asymmetry distribution with variance

$$\sigma_\epsilon^2 = \left\langle \epsilon_{\text{asym}}^2 \right\rangle, \tag{699}$$

the characteristic resolved boundary curvature is approximately

$$|\Omega_{k,\mathrm{res}}|_{\mathrm{char}} \sim |K_2|\sigma_\epsilon^2. \tag{700}$$

The present curvature is then

$$|\Omega_{k,0}|_{\mathrm{char}} \sim T_k^{(\mathrm{res}\to 0)}|K_2|\sigma_\epsilon^2. \tag{701}$$

The observational condition becomes

$$\sigma_\epsilon^2 < \frac{\Omega_{k,\mathrm{obs}}^{\max}}{T_k^{(\mathrm{res}\to 0)}|K_2|}. \tag{702}$$

This formulation converts the qualitative balanced-state proposal into a constraint on the width of the pre-singularity asymmetry distribution.

118 Second-Leg Curvature Evolution

During the second leg,

$$\dot{R}_E^{(2)} < 0, \tag{703}$$

while the baseline global background satisfies

$$H_2 > 0. \tag{704}$$

The background curvature transfer remains

$$T_k^{(2)} = \exp\left\{2\int_{N_{\mathrm{O}}^+}^{N_{\mathrm{C}}^-} \left[\epsilon_H^{(2)}(N) - 1\right] \mathrm{d}N\right\}. \tag{705}$$

The second leg suppresses curvature only when

$$\int_{N_{\mathrm{O}}^+}^{N_{\mathrm{C}}^-} \left[\epsilon_H^{(2)}(N) - 1\right] \mathrm{d}N < 0. \tag{706}$$

Equivalently, the e-fold-weighted average must satisfy

$$\left\langle \epsilon_H^{(2)} \right\rangle_N < 1. \tag{707}$$

The inward *Electro* trajectory does not by itself create this condition.

Thus,

$$\boxed{\dot{R}_E^{(2)} < 0 \quad \not\Rightarrow \quad T_k^{(2)} < 1.} \tag{708}$$

If the chasing sector dominates the global background during the second leg, it may produce

$$\epsilon_H^{(2)} < 1. \tag{709}$$

In that case, the metric background is accelerated even though the active *Electro* sector moves inward.

This would be a distinctive sectoral configuration:

$$\boxed{\dot{R}_E^{(2)} < 0, \qquad \ddot{a}_2 > 0.} \tag{710}$$

It would suppress curvature through an inflation-like geometric mechanism, but during the inward sectoral leg rather than through an identification of the first leg with inflation.

119 Third-Leg Curvature Growth

The central transition supplies the initial curvature for the observable third leg:

$$\Omega_{k,\mathrm{C}}^{+}. \tag{711}$$

Its evolution to the present is

$$|\Omega_{k,0}| = T_k^{(3)} \left|\Omega_{k,\mathrm{C}}^{+}\right|, \tag{712}$$

where

$$T_k^{(3)} = \exp\left\{2\int_{N_\mathrm{C}^+}^{N_0} \left[\epsilon_H^{(3)}(N) - 1\right] \mathrm{d}N\right\}. \tag{713}$$

During radiation domination,

$$\epsilon_H = 2, \tag{714}$$

and therefore

$$|\Omega_k| \propto a^2. \tag{715}$$

During matter domination,

$$\epsilon_H = \frac{3}{2}, \tag{716}$$

and therefore

$$|\Omega_k| \propto a. \tag{717}$$

During a sufficiently accelerated late-time interval,

$$\epsilon_H < 1, \tag{718}$$

the curvature fraction decreases.

The net third-leg transfer depends on the complete expansion history.

The required central-output curvature is

$$\boxed{\left|\Omega^{+}_{k,\mathrm{C}}\right| < \frac{\Omega^{\max}_{k,\mathrm{obs}}}{T^{(3)}_k}.} \tag{719}$$

Because radiation- and matter-dominated intervals amplify $|\Omega_k|$, the third-leg initial curvature generally must be smaller than the present observational bound.

120 Limited Accelerated Subintervals

The first *Electro* leg need not be identical to inflation in order to contain a period of accelerated metric expansion.

Let

$$\mathcal{I}_{\mathrm{acc}} \subset \mathcal{I}_1 \cup \mathcal{I}_{\mathrm{O}} \cup \mathcal{I}_2 \cup \mathcal{I}_{\mathrm{C}} \tag{720}$$

be the subset over which

$$\epsilon_H < 1. \tag{721}$$

Define the effective curvature-suppression exponent

$$\mathcal{A}_k = 2 \int_{\mathcal{I}_{\mathrm{acc}}} (1 - \epsilon_H)\, \mathrm{d}N. \tag{722}$$

The accelerated subinterval contributes

$$T_{k,\mathrm{acc}} = e^{-\mathcal{A}_k}. \tag{723}$$

The decelerating intervals contribute

$$\mathcal{D}_k = 2\int_{\mathcal{I}_{\mathrm{dec}}} (\epsilon_H - 1)\,\mathrm{d}N, \tag{724}$$

with

$$T_{k,\mathrm{dec}} = e^{\mathcal{D}_k}. \tag{725}$$

The net pre-third-leg transfer is

$$T_k^{(\mathrm{pre3})} = \exp\left(\mathcal{D}_k - \mathcal{A}_k\right). \tag{726}$$

Net curvature suppression occurs when

$$\mathcal{A}_k > \mathcal{D}_k. \tag{727}$$

This formulation allows a mixed history.

The first leg as a whole remains a broader *Electro* construction phase, while a limited metric subinterval performs part of the curvature suppression.

The terminology must remain precise:

$$\boxed{\text{The full first leg is not inflation, but a metric subinterval with } \epsilon_H < 1 \text{ is inflationary in the geometric sense.}} \tag{728}$$

121 Noninflationary Preservation

A different possibility is that the boundary map produces sufficiently small curvature and the hidden legs approximately preserve it.

Preservation requires

$$T_k^{(\mathrm{pre3})} \approx 1. \tag{729}$$

This occurs when

$$\int_{N_{\rm res}}^{N_{\rm C}^{+}} (\epsilon_H - 1)\, \mathrm{d}N \approx 0. \tag{730}$$

One simple realization is a near-coasting background,

$$\epsilon_H \approx 1, \tag{731}$$

or equivalently,

$$w_{\rm eff} \approx -\frac{1}{3}. \tag{732}$$

Another possibility is cancellation between curvature-growing and curvature-suppressing intervals:

$$\mathcal{D}_k \approx \mathcal{A}_k. \tag{733}$$

Such cancellation may be dynamical or tuned.

The distinction can be tested through parameter sensitivity.

If a small change in one duration or equation-of-state parameter produces

$$|\Delta \ln |\Omega_{k,0}|| \gg 1, \tag{734}$$

then the preservation is fragile.

A robust preservation mechanism requires a finite neighborhood of parameters for which

$$|\Omega_{k,0}| < \Omega_{k,\rm obs}^{\rm max}. \tag{735}$$

122 Modified Curvature Transfer

If the effective gravitational equations differ from standard general relativity during the hidden history, the curvature evolution may take the form

$$\frac{\mathrm{d} \ln |\Omega_k|}{\mathrm{d}N} = 2\,(\epsilon_H - 1) + \mathcal{S}_k. \tag{736}$$

Here,

$$\mathcal{S}_k = \mathcal{S}_k\,(\mathit{Electro}, \text{scaffold}, \text{chasing sector}, \text{geometry}) \tag{737}$$

is a derived modification.

The integrated transfer becomes

$$T_{k,\mathrm{mod}} = \exp\left\{\int \left[2\left(\epsilon_H - 1\right) + \mathcal{S}_k\right] \mathrm{d}N\right\}. \tag{738}$$

Curvature suppression in a decelerating interval requires

$$\mathcal{S}_k < -2\left(\epsilon_H - 1\right) \tag{739}$$

on average.

A physically acceptable source must satisfy:

1. it follows from a covariant action or equivalent closed field equations;
2. it respects total energy–momentum conservation;
3. it has a well-defined standard-gravity limit;
4. it does not introduce a ghost;
5. it does not introduce a gradient instability;
6. it does not generate unacceptable anisotropy;
7. it is not chosen independently at every time; and
8. it produces testable consequences beyond the desired curvature suppression.

The symbol $\mathcal{S}_k$ therefore denotes unresolved derived dynamics, not an available free function.

123 Transition-Assisted Curvature Control

The finite outer and central transitions may influence curvature through their background equations.

For transition j,

$$T_k^{(j)} = \exp\left\{2\int_{\mathcal{I}_j} \left[\epsilon_H - 1\right] \mathrm{d}N\right\} \tag{740}$$

in the standard classical regime.

If a transition is short in e-fold time,

$$|\Delta N_j| \ll 1, \tag{741}$$

then ordinarily

$$T_k^{(j)} \approx 1. \tag{742}$$

A short smooth reversal therefore cannot provide enormous curvature suppression unless

$$|\epsilon_H - 1| \tag{743}$$

becomes correspondingly large or the standard evolution law breaks down.

Such behavior must be tested for:

- finite energy density;
- finite pressure;
- finite $\dot{H}$;
- perturbative stability;
- absence of a hidden surface layer; and
- compatibility with the hot-state handoff.

The central transition may reorganize perturbative curvature,

$$\mathcal{R}^- \longrightarrow \mathcal{R}^+, \tag{744}$$

without changing the homogeneous spatial-curvature index.

The model must not confuse:

$$\boxed{\Omega_k \neq \mathcal{R}.} \tag{745}$$

The former describes homogeneous background curvature.

The latter describes scalar curvature perturbations.

124 Horizon–Flatness Compatibility

The horizon and flatness calculations depend on the same background history.

The causal integral is

$$\chi_{\mathrm{causal}} = \int \frac{c_{\mathrm{eff}}(t)}{a(t)} \mathrm{d}t, \tag{746}$$

while the curvature transfer is

$$\ln T_k = 2\int (\epsilon_H - 1)\,\mathrm{d}N. \tag{747}$$

A proposed scale-factor history must be inserted into both equations.

Define the joint success region

$$\mathcal{V}_{\rm joint} = \left\{\boldsymbol{\theta} : \mathcal{Q}_{\rm hor}(\boldsymbol{\theta}) \geq 1 \quad \text{and} \quad |\Omega_{k,0}(\boldsymbol{\theta})| < \Omega_{k,\rm obs}^{\max}\right\}. \tag{748}$$

Here, $\boldsymbol{\theta}$ contains the common background and transition parameters.

A viable incomplete-history model requires

$$\mathcal{V}_{\rm joint} \neq \varnothing. \tag{749}$$

A robust model requires more:

$$\mu\left(\mathcal{V}_{\rm joint}\right) > 0 \tag{750}$$

under the stated parameter measure.

A background that meets the flatness requirement but fails the horizon test is insufficient.

A background that meets the horizon requirement but amplifies curvature beyond the observational limit is also insufficient.

124.1 Preparation versus stretching

Inflation addresses horizon and flatness through one accelerated background mechanism.

The Triple Bounce model may divide the functions:

$$\begin{aligned} \text{first and second legs} &\longrightarrow \text{causal preparation and structural memory,} \\ \text{balanced boundary or limited acceleration} &\longrightarrow \text{near-flatness.} \end{aligned} \tag{751}$$

This division is acceptable only if the mechanisms operate on one consistent solution.

125 Curvature, Conformal Time, and Competing Requirements

The same expansion history may affect conformal reach and curvature in different ways.

During a rapidly accelerated interval,

$$|\Omega_k| \tag{752}$$

can be strongly suppressed.

However, the forward comoving distance accumulated over a fixed post-boundary interval,

$$\Delta\chi = \int \frac{c_{\rm eff}}{a}\mathrm{d}t, \tag{753}$$

may approach a finite value as $a(t)$ grows rapidly.

Inflation addresses the horizon problem by stretching an initially connected domain rather than by requiring unlimited future-directed conformal propagation during inflation.

The Triple Bounce proposal may combine:

- direct first-leg causal propagation;
- shared boundary-state correlation;
- second-leg repeated contact;
- scaffold-correlated retention;
- limited accelerated stretching; and
- central conversion into third-leg perturbations.

The contribution of each effect must be separated.

A large causal integral should not be claimed from accelerated expansion without calculation.

Likewise, curvature suppression should not be claimed from a long causal history without evaluating ϵ_H.

126 Parameter Sensitivity

Let

$$\boldsymbol{\theta}_k = (\epsilon_{\rm asym}, K_2, \Delta N_1, \Delta N_2, \Delta N_{\rm acc}, \bar{\epsilon}_1, \bar{\epsilon}_2, \tau_{\rm O}, \tau_{\rm C}, \ldots) \tag{754}$$

denote the parameters controlling the curvature history.

Define the logarithmic sensitivity of the present curvature to parameter θ_a as

$$\Delta_a^{(k)} = \left| \frac{\partial \ln |\Omega_{k,0}|}{\partial \ln |\theta_a|} \right|. \tag{755}$$

A maximum local sensitivity measure is

$$\Delta_{\max}^{(k)} = \max_a \Delta_a^{(k)}. \tag{756}$$

Large values,

$$\Delta_{\max}^{(k)} \gg 1, \tag{757}$$

indicate that small fractional changes in a parameter produce large fractional changes in the curvature output.

A finite basin may still be narrow even when no single local derivative is extremely large.

Therefore, both the local sensitivity and the global basin fraction f_{flat} should be reported.

126.1 Transition sensitivity

The sensitivity to the reversal widths is

$$\Delta_{\tau_{\text{O}}}^{(k)} = \left| \frac{\partial \ln |\Omega_{k,0}|}{\partial \ln \tau_{\text{O}}} \right|, \tag{758}$$

$$\Delta_{\tau_{\text{C}}}^{(k)} = \left| \frac{\partial \ln |\Omega_{k,0}|}{\partial \ln \tau_{\text{C}}} \right|. \tag{759}$$

These quantities test whether near-flatness depends on precisely timed reversals.

127 Scenario Classification

The table separates the physical possibilities that could otherwise be blurred together under the phrase *the first leg explains flatness.*

Table 3: Principal Triple Bounce near-flatness scenarios.

Scenario	**Primary mechanism**	**Metric status**	**Main scientific burden**
Balanced-boundary model	Pre-singularity symmetry concentrates resolved boundary curvature near zero.	Hidden legs may remain noninflationary.	Derive the boundary measure and show a broad near-flat basin.
Preservation model	The total hidden-history curvature transfer remains close to unity.	Approximately coasting or dynamically balanced.	Show that preservation is stable rather than finely canceled.
First-leg accelerated subinterval	A limited part of the first leg suppresses curvature.	That subinterval is inflationary in the metric sense.	Derive its cause, duration, and compatibility with the broader *Electro* leg.
Second-leg accelerated background	The chasing sector accelerates the metric while *Electro* moves inward.	Inflation-like geometry with inward sector motion.	Derive the sector interactions and observational consequences.
Transition-assisted model	Finite reversal dynamics suppress the curvature fraction.	Depends on transition equation of state.	Avoid singular stress, instability, and arbitrary jumps.
Modified-curvature model	A derived source $\mathcal{S}_k$ changes standard curvature evolution.	Nonstandard gravitational regime.	Provide a covariant stable theory and recover general relativity.
Hybrid model	Boundary suppression, preservation, and limited acceleration cooperate.	Mixed.	Prevent excess parameter freedom and retain a distinctive prediction.

128 Numerical Evaluation Program

The first numerical flatness study should proceed through a common background integration.

128.1 Step 1: specify the boundary ensemble

Choose a parameterized distribution

$$P\left(\boldsymbol{\lambda}_{\mathrm{pre}}\right). \tag{760}$$

Map each sample into

$$\{\Omega_{k,\mathrm{res}}, \Omega_{\sigma,\mathrm{res}}, a_{\mathrm{res}}, H_{\mathrm{res}}\}. \tag{761}$$

128.2 Step 2: solve one continuous background

Integrate

$$\{a(t), H(t), \rho_i(t), p_i(t), Q_i(t)\} \tag{762}$$

across:

$$\mathcal{I}_1, \quad \mathcal{I}_{\mathrm{O}}, \quad \mathcal{I}_2, \quad \mathcal{I}_{\mathrm{C}}, \quad \mathcal{I}_3. \tag{763}$$

128.3 Step 3: calculate curvature transfer

Evaluate

$$T_k^{(\mathrm{res}\to 0)} = \exp\left\{2\int_{N_{\mathrm{res}}}^{N_0} (\epsilon_H - 1)\,\mathrm{d}N\right\}. \tag{764}$$

When modified dynamics are used, include the derived source $\mathcal{S}_k$.

128.4 Step 4: calculate the horizon statistic

Using the same $a(t)$, compute

$$\mathcal{Q}_{\mathrm{hor}} = \frac{2\chi_{\mathrm{ret}}^{(123)}}{\Delta\chi_{\mathrm{max}}}. \tag{765}$$

128.5 Step 5: impose joint acceptance

Accept a sample only when

$$\mathcal{Q}_{\rm hor} \geq 1 \tag{766}$$

and

$$|\Omega_{k,0}| < \Omega_{k,\rm obs}^{\rm max}. \tag{767}$$

Later perturbative acceptance must also require:

$$\{\mathcal{P}_{\mathcal{R}}, \beta_{\rm iso}, \mathcal{P}_T, f_{\rm NL}\} \in \mathcal{V}_{\rm obs}. \tag{768}$$

128.6 Step 6: map the successful basin

Report:

- $f_{\rm flat}$;
- the joint horizon–flatness basin fraction;
- $\Delta_{\rm max}^{(k)}$;
- which interval dominates curvature growth;
- which interval dominates curvature suppression; and
- which assumptions control the result.

129 Observational Distinction and Degeneracy

A successful near-flatness mechanism may still be observationally degenerate with inflation.

The measured value of

$$\Omega_{k,0} \tag{769}$$

alone does not reveal how the small value originated.

Potential historical distinctions may arise through correlations among:

- residual curvature;
- large-scale power suppression or enhancement;

- statistical anisotropy;
- isocurvature;
- tensor power;
- transition-generated oscillatory features;
- inherited-web phase correlations; and
- late-time expansion parameters.

A generic joint signature may be written as

$$\mathcal{O}_{\text{flat,TB}} = \mathcal{O}\left(\Omega_{k,0}, \mathcal{P}_{\mathcal{R}}(k), g_*(k), \beta_{\text{iso}}(k), \mathcal{P}_T(k)\right). \tag{770}$$

The model should not treat a curvature value consistent with zero as direct evidence for the Triple Bounce history.

Near-flatness is a recovery requirement.

Historical confirmation requires additional nondegenerate structure.

130 Near-Flatness-Claim Rejection Criteria

The Triple Bounce near-flatness explanation fails or requires revision if any of the following conditions is unavoidable.

1. No explicit singular-opening boundary law or statistical prescription constrains the resolved boundary curvature.
2. The curvature distribution has a small mean but a broad variance.
3. The successful boundary basin has effectively zero measure under every reasonable stated measure.
4. The hidden history is everywhere decelerating under standard Friedmann–Lemaître–Robertson–Walker dynamics, strongly amplifies curvature, and supplies no explanation for the required extremely small boundary value.
5. The model claims that inward or outward *Electro* motion directly suppresses Ω_k without deriving the metric effect.
6. A long interval satisfies $\epsilon_H < 1$, but the framework avoids the term inflation only by semantic reclassification.
7. A limited accelerated interval is introduced with no dynamical cause or transition into and out of it.

8. The second leg amplifies curvature beyond recovery.
9. The reversals produce arbitrary jumps in homogeneous curvature despite continuous a, H, and fixed k.
10. A modified source $\mathcal{S}_k$ is selected specifically to cancel curvature growth but is not derived from a consistent theory.
11. The near-flat solution contains a ghost, gradient instability, divergent pressure, or divergent anisotropic stress.
12. The curvature result uses a different background from the horizon result.
13. The joint horizon–flatness success region is empty.
14. The joint success region exists only at an isolated measure-zero point.
15. The present curvature is acceptable only after extreme cancellation among independently tuned intervals.
16. The central-output curvature violates the third-leg observational requirement after radiation- and matter-era growth.
17. The model confuses homogeneous curvature Ω_k with the scalar perturbation $\mathcal{R}$.
18. No distinctive observable remains after fitting the same background and perturbation data as inflation.

131 Formal Summary of the Near-Flatness Test

The complete curvature transfer is

$$\boxed{|\Omega_{k,0}| = T_k^{(1)} T_k^{(\mathrm{O})} T_k^{(2)} T_k^{(\mathrm{C})} T_k^{(3)} \, |\Omega_{k,\mathrm{res}}| \, .} \tag{771}$$

The standard evolution law is

$$\boxed{\frac{\mathrm{d} \ln |\Omega_k|}{\mathrm{d}N} = 2 \left(\epsilon_H - 1 \right) .} \tag{772}$$

Therefore,

$$\boxed{H > 0 \text{ and } \epsilon_H \geq 1 \quad \Longrightarrow \quad \frac{\mathrm{d}|\Omega_k|}{\mathrm{d}t} \geq 0} \tag{773}$$

under standard fixed-k Friedmann–Lemaître–Robertson–Walker evolution.

The balanced-boundary requirement is

$$\boxed{P\left(\Omega_{k,\mathrm{res}}\right)\ \text{must be concentrated near zero.}} \tag{774}$$

A symmetry-protected response may be

$$\boxed{\Omega_{k,\mathrm{res}} \approx K_2 \epsilon_{\mathrm{asym}}^2,} \tag{775}$$

but it must be derived from the pre-singularity theory.

A limited accelerated contribution is

$$\boxed{T_{k,\mathrm{acc}} = \exp\left[-2\int_{\mathcal{I}_{\mathrm{acc}}} \left(1-\epsilon_H\right) \mathrm{d}N\right].} \tag{776}$$

A modified-curvature alternative is

$$\boxed{\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} = 2\left(\epsilon_H - 1\right) + \mathcal{S}_k,} \tag{777}$$

where $\mathcal{S}_k$ must be derived rather than freely chosen.

The present observational requirement is

$$\boxed{|\Omega_{k,0}| < \Omega_{k,\mathrm{obs}}^{\max}.} \tag{778}$$

The joint horizon–flatness requirement is

$$\boxed{\mathcal{Q}_{\mathrm{hor}} \geq 1 \qquad \text{and} \qquad |\Omega_{k,0}| < \Omega_{k,\mathrm{obs}}^{\max}} \tag{779}$$

for the same background solution.

The decisive conclusion is

$$\boxed{\begin{array}{l}\text{A noninflationary hidden history may preserve near-flatness,}\\ \text{but standard decelerating expansion cannot dynamically suppress}\\ \text{the curvature fraction without a sufficiently flat boundary,}\\ \text{an accelerated subinterval, or derived modified dynamics.}\end{array}} \tag{780}$$

132 Flatness Decision Status

The Triple Bounce framework now possesses a precise flatness decision tree.

1. Derive the pre-singularity curvature distribution.
2. Determine whether the balanced state suppresses the leading curvature response.
3. Propagate the complete boundary ensemble through the first leg.
4. Match the background regularly across the outer reversal.
5. Propagate curvature through the globally expanding second leg.
6. Match the background regularly across the central reversal.
7. Propagate the resulting curvature through the observable third leg.
8. Calculate the full present distribution of $\Omega_{k,0}$.
9. Evaluate the near-flat basin fraction and parameter sensitivity.
10. Use the same solutions to calculate $\mathcal{Q}_{\rm hor}$.
11. Reject solutions that fail either the horizon or flatness condition.
12. Determine whether the surviving solutions require metric acceleration.
13. Classify any accelerated interval honestly as inflationary in the geometric sense.
14. Identify any residual observable that distinguishes the surviving history from standard inflationary preparation.

The incomplete-history proposal has therefore not assumed a flatness solution.

It has defined the conditions under which one could exist.

133 Central Perturbation Matching

The central reversal is the decisive perturbative transition in the Triple Bounce framework.

The first two legs may establish:

- large-scale causal correlation;
- a persistent scaffold;
- counter-directed *Electro* history;
- entropy and relative-density modes;
- tensor and vector excitations;

- directional memory; and
- a nonequilibrium central state.

None of these quantities is directly observable in its precentral form.

The central transition must convert them into the perturbation variables that govern the third-leg radiation, matter, and metric evolution.

The fundamental map is

$$\boxed{\mathcal{T}_{\rm C} : \mathcal{I}_{\rm C}^{-} \longrightarrow \mathcal{I}_{\rm C}^{+}.} \tag{781}$$

The incoming state contains the two-pass inherited history,

$$\mathcal{I}_{\rm C}^{-} = \left\{ g_{\mu\nu}^{-}, T_{\mu\nu}^{-}, \mathit{Electro}^{-}, \mathcal{W}_{12}, \boldsymbol{\mathcal{P}}_{12}, \mathcal{M}_{ij}^{(12)}, \delta X_{A}^{-}, \Pi_{A}^{-}, \dots \right\}. \tag{782}$$

The outgoing state must contain acceptable third-leg perturbations,

$$\mathcal{I}_{\rm C}^{+} = \{ \mathcal{R}_{\boldsymbol{k}}, S_{A,\boldsymbol{k}}, h_{\lambda,\boldsymbol{k}}, V_{A,\boldsymbol{k}}, \delta_i, \theta_i, \Phi, \Psi, \dots \}. \tag{783}$$

A viable transition must satisfy two simultaneous requirements:

$$\boxed{\begin{array}{l}\text{preserve sufficient information from the hidden history,}\\ \text{erase or suppress observationally unacceptable information.}\end{array}} \tag{784}$$

Complete erasure would eliminate the distinctive Triple Bounce history.

Complete preservation would likely retain excessive anisotropy, isocurvature, or multipass structure.

The transition must therefore be selective.

134 The Inherited Two-Pass Perturbation State

The state immediately before the central transition may be decomposed into four perturbative sectors:

$$\boldsymbol{X}_{\rm C}^{-} = \left(\boldsymbol{X}_{\rm geom}^{-}, \boldsymbol{X}_{\rm matter}^{-}, \boldsymbol{X}_{\rm memory}^{-}, \boldsymbol{X}_{\rm transition}^{-} \right)^{\mathsf{T}}. \tag{785}$$

The geometric sector contains

$$\boldsymbol{X}_{\rm geom}^{-} = \left(\mathcal{R}^{-}, \Phi^{-}, \Psi^{-}, h_{\lambda}^{-}, V_{A}^{-} \right). \tag{786}$$

The effective matter and energetic sector contains

$$\boldsymbol{X}^{-}_{\mathrm{matter}} = (\delta\rho_E, \delta\rho_{\mathrm{sc}}, \delta\rho_{\mathrm{ch}}, \delta p_E, \delta p_{\mathrm{sc}}, \delta p_{\mathrm{ch}}, \theta_E, \theta_{\mathrm{sc}}, \theta_{\mathrm{ch}})\,. \tag{787}$$

The memory sector contains

$$\boldsymbol{X}^{-}_{\mathrm{memory}} = \left(\mathcal{W}_{12}, \boldsymbol{\mathcal{P}}_{12}, \mathcal{M}^{(12)}_{ij}\right). \tag{788}$$

The transition sector contains the variables that control the reversal,

$$\boldsymbol{X}^{-}_{\mathrm{transition}} = (\delta\Xi_{\mathrm{C}}, \Pi_{\Xi_{\mathrm{C}}}, \delta\Gamma_{\mathrm{C}}, \ldots)\,, \tag{789}$$

where Ξ_{C} is a provisional central-transition order parameter and Γ_{C} represents one or more effective interaction rates.

The two-pass state may contain nonzero cross-correlations:

$$\left\langle X^{-}_{A}(\boldsymbol{k}) X^{-*}_{B}(\boldsymbol{k}')\right\rangle \neq 0 \tag{790}$$

for variables belonging to different sectors.

These cross-correlations are part of the inherited history.

They must be included in the outgoing covariance matrix.

135 Effective Multisector Perturbation Action

For the perturbative action in this section, natural units

$$c = \hbar = 1 \tag{791}$$

are used.

Let

$$\boldsymbol{q} = (q_1, q_2, \ldots, q_N)^{\mathsf{T}} \tag{792}$$

contain the independent scalar perturbation variables after the constraint variables have been eliminated.

A general quadratic action is

$$S^{(2)}_{\mathrm{s}} = \frac{1}{2}\int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \left[\dot{\boldsymbol{q}}^{\mathsf{T}}\mathsf{K}\dot{\boldsymbol{q}} - \frac{1}{a^2}\partial_i \boldsymbol{q}^{\mathsf{T}}\mathsf{G}\partial_i \boldsymbol{q} - \boldsymbol{q}^{\mathsf{T}}\mathsf{M}^2\boldsymbol{q} + 2\dot{\boldsymbol{q}}^{\mathsf{T}}\mathsf{B}\boldsymbol{q}\right]. \tag{793}$$

Here:

- K is the kinetic matrix;
- G is the gradient matrix;
- M^2 is the effective mass matrix; and
- B describes derivative mixing.

All matrices may depend on

$$(t, k, \mathit{Electro}, \mathcal{W}_{12}, \Xi_{\mathrm{C}}, \rho_i, p_i)\,. \tag{794}$$

The canonical momentum is

$$\boldsymbol{\Pi} = \frac{\partial \mathcal{L}^{(2)}}{\partial \dot{\boldsymbol{q}}} = a^3 \left(\mathsf{K}\dot{\boldsymbol{q}} + \mathsf{B}\boldsymbol{q}\right). \tag{795}$$

The phase-space state is

$$\boldsymbol{Y} = \begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{\Pi} \end{pmatrix}. \tag{796}$$

Its linear evolution is

$$\frac{\mathrm{d}\boldsymbol{Y}}{\mathrm{d}t} = \mathsf{A}(t,k)\boldsymbol{Y}. \tag{797}$$

The central transfer matrix is the corresponding time-ordered evolution operator:

$$\mathsf{M}_{\mathrm{C}}(k) = \mathcal{T} \exp \left[\int_{t_{\mathrm{C}}^{-}}^{t_{\mathrm{C}}^{+}} \mathsf{A}(t,k)\mathrm{d}t \right]. \tag{798}$$

This construction ensures that the transfer matrix is derived from one transition theory rather than independently selected at each scale.

136 Canonical Normalization and Regularity

The kinetic matrix must be positive definite:

$$\lambda_a\left(\mathsf{K}\right) > 0. \tag{799}$$

A canonical basis may be introduced through a matrix L satisfying

$$\mathsf{K} = \mathsf{L}^{\mathsf{T}}\mathsf{L}. \tag{800}$$

Define canonically normalized variables

$$\boldsymbol{v} = a^{3/2}\mathsf{L}\boldsymbol{q}. \tag{801}$$

The mode equation takes the schematic form

$$\boldsymbol{v}_{\boldsymbol{k}}'' + \left[k^2\mathsf{C}_s^2 + \Omega^2(\eta, k)\right]\boldsymbol{v}_{\boldsymbol{k}} = 0, \tag{802}$$

where C_s^2 is the matrix of effective propagation speeds and Ω^2 contains background, mass, and mixing terms.

Regularity requires:

$$\|\mathsf{K}\| < \infty, \qquad \left\|\mathsf{K}^{-1}\right\| < \infty, \qquad \left\|\Omega^2\right\| < \infty \tag{803}$$

through the central transition.

If an eigenvalue of K passes through zero, the quadratic description becomes strongly coupled or singular.

Such a crossing requires a nonlinear or alternative description.

It cannot be ignored within the linear transfer calculation.

137 Selection of the Central Transition Hypersurface

Perturbation matching depends on the physical hypersurface defining the transition.

Let the central transition be triggered when a scalar condition

$$F\left(x^\mu, X_A\right) = 0 \tag{804}$$

is satisfied.

The transition hypersurface is

$$\Sigma_{\mathrm{C}} : F = 0. \tag{805}$$

Candidate physical choices include:

- uniform total density;

- uniform *Electro* compression;
- constant transition-order parameter;
- constant interaction rate;
- constant effective temperature;
- constant central potential; or
- another invariant condition derived from the microphysics.

A perturbation in the transition time is

$$\delta t_{\rm C} = -\frac{\delta F}{\dot{F}}. \tag{806}$$

Spatial variation in $\delta t_{\rm C}$ can itself generate curvature.

For example, a transition occurring at slightly different times in different regions may produce

$$\Delta\mathcal{R}_{\rm mod} \sim H_{\rm C}\delta t_{\rm C}. \tag{807}$$

This contribution is physical only if F is physically defined.

Selecting the transition surface after inspecting the desired spectrum would introduce hidden parameter freedom.

138 Gauge-Invariant Matching at the Central Transition

The curvature perturbation on uniform-total-density hypersurfaces is

$$\zeta = -\psi - H\frac{\delta\rho}{\dot{\rho}}. \tag{808}$$

The comoving curvature perturbation is

$$\mathcal{R} = \psi - \frac{H}{\rho+p}\delta q, \tag{809}$$

up to the sign convention used for the scalar momentum potential δq.

For a regular adiabatic transition on a uniform-density hypersurface,

$$[\zeta]_{\rm C} \approx 0 \tag{810}$$

on sufficiently large scales.

The Triple Bounce central transition may be nonadiabatic, so the more general relation is

$$\zeta^{+} = \zeta^{-} + \Delta\zeta_{\rm nad} + \Delta\zeta_{\rm mem} + \Delta\zeta_{\rm mod}, \tag{811}$$

where:

- $\Delta\zeta_{\rm nad}$ is generated by nonadiabatic pressure;
- $\Delta\zeta_{\rm mem}$ is generated by inherited scaffold conversion; and
- $\Delta\zeta_{\rm mod}$ is generated by spatial modulation of the transition hypersurface.

The corresponding comoving curvature transfer is

$$\mathcal{R}^{+} = T^{\rm C}_{\mathcal{R}\mathcal{R}}\mathcal{R}^{-} + \sum_{A} T^{\rm C}_{\mathcal{R}S_A} S_A^{-} + T^{\rm C}_{\mathcal{R}\mathcal{W}}\mathcal{W}_{12} + T^{\rm C}_{\mathcal{R}\Xi}\delta\Xi_{\rm C} + N^{\rm C}_{\mathcal{R}}. \tag{812}$$

Every coefficient must follow from the common central-transition theory.

139 Adiabatic and Entropy Bases

A useful effective realization treats the central background as a trajectory through a multisector state space.

Let the background variables be

$$\boldsymbol{\varphi} = \left(\varphi^{1}, \varphi^{2}, \ldots, \varphi^{N}\right). \tag{813}$$

The state-space speed is

$$\dot{\sigma} = \sqrt{G_{AB}\dot{\varphi}^{A}\dot{\varphi}^{B}}, \tag{814}$$

where G_{AB} is an effective field-space metric.

The adiabatic basis vector is

$$e_{\sigma}^{A} = \frac{\dot{\varphi}^{A}}{\dot{\sigma}}. \tag{815}$$

Entropy basis vectors satisfy

$$G_{AB}e_{\sigma}^{A}e_{s_I}^{B} = 0. \tag{816}$$

The perturbations decompose as

$$Q^A = e^A_\sigma Q_\sigma + \sum_I e^A_{s_I} Q_{s_I}. \tag{817}$$

The curvature perturbation is related schematically to the adiabatic mode by

$$\mathcal{R} = \frac{H}{\dot{\sigma}} Q_\sigma. \tag{818}$$

A turning background trajectory causes entropy-to-curvature conversion:

$$\dot{\mathcal{R}} \propto \omega_I Q_{s_I}, \tag{819}$$

where ω_I is the turn rate toward entropy direction I.

The central reversal is expected to involve strong trajectory bending.

Thus,

$$\omega_I \neq 0 \tag{820}$$

may be a primary mechanism converting the two-pass entropy structure into third-leg curvature.

This field-space construction is illustrative.

A later theory may realize the relevant degrees of freedom through fluids, kinetic distributions, vector fields, or geometric variables rather than canonical scalar fields.

140 Transfer Functions in the Adiabatic–Entropy Basis

On large scales, a two-sector transfer may be written as

$$\begin{pmatrix} \mathcal{R} \\ S \end{pmatrix}^+_{\mathrm{C}} = \begin{pmatrix} T_{\mathcal{R}\mathcal{R}} & T_{\mathcal{R}S} \\ T_{S\mathcal{R}} & T_{SS} \end{pmatrix}_{\mathrm{C}} \begin{pmatrix} \mathcal{R} \\ S \end{pmatrix}^-_{\mathrm{C}} + \begin{pmatrix} N_{\mathcal{R}} \\ N_S \end{pmatrix}_{\mathrm{C}}. \tag{821}$$

In a larger multisector system,

$$\begin{pmatrix} \mathcal{R}^+ \\ \boldsymbol{S}^+ \end{pmatrix} = \begin{pmatrix} T_{\mathcal{R}\mathcal{R}} & \boldsymbol{T}_{\mathcal{R}S} \\ \boldsymbol{T}_{S\mathcal{R}} & \mathsf{T}_{SS} \end{pmatrix} \begin{pmatrix} \mathcal{R}^- \\ \boldsymbol{S}^- \end{pmatrix} + \begin{pmatrix} N_{\mathcal{R}} \\ \boldsymbol{N}_S \end{pmatrix}. \tag{822}$$

The final curvature spectrum is

$$
\begin{aligned}
\mathcal{P}_{\mathcal{R}}^{+} = {} & \left|T_{\mathcal{R}\mathcal{R}}\right|^{2}\mathcal{P}_{\mathcal{R}}^{-} + \boldsymbol{T}_{\mathcal{R}S}\mathsf{P}_{SS}^{-}\boldsymbol{T}_{\mathcal{R}S}^{\dagger} \\
& + 2\,\mathrm{Re}\left[T_{\mathcal{R}\mathcal{R}}\boldsymbol{T}_{\mathcal{R}S}^{\dagger}\boldsymbol{\mathcal{P}}_{\mathcal{R}S}^{-}\right] + \mathcal{P}_{N_{\mathcal{R}}}.
\end{aligned} \tag{823}
$$

The outgoing entropy covariance is

$$
\begin{aligned}
\mathsf{P}_{SS}^{+} = {} & \boldsymbol{T}_{S\mathcal{R}}\mathcal{P}_{\mathcal{R}}^{-}\boldsymbol{T}_{S\mathcal{R}}^{\dagger} + \mathsf{T}_{SS}\mathsf{P}_{SS}^{-}\mathsf{T}_{SS}^{\dagger} \\
& + \text{cross terms} + \mathsf{P}_{N_S N_S}.
\end{aligned} \tag{824}
$$

A viable central transfer should produce

$$
\mathcal{P}_{\mathcal{R}}^{+} \gg \mathcal{P}_{S,\text{visible}}^{+} \tag{825}
$$

over the observationally constrained scales.

141 Scaffold-Memory Conversion

The inherited scaffold is represented by

$$
\left\{\mathcal{W}_{12}, \boldsymbol{\mathcal{P}}_{12}, \mathcal{M}_{ij}^{(12)}\right\}. \tag{826}
$$

These variables may contribute to scalar, vector, and tensor perturbations.

Decompose the scaffold tensor into trace, longitudinal, vector, and transverse-traceless parts:

$$
\mathcal{M}_{ij}^{(12)} = \frac{1}{3}\delta_{ij}\mathcal{M}_{\text{tr}} + \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\mathcal{M}_{\text{L}} + \partial_{(i}\mathcal{M}_{j)}^{\text{V}} + \mathcal{M}_{ij}^{\text{TT}}. \tag{827}
$$

The scalar trace and longitudinal parts may source

$$
\mathcal{R}, \qquad \Phi, \qquad \Psi. \tag{828}
$$

The transverse vector component may source

$$
V_i. \tag{829}
$$

The transverse-traceless component may source

$$h_{ij}^{\mathrm{TT}}. \tag{830}$$

The central conversion may be written as

$$\mathcal{R}^{+} \supset T_{\mathcal{R}\mathcal{W}}\mathcal{W}_{12} + T_{\mathcal{R}\mathcal{M}_{\mathrm{L}}}\mathcal{M}_{\mathrm{L}}, \tag{831}$$

$$V_i^{+} \supset T_{V\mathcal{M}}\mathcal{M}_i^{\mathrm{V}}, \tag{832}$$

$$h_{ij}^{+} \supset T_{h\mathcal{M}}\mathcal{M}_{ij}^{\mathrm{TT}}. \tag{833}$$

The scalar conversion must be strong enough to preserve useful large-scale information.

The vector and tensor conversions must be weak enough to satisfy observational limits.

This creates a selective-transfer condition:

$$\boxed{|T_{\mathcal{R}\mathcal{W}}| \text{ may be significant,} \qquad |T_{V\mathcal{M}}|\,, |T_{h\mathcal{M}}| \text{ must remain controlled.}} \tag{834}$$

142 Statistical Homogeneity and Isotropy of the Inherited State

A primordial pathway may contain structure without selecting a unique observable center.

For statistical homogeneity,

$$\langle X(\boldsymbol{x})X(\boldsymbol{y})\rangle = C_X\left(\boldsymbol{x}-\boldsymbol{y}\right). \tag{835}$$

For statistical isotropy,

$$C_X\left(\boldsymbol{x}-\boldsymbol{y}\right) = C_X\left(|\boldsymbol{x}-\boldsymbol{y}|\right). \tag{836}$$

A weak anisotropic correction may be written as

$$\mathcal{P}_X(\boldsymbol{k}) = \mathcal{P}_X^{(0)}(k)\left[1 + g_X(k)\left(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}}\right)^2 + \cdots\right]. \tag{837}$$

The central transfer gives

$$g_{\mathcal{R}}^{+}(k) = \mathcal{G}_{\mathrm{C}}\left[g_{\mathcal{R}}^{-}, g_{\mathcal{W}}, g_{\mathcal{M}}, \mathsf{M}_{\mathrm{C}}\right]. \tag{838}$$

A viable model requires

$$\left| g^{+}_{\mathcal{R}}(k) \right| < g_{\max}(k). \tag{839}$$

The transition may reduce anisotropy through angular averaging, decoherence of small-scale directional modes, or preferential transfer of the isotropic scaffold component.

Any such process must be derived.

It cannot be represented only by deleting the anisotropic terms from the output.

143 Scale Dependence and Central Transfer Kernels

The central transition has a finite duration

$$\Delta t_{\mathrm{C}} = t^{+}_{\mathrm{C}} - t^{-}_{\mathrm{C}}. \tag{840}$$

A characteristic transition scale is

$$k_{\mathrm{C}} \sim \frac{a_{\mathrm{C}}}{c_{\mathrm{eff,C}} \Delta t_{\mathrm{C}}}. \tag{841}$$

Modes satisfying

$$k \ll k_{\mathrm{C}} \tag{842}$$

experience the transition as rapid relative to their oscillation period.

Modes satisfying

$$k \gg k_{\mathrm{C}} \tag{843}$$

may evolve adiabatically through it or experience dispersive damping, depending on the microphysics.

A generic transfer kernel may be expanded as

$$T^{\mathrm{C}}_{AB}(k) = T^{(0)}_{AB} + T^{(2)}_{AB} \left(\frac{k}{k_{\mathrm{C}}} \right)^2 + T^{(4)}_{AB} \left(\frac{k}{k_{\mathrm{C}}} \right)^4 + \cdots \tag{844}$$

when the long-wavelength expansion is regular.

Alternatively, a finite transition may produce oscillatory structure,

$$T^{\mathrm{C}}_{AB}(k) = \overline{T}_{AB}(k) \left[1 + A_{AB}(k) \cos\left(2k\Delta\eta_{\mathrm{C}} + \varphi_{AB} \right) \right]. \tag{845}$$

Such oscillations could become a distinctive historical signature.

Their amplitude and phase must be predicted by the transition theory.

144 Generation of the Scalar Spectrum

The outgoing scalar spectrum is parameterized as

$$\mathcal{P}_{\mathcal{R}}^{+}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1} \left[1 + \Delta_{\mathrm{C}}(k) + \Delta_{\mathrm{mem}}(k)\right]. \tag{846}$$

Here:

- $\Delta_{\mathrm{C}}(k)$ is the central-transition correction; and
- $\Delta_{\mathrm{mem}}(k)$ is the inherited-memory correction.

The spectral index is

$$n_s(k) - 1 = \frac{\mathrm{d}\ln \mathcal{P}_{\mathcal{R}}}{\mathrm{d}\ln k}. \tag{847}$$

The running is

$$\alpha_s = \frac{\mathrm{d}n_s}{\mathrm{d}\ln k}. \tag{848}$$

The central theory must determine:

$$\{A_s, n_s, \alpha_s, \Delta_{\mathrm{C}}, \Delta_{\mathrm{mem}}\}. \tag{849}$$

These quantities cannot all be free fit functions.

A minimally predictive parameterization should use a finite set of transition parameters,

$$\boldsymbol{\theta}_{\mathrm{C}} = \left(\tau_{\mathrm{C}}, \Gamma_{\mathrm{C}}, \omega_{\mathrm{C}}, c_{s,\mathrm{C}}, m_{\mathrm{eff},\mathrm{C}}, \lambda_{\mathrm{mem}}, \ldots\right), \tag{850}$$

from which the complete transfer kernel is calculated.

145 Acoustic Phase Coherence

The photon–baryon fluid later undergoes acoustic oscillation.

For an idealized mode,

$$\Theta_0(k,\eta) + \Phi(k,\eta) = A_k \cos\left[k r_s(\eta)\right] + B_k \sin\left[k r_s(\eta)\right], \tag{851}$$

where:

- Θ_0 is the photon-temperature monopole;
- $r_s(\eta)$ is the sound horizon; and
- A_k and B_k encode the initial phase.

The observed acoustic peak sequence requires a coherent relation among these coefficients across the ensemble of modes (Hu and Dodelson, 2002; Planck Collaboration, 2020a).

Define the phase

$$\varphi_k = \arctan\left(\frac{B_k}{A_k}\right). \tag{852}$$

A phase-coherent initial state has a narrow phase distribution,

$$\mathrm{Var}\left(\varphi_k\right) \ll 1 \tag{853}$$

at fixed k.

A viable central transfer must leave the early third leg dominated by one regular scalar mode.

Let the outgoing perturbation be decomposed as

$$\mathcal{R}^+ = C_{\mathrm{grow}}(k)\mathcal{R}_{\mathrm{grow}} + C_{\mathrm{dec}}(k)\mathcal{R}_{\mathrm{dec}}. \tag{854}$$

Acoustic coherence requires

$$\left|\frac{C_{\mathrm{dec}}(k)}{C_{\mathrm{grow}}(k)}\right| \ll 1 \tag{855}$$

over the scales responsible for the observed acoustic peaks.

A continuous active source operating throughout the later acoustic era could destroy coherence.

Therefore, the principal Triple Bounce sourcing must become effectively passive before standard photon–baryon oscillation is established.

146 Non-Gaussianity Generated at the Central Transition

The central transition is nonlinear and nonequilibrium.

It may generate non-Gaussian perturbations even when the incoming state is approximately Gaussian.

Expand the outgoing curvature perturbation as

$$\mathcal{R}_{\boldsymbol{k}}^{+} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} T^{(1)}\left(\boldsymbol{k}, \boldsymbol{p}\right) X_{\boldsymbol{p}}^{-} + \frac{1}{2} \int \frac{\mathrm{d}^3 p \, \mathrm{d}^3 q}{(2\pi)^6} T^{(2)}\left(\boldsymbol{k}, \boldsymbol{p}, \boldsymbol{q}\right) X_{\boldsymbol{p}}^{-} X_{\boldsymbol{q}}^{-} + \cdots . \tag{856}$$

The quadratic kernel contributes to the bispectrum,

$$\left\langle \mathcal{R}_{\boldsymbol{k}_1}^{+} \mathcal{R}_{\boldsymbol{k}_2}^{+} \mathcal{R}_{\boldsymbol{k}_3}^{+} \right\rangle = (2\pi)^3 \delta_D \left(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3\right) B_{\mathcal{R}}^{+}\left(k_1, k_2, k_3\right). \tag{857}$$

The outgoing bispectrum contains:

$$B_{\mathcal{R}}^{+} = B_{\text{inherited}} + B_{\text{conversion}} + B_{\text{transition}}. \tag{858}$$

A local nonlinear representation is

$$\mathcal{R} = \mathcal{R}_g + \frac{3}{5} f_{\text{NL}}^{\text{local}} \left(\mathcal{R}_g^2 - \left\langle \mathcal{R}_g^2 \right\rangle \right) + \cdots . \tag{859}$$

The transition may also generate equilateral, orthogonal, oscillatory, or anisotropic bispectrum shapes.

The model must predict both the amplitude and configuration dependence.

A small total bispectrum may arise through weak interactions or through cancellation.

Cancellation among large unrelated terms would require a stability and naturalness analysis.

147 Tensor Perturbations from the Central Reversal

The transverse-traceless metric perturbation satisfies

$$h_\lambda'' + 2\mathcal{H} h_\lambda' + c_T^2 k^2 h_\lambda = \frac{2a^2}{M_{\text{Pl}}^2} \Pi_\lambda^{\text{TT}}. \tag{860}$$

The central transition may source tensors through:

- time-dependent background geometry;
- anisotropic stress;

- scaffold tensor memory;
- collisions in the central state;
- phase transitions; and
- nonlinear scalar coupling.

The outgoing tensor mode is

$$h_\lambda^+(k) = T_{hh}^{\mathrm{C}}(k)h_\lambda^-(k) + T_{h\mathcal{M}}^{\mathrm{C}}(k)\mathcal{M}_\lambda^{\mathrm{TT}}(k) + N_{h,\lambda}^{\mathrm{C}}(k). \tag{861}$$

Its power spectrum is

$$\mathcal{P}_T^+(k) = \sum_\lambda \frac{k^3}{2\pi^2}\left\langle \left|h_\lambda^+(k)\right|^2 \right\rangle. \tag{862}$$

The tensor-to-scalar ratio is

$$r_{\mathrm{C}}(k) = \frac{\mathcal{P}_T^+(k)}{\mathcal{P}_{\mathcal{R}}^+(k)}. \tag{863}$$

The central reversal need not satisfy the standard single-field inflationary consistency relation.

This may provide a distinctive test.

However, the tensor spectrum must remain within observational bounds and must not be independently adjusted after the scalar spectrum is fitted.

148 Vector and Vorticity Perturbations

The multipass and junction structure may generate vector perturbations.

Let

$$V_i, \qquad \partial_i V_i = 0 \tag{864}$$

represent a transverse metric or velocity vector mode.

The outgoing vector field is

$$V_i^+ = T_{VV}^{\mathrm{C}} V_i^- + T_{V\mathcal{P}}^{\mathrm{C}} \mathcal{P}_i^{\mathrm{T}} + T_{V\mathcal{M}}^{\mathrm{C}} \mathcal{M}_i^{\mathrm{V}} + N_{V,i}^{\mathrm{C}}. \tag{865}$$

The vorticity is

$$\omega_i = \frac{1}{2}\epsilon_{ijk}\partial_j v_k. \tag{866}$$

The model may permit a small primordial vorticity component.

It must satisfy:

$$\mathcal{P}_V(k) < \mathcal{P}_{V,\max}(k), \tag{867}$$

and

$$\mathcal{P}_\omega(k) < \mathcal{P}_{\omega,\max}(k) \tag{868}$$

over the constrained scales.

A detectable but small correlated vector component could become a distinctive prediction.

An uncontrolled large component would violate early-universe isotropy.

149 Third-Leg Initial Covariance Matrix

The complete linear third-leg perturbation state may be represented by

$$\boldsymbol{Z}_{\boldsymbol{k}}^{+} = \left(\mathcal{R}_{\boldsymbol{k}}, S_{1,\boldsymbol{k}}, \ldots, S_{n,\boldsymbol{k}}, h_{+,\boldsymbol{k}}, h_{\times,\boldsymbol{k}}, V_{1,\boldsymbol{k}}, \ldots\right)^{\mathsf{T}}. \tag{869}$$

Its covariance matrix is

$$\left\langle \boldsymbol{Z}_{\boldsymbol{k}}^{+} \boldsymbol{Z}_{\boldsymbol{k}'}^{+\dagger} \right\rangle = (2\pi)^3 \delta_D \left(\boldsymbol{k} - \boldsymbol{k}'\right) \mathsf{P}_Z^{+}(k). \tag{870}$$

The covariance matrix contains:

$$\mathsf{P}_Z^{+} = \begin{pmatrix} \mathcal{P}_{\mathcal{R}} & \boldsymbol{\mathcal{P}}_{\mathcal{R}S} & \boldsymbol{\mathcal{P}}_{\mathcal{R}h} & \boldsymbol{\mathcal{P}}_{\mathcal{R}V} \\ \boldsymbol{\mathcal{P}}_{S\mathcal{R}} & \mathsf{P}_{SS} & \mathsf{P}_{Sh} & \mathsf{P}_{SV} \\ \boldsymbol{\mathcal{P}}_{h\mathcal{R}} & \mathsf{P}_{hS} & \mathsf{P}_{hh} & \mathsf{P}_{hV} \\ \boldsymbol{\mathcal{P}}_{V\mathcal{R}} & \mathsf{P}_{VS} & \mathsf{P}_{Vh} & \mathsf{P}_{VV} \end{pmatrix}. \tag{871}$$

This covariance matrix, together with higher-order correlation functions, is the correct initial statistical input to the third-leg radiation-transfer calculation.

The model must not specify only

$$\mathcal{P}_{\mathcal{R}}(k) \tag{872}$$

while leaving all cross-correlations undefined.

150 Required Third-Leg Perturbation State

The derived central transfer must yield an early-third-leg state satisfying the following qualitative requirements.

150.1 Scalar amplitude

$$\mathcal{P}_{\mathcal{R}}(k_*) = A_s \tag{873}$$

within the observationally allowed range.

150.2 Spectral shape

$$n_s, \qquad \alpha_s, \qquad \Delta_{\mathrm{C}}(k), \qquad \Delta_{\mathrm{mem}}(k) \tag{874}$$

must remain compatible with the measured primordial spectrum.

150.3 Adiabatic dominance

$$\beta_{\mathrm{iso}}(k) < \beta_{\mathrm{iso,max}}(k). \tag{875}$$

150.4 Small statistical anisotropy

$$|g_{\mathcal{R}}(k)| < g_{\max}(k). \tag{876}$$

150.5 Acceptable non-Gaussianity

$$f_{\mathrm{NL}}^{\mathrm{shape}} \in \mathcal{V}_{f_{\mathrm{NL}}}. \tag{877}$$

150.6 Controlled tensor and vector output

$$r(k) < r_{\max}(k), \qquad \mathcal{P}_V(k) < \mathcal{P}_{V,\max}(k). \tag{878}$$

150.7 Phase coherence

$$\mathrm{Var}\,(\varphi_k) \ll 1. \tag{879}$$

The combined requirement is

$$\boxed{\mathsf{P}_Z^+(k) \in \mathcal{V}_{\rm obs}} \tag{880}$$

over the full observational wavenumber range.

151 Central-Matching Test Models

The central perturbation analysis should proceed through five nested models.

151.1 Model A: adiabatic continuity

Assume

$$\mathcal{R}^+ = \mathcal{R}^-, \qquad S_A^+ = 0, \qquad N_{\mathcal{R}}^{\rm C} = 0. \tag{881}$$

This is the simplest benchmark.

It tests whether the incoming curvature spectrum is already acceptable.

151.2 Model B: entropy-to-curvature conversion

Include

$$\boldsymbol{T}_{\mathcal{R}S} \neq 0. \tag{882}$$

This tests whether precentral relative modes generate the dominant third-leg curvature spectrum.

151.3 Model C: scaffold-memory conversion

Include

$$T_{\mathcal{R}\mathcal{W}} \neq 0. \tag{883}$$

This tests whether inherited geometry can seed scalar structure while remaining statistically isotropic.

151.4 Model D: modulated central transition

Allow

$$\delta t_{\rm C} \neq 0. \tag{884}$$

This tests curvature generation from spatial variation in the transition timing.

151.5 Model E: complete finite-width nonlinear transition

Solve the full coupled background and perturbation evolution through the central interval.

Model E must include:

- the scalar kinetic and gradient matrices;
- entropy-to-curvature conversion;
- scaffold-memory transfer;
- tensor and vector sourcing;
- transition modulation;
- nonlinear kernels;
- radiation and matter production; and
- the complete output covariance matrix.

Only Model E can support a complete physical replacement claim.

152 Numerical Implementation

A numerical central-transition calculation should follow one common background and perturbation system.

152.1 Step 1: construct the background

Solve

$$\{a(t), H(t), \rho_i(t), p_i(t), Q_i(t), R_E(t), \Xi_{\mathrm{C}}(t)\} \tag{885}$$

through

$$t_{\mathrm{C}}^{-} \leq t \leq t_{\mathrm{C}}^{+}. \tag{886}$$

152.2 Step 2: verify stability

Evaluate the eigenvalues of

$$\mathsf{K}, \qquad \mathsf{K}^{-1}\mathsf{G}, \qquad \mathsf{M}^2. \tag{887}$$

Reject ghost or uncontrolled gradient-instability regions.

152.3 Step 3: evolve a basis of perturbations

For each k, evolve independent basis initial conditions

$$\boldsymbol{Y}_A^- \tag{888}$$

through the transition.

The outgoing basis vectors determine the columns of

$$\mathsf{M}_{\mathrm{C}}(k). \tag{889}$$

152.4 Step 4: propagate the covariance

Given the incoming covariance

$$\mathsf{P}_Z^-(k), \tag{890}$$

compute

$$\mathsf{P}_Z^+(k) = \mathsf{M}_{\mathrm{C}}(k)\mathsf{P}_Z^-(k)\mathsf{M}_{\mathrm{C}}^\dagger(k) + \mathsf{P}_{N_{\mathrm{C}}}(k). \tag{891}$$

152.5 Step 5: calculate higher-order correlations

Compute the bispectrum and, where relevant, the trispectrum from the nonlinear transition kernels.

152.6 Step 6: initialize third-leg radiation evolution

Use the derived state

$$\mathcal{I}_{\mathrm{C}}^+ \tag{892}$$

as the input to a third-leg Einstein–Boltzmann solver.

No perturbation variable should be reset by hand after the central calculation.

153 Central-Perturbation-Mechanism Rejection Criteria

The central perturbation hypothesis fails or requires revision if any of the following conditions is unavoidable.

1. No closed perturbative action or equivalent evolution system can be defined through the central transition.
2. The kinetic matrix develops a negative eigenvalue.
3. The kinetic matrix becomes singular over the relevant transition interval.
4. The gradient matrix produces an uncontrolled instability.
5. The transition hypersurface is selected only after the desired spectrum is known.
6. The outgoing curvature perturbation diverges.
7. The scalar amplitude is obtained only by arbitrary normalization of the transfer matrix.
8. The spectral tilt requires an independent free function of wavenumber.
9. The inherited scaffold produces excessive statistical anisotropy.
10. Most inherited power is transferred into visible isocurvature modes.
11. The central transition generates tensor power beyond observational limits.
12. The vector or vorticity component is too large.
13. The decaying scalar mode remains comparable to the growing mode and destroys acoustic phase coherence.
14. The transition acts as a persistent active source during the photon–baryon acoustic era.
15. The bispectrum or trispectrum exceeds observational limits.
16. Acceptable non-Gaussianity occurs only through unstable cancellation among unrelated large terms.
17. The complete output covariance matrix is not positive semidefinite.
18. The derived perturbation state violates the Einstein constraint equations.
19. The background used for perturbation matching differs from the background used for the horizon and flatness calculations.

20. The third-leg perturbations must be manually replaced with standard inflationary initial conditions after the transition.
21. The transfer erases all hidden-history information, leaving no possible distinctive signature.
22. The transfer preserves so much directional memory that statistical homogeneity or isotropy is lost.

154 Formal Summary of Central Perturbation Matching

The inherited precentral state is

$$\boxed{\mathcal{I}_{\mathrm{C}}^{-} = \{\text{geometry}, \text{multisector perturbations}, \text{two-pass memory}, \text{transition variables}\}.} \tag{893}$$

The effective quadratic action is

$$\boxed{S^{(2)} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\left[\dot{\boldsymbol{q}}^{\mathsf{T}}\mathsf{K}\dot{\boldsymbol{q}} - \frac{1}{a^2}\partial_i\boldsymbol{q}^{\mathsf{T}}\mathsf{G}\partial_i\boldsymbol{q} - \boldsymbol{q}^{\mathsf{T}}\mathsf{M}^2\boldsymbol{q} + 2\dot{\boldsymbol{q}}^{\mathsf{T}}\mathsf{B}\boldsymbol{q}\right].} \tag{894}$$

The central linear transfer is

$$\boxed{\boldsymbol{X}_{\mathrm{C}}^{+} = \mathsf{M}_{\mathrm{C}}(k)\boldsymbol{X}_{\mathrm{C}}^{-} + \boldsymbol{N}_{\mathrm{C}}(k).} \tag{895}$$

The outgoing curvature perturbation is

$$\boxed{\mathcal{R}^{+} = T_{\mathcal{R}\mathcal{R}}\mathcal{R}^{-} + \boldsymbol{T}_{\mathcal{R}S}\boldsymbol{S}^{-} + T_{\mathcal{R}\mathcal{W}}\mathcal{W}_{12} + T_{\mathcal{R}\Xi}\delta\Xi_{\mathrm{C}} + N_{\mathcal{R}}.} \tag{896}$$

The outgoing covariance is

$$\boxed{\mathsf{P}_{Z}^{+} = \mathsf{M}_{\mathrm{C}}\mathsf{P}_{Z}^{-}\mathsf{M}_{\mathrm{C}}^{\dagger} + \mathsf{P}_{N_{\mathrm{C}}}.} \tag{897}$$

Acoustic phase coherence requires

$$\boxed{\left|\frac{C_{\mathrm{dec}}(k)}{C_{\mathrm{grow}}(k)}\right| \ll 1, \qquad \mathrm{Var}\left(\varphi_k\right) \ll 1.} \tag{898}$$

The complete observational requirement is

$$\boxed{\{\mathcal{P}_{\mathcal{R}}, \mathsf{P}_{SS}, \mathcal{P}_{T}, \mathcal{P}_{V}, f_{\mathrm{NL}}, g_{\mathcal{R}}, \varphi_k\} \in \mathcal{V}_{\mathrm{obs}}.} \tag{899}$$

The central perturbation question is therefore

$$\boxed{\begin{array}{l}\text{Can one finite, stable central-transition theory convert the} \\ \text{correlated two-pass state into predominantly adiabatic, nearly} \\ \text{scale-invariant, statistically isotropic, phase-coherent} \\ \text{third-leg perturbations while retaining a calculable historical} \\ \text{signature?}\end{array}} \tag{900}$$

155 Perturbation-Transfer Status

The central perturbation problem is now reduced to a defined sequence.

1. Specify the independent precentral degrees of freedom.
2. Construct the quadratic and leading nonlinear perturbation actions.
3. Identify the physical central-transition hypersurface.
4. Verify no-ghost and no-gradient-instability conditions.
5. Construct the adiabatic and entropy bases.
6. Calculate the finite-width central transfer matrix.
7. Calculate scaffold-memory conversion into scalar, vector, and tensor modes.
8. Propagate the complete incoming covariance matrix.
9. Calculate the outgoing scalar spectrum and spectral tilt.
10. Calculate isocurvature and cross-correlations.
11. Calculate tensor and vector spectra.
12. Calculate the bispectrum and higher-order correlations.
13. Verify acoustic phase coherence.
14. Verify statistical homogeneity and isotropy.
15. Initialize the third-leg radiation-transfer equations using the derived output.
16. Reject any model requiring manual replacement of the outgoing perturbations.
17. Identify a residual feature associated with the hidden two-pass history.

The framework has therefore not yet derived the primordial spectrum.

It has specified the complete mathematical object that must be derived.

156 Propagation of the Central Output into the Cosmic Microwave Background

The preceding transition analysis defines the earliest resolved third-leg perturbation state immediately after the central reversal,

$$\mathcal{I}_{\mathrm{C}}^{+}. \tag{901}$$

This symbol denotes boundary data for the third leg. It does not assert that a standard radiation plasma, stable ordinary matter, or the complete late-time species inventory exists instantaneously at the reversal. A derived thermal handoff,

$$\mathcal{H}_{\mathrm{th}} : \mathcal{I}_{\mathrm{C}}^{+} \longrightarrow \mathcal{I}_{\mathrm{th}}, \tag{902}$$

must connect the central output to the earliest state for which a conventional species description and Einstein–Boltzmann evolution are valid. Paper II tests this required handoff without assigning the origin of the opaque early plasma era to the Triple Bounce mechanism.

Once the handoff is defined, the third-leg state must evolve through:

1. the early radiation-dominated plasma;
2. neutrino decoupling;
3. electron–positron annihilation;
4. primordial nucleosynthesis;
5. matter–radiation equality;
6. photon–baryon acoustic evolution;
7. recombination;
8. photon free streaming;
9. reionization;
10. late-time gravitational evolution; and
11. gravitational lensing.

The observed cosmic microwave background therefore tests not only the central transition but the entire third-leg transfer from

$$\mathcal{I}_{\mathrm{C}}^{+} \xrightarrow{\mathcal{H}_{\mathrm{th}}} \mathcal{I}_{\mathrm{th}} \longrightarrow \mathcal{I}_{*} \longrightarrow \mathcal{O}_{\mathrm{CMB}}, \tag{903}$$

where:

- $\mathcal{I}_{\mathrm{th}}$ is the earliest derived state for which the standard thermal species description is valid;
- $\mathcal{I}_*$ is the perturbation state at last scattering; and
- $\mathcal{O}_{\mathrm{CMB}}$ is the observed temperature and polarization data.

A viable Triple Bounce perturbation model must reproduce the measured cosmic microwave background spectra from the derived central output and thermal handoff. It may not replace either step with a standard inflationary spectrum or an independently selected hot-state boundary condition after the transition. If the handoff cannot be derived consistently, the downstream cosmic microwave background calculation is incomplete.

The total prediction can be represented as

$$\boxed{\mathcal{O}_{\mathrm{CMB}}^{\mathrm{TB}} = \mathcal{L}_{\mathrm{obs}} \circ \mathcal{B}_3 \circ \mathcal{H}_{\mathrm{th}} \circ \mathcal{T}_{\mathrm{C}} \left[\mathcal{I}_{\mathrm{C}}^{-}\right],} \tag{904}$$

where:

- $\mathcal{T}_{\mathrm{C}}$ is the central transition map;
- $\mathcal{H}_{\mathrm{th}}$ is the derived handoff from central output to a standard thermal species description;
- $\mathcal{B}_3$ is the subsequent third-leg Einstein–Boltzmann evolution operator; and
- $\mathcal{L}_{\mathrm{obs}}$ is the line-of-sight and observational projection operator.

157 Einstein–Boltzmann Evolution

157.1 Species distribution functions

Each particle species i is described by a phase-space distribution

$$f_i = f_i\left(x^\mu, p^\nu\right). \tag{905}$$

Its evolution is governed by a Boltzmann equation,

$$\frac{\mathrm{D} f_i}{\mathrm{D}\lambda} = \mathcal{C}_i\left[f_j\right], \tag{906}$$

where:

- λ is an affine or phase-space evolution parameter; and
- $\mathcal{C}_i$ is the collision operator.

The stress–energy tensor is obtained from the distribution function,

$$T_i^{\mu\nu} = \int \frac{\mathrm{d}^3 p}{\sqrt{-g}\, p^0} p^\mu p^\nu f_i. \tag{907}$$

The metric perturbations are sourced through the linearized Einstein equations,

$$\delta G_{\mu\nu} = \frac{8\pi G}{c^4} \delta T_{\mu\nu}^{\mathrm{tot}}. \tag{908}$$

157.2 Third-leg species

A minimum third-leg calculation includes:

$$\{\gamma, b, c, \nu, \mathrm{ch}, \mathrm{other}\}, \tag{909}$$

where:

- γ denotes photons;
- b denotes baryonic matter;
- c denotes cold dark matter or the appropriate scaffold component;
- ν denotes neutrinos or other free-streaming relativistic species;
- ch denotes the chasing sector; and
- other denotes any additional derived Triple Bounce sector.

Every additional sector must contribute consistently to:

$$\{H(a), \Phi, \Psi, \delta\rho, \delta p, \theta, \sigma\}. \tag{910}$$

A sector may not influence the background while being omitted from the perturbation equations unless a controlled approximation justifies the omission.

158 Scalar Perturbation Equations in the Third Leg

In conformal Newtonian gauge, the scalar-perturbed metric is

$$\mathrm{d}s^2 = a^2(\eta)\left[-\left(1+2\Psi\right)\mathrm{d}\eta^2 + \left(1-2\Phi\right)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]. \tag{911}$$

For a fluid component i, define the density contrast

$$\delta_i = \frac{\delta\rho_i}{\bar{\rho}_i}, \tag{912}$$

and the velocity divergence

$$\theta_i = \mathrm{i}k_j v_i^j. \tag{913}$$

A generic continuity equation is

$$\begin{aligned}\delta_i' = &-\left(1+w_i\right)\left(\theta_i - 3\Phi'\right)\\ &-3\mathcal{H}\left(c_{s,i}^2 - w_i\right)\delta_i + \mathcal{Q}_{\delta_i},\end{aligned} \tag{914}$$

where $\mathcal{Q}_{\delta_i}$ represents perturbative sector interactions.

The Euler equation is schematically

$$\begin{aligned}\theta_i' = &-\mathcal{H}\left(1-3w_i\right)\theta_i - \frac{w_i'}{1+w_i}\theta_i\\ &+\frac{c_{s,i}^2}{1+w_i}k^2\delta_i - k^2\sigma_i + k^2\Psi + \mathcal{Q}_{\theta_i}.\end{aligned} \tag{915}$$

The interaction terms must obey total energy–momentum conservation.

The scalar Einstein constraints include a generalized Poisson equation,

$$k^2\Phi + 3\mathcal{H}\left(\Phi' + \mathcal{H}\Psi\right) = 4\pi G a^2 \delta\rho_{\mathrm{tot}}, \tag{916}$$

and an anisotropic-stress relation,

$$k^2\left(\Phi - \Psi\right) = 12\pi G a^2\left(\rho + p\right)\sigma_{\mathrm{tot}}, \tag{917}$$

up to sign and normalization conventions.

The central output must satisfy these constraint equations at the initial integration time.

159 Photon Boltzmann Hierarchy

The photon brightness perturbation may be expanded in Legendre multipoles,

$$\Theta\left(\boldsymbol{k}, \mu, \eta\right) = \sum_{\ell=0}^{\infty} (-\mathrm{i})^{\ell} (2\ell+1)\Theta_\ell(k,\eta) P_\ell(\mu), \tag{918}$$

where:

- $\mu = \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}}$; and
- P_ℓ is a Legendre polynomial.

The first few multipoles correspond approximately to:

$$\Theta_0 : \text{temperature monopole}, \tag{919}$$

$$\Theta_1 : \text{velocity or dipole}, \tag{920}$$

$$\Theta_2 : \text{anisotropic stress and polarization source}. \tag{921}$$

The hierarchy has the schematic form

$$\Theta_\ell' = \frac{k}{2\ell+1} \left[\ell\Theta_{\ell-1} - (\ell+1)\Theta_{\ell+1}\right] + \mathcal{G}_\ell + \mathcal{C}_\ell, \tag{922}$$

where:

- $\mathcal{G}_\ell$ contains metric sources; and
- $\mathcal{C}_\ell$ contains Thomson-scattering terms.

Before recombination, rapid Thomson scattering strongly couples photons and baryons.

After recombination, the collision rate drops and the photon hierarchy free streams.

160 Photon–Baryon Tight Coupling

The differential optical depth is

$$\dot{\tau} = a n_e \sigma_T c, \tag{923}$$

where:

- n_e is the free-electron number density; and

- σ_T is the Thomson cross section.

In conformal-time notation, one often uses

$$\tau' = a n_e \sigma_T \tag{924}$$

in units with $c = 1$.

The baryon-loading ratio is

$$R_b = \frac{3\rho_b}{4\rho_\gamma}. \tag{925}$$

The photon–baryon sound speed is

$$c_s^2 = \frac{c^2}{3\left(1 + R_b\right)}. \tag{926}$$

In the tight-coupling regime,

$$\tau' \gg k, \qquad \tau' \gg \mathcal{H}, \tag{927}$$

the photon and baryon velocities satisfy approximately

$$\theta_\gamma \approx \theta_b. \tag{928}$$

A simplified acoustic equation is

$$\left(\Theta_0 + \Phi\right)'' + \frac{R_b'}{1 + R_b}\left(\Theta_0 + \Phi\right)' + k^2 c_s^2 \left(\Theta_0 + \Phi\right) = \mathcal{S}_{\text{grav}}, \tag{929}$$

where $\mathcal{S}_{\text{grav}}$ contains gravitational driving.

The solution has the form

$$\Theta_0 + \Phi = A_k \cos\left(k r_s\right) + B_k \sin\left(k r_s\right) + \text{driven contribution}. \tag{930}$$

The coefficients A_k and B_k are determined by the central-output initial conditions.

They are not independent late-time fitting parameters.

161 Sound Horizon and Acoustic Scale

The comoving sound horizon at conformal time η is

$$r_s(\eta) = \int_{\eta_{\text{th}}}^{\eta} c_s(\tilde{\eta}) \mathrm{d}\tilde{\eta}. \tag{931}$$

Equivalently,

$$r_s(a) = \int_{a_{\text{th}}}^{a} \frac{c_s(a')}{a'^2 H(a')} \mathrm{d}a'. \tag{932}$$

Here η_{th} and a_{th} mark the derived thermal handoff. They need not coincide with the instantaneous central reversal.

At last scattering,

$$r_{s*} = r_s(\eta_*). \tag{933}$$

The transverse comoving distance to last scattering is

$$D_M(z_*). \tag{934}$$

The acoustic angular scale is

$$\theta_* = \frac{r_{s*}}{D_M(z_*)}. \tag{935}$$

The corresponding multipole scale is approximately

$$\ell_A = \pi \frac{D_M(z_*)}{r_{s*}}. \tag{936}$$

The Triple Bounce background must reproduce both:

$$r_{s*} \qquad \text{and} \qquad D_M(z_*), \tag{937}$$

using the same third-leg expansion model.

A successful fit cannot compensate an arbitrary sound horizon with an independently arbitrary distance.

162 Peak Positions and Phase Coherence

In an idealized acoustic system, compression and rarefaction extrema occur near

$$kr_{s*} \approx m\pi, \qquad m = 1, 2, 3, \ldots \tag{938}$$

for a coherent cosine-like adiabatic mode.

The observed peak positions may be represented as

$$\ell_m \approx \ell_A \left(m - \varphi_m\right), \tag{939}$$

where φ_m includes gravitational driving, baryon loading, free-streaming species, and other phase shifts.

A central transition that produces a significant sine-mode component changes the phase.

Define the outgoing acoustic phase

$$\varphi_{\mathrm{TB}}(k) = \mathrm{atan2}\left(B_k, A_k\right), \tag{940}$$

and compare it with the phase $\varphi_{\mathrm{reg}}(k)$ of the regular growing adiabatic mode evolved through the same third-leg background,

$$\Delta\varphi_{\mathrm{ac}}(k) = \varphi_{\mathrm{TB}}(k) - \varphi_{\mathrm{reg}}(k). \tag{941}$$

Acoustic coherence does not require a universally vanishing sine coefficient in every convention. It requires the derived phase residual, decaying-mode fraction, and correlated entropy contribution to remain inside the joint observationally allowed region across the acoustic range, without mode-by-mode retuning.

The peak test therefore constrains:

$$\left\{T_{\mathcal{R}\mathcal{R}}, \boldsymbol{T}_{\mathcal{R}S}, T_{\mathcal{R}\mathcal{W}}, T_{\mathcal{R}\Xi}, C_{\mathrm{dec}}/C_{\mathrm{grow}}\right\}. \tag{942}$$

The central transition cannot generate an arbitrary mixture of growing and decaying modes while retaining the measured peak sequence.

163 Peak Heights and Baryon Loading

Baryons alter the equilibrium point of the photon–baryon oscillator.

This enhances compressional peaks relative to rarefaction peaks.

The peak-height pattern therefore constrains:

- the baryon density;
- the photon density;
- the gravitational potentials;
- the matter–radiation equality epoch;
- free-streaming relativistic species;
- the primordial tilt; and
- the adiabatic–isocurvature mixture.

The baryon-loading parameter at last scattering is

$$R_{b*} = \frac{3\rho_b(\eta_*)}{4\rho_\gamma(\eta_*)}. \tag{943}$$

The matter–radiation equality condition is

$$\rho_m(a_{\rm eq}) = \rho_r(a_{\rm eq}). \tag{944}$$

The associated comoving scale is

$$k_{\rm eq} = \frac{a_{\rm eq} H_{\rm eq}}{c}. \tag{945}$$

A Triple Bounce model must reproduce the peak heights with one derived set of third-leg densities.

It cannot choose one baryon density for nucleosynthesis and another for the cosmic microwave background.

164 Photon Diffusion and the Damping Tail

Before recombination, photons diffuse through the plasma over a finite distance.

This suppresses small-scale anisotropies.

The comoving diffusion scale may be written schematically as

$$k_D^{-2}(\eta) = \int^{\eta} \frac{\mathrm{d}\tilde{\eta}}{6\tau'} \frac{R_b^2 + \frac{16}{15}(1+R_b)}{(1+R_b)^2}. \tag{946}$$

The primordial perturbation is damped approximately by

$$\exp\left[-\frac{k^2}{k_D^2}\right]. \tag{947}$$

The observed high-multipole damping tail constrains:

- the recombination history;
- the free-electron abundance;
- the expansion rate before recombination;
- the helium fraction;
- the primordial spectrum at small scales; and
- additional scattering or energy injection.

An inherited-memory correction that rises strongly at high wavenumber may be excluded by the damping tail even if it is hidden at low multipoles.

165 Visibility Function and Recombination

The optical depth from conformal time η to the present is

$$\tau(\eta) = \int_{\eta}^{\eta_0} a(\tilde{\eta}) n_e(\tilde{\eta}) \sigma_T \mathrm{d}\tilde{\eta}. \tag{948}$$

The visibility function is

$$g(\eta) = -\tau'(\eta) e^{-\tau(\eta)}. \tag{949}$$

It gives the probability density that an observed photon last scattered near η.

The visibility function peaks near recombination and has finite width.

The cosmic microwave background is therefore not emitted from an infinitely thin boundary. The Triple Bounce framework must retain this standard radiative interpretation:

$$\boxed{\text{cosmic microwave background photons are radiation released from a finite last-scattering region, not dark energy and not a physical outer wall.}} \tag{950}$$

A modified third-leg expansion rate changes

$$H(z), \qquad n_e(z), \qquad \tau(z), \qquad g(z), \tag{951}$$

and must therefore be tested through the full recombination calculation.

166 Line-of-Sight Solution

The observed temperature multipole transfer function may be written as

$$\Delta_\ell^T(k) = \int_0^{\eta_0} \mathrm{d}\eta \, S_T(k,\eta) j_\ell \left[k(\eta_0 - \eta)\right], \tag{952}$$

where:

- $S_T(k,\eta)$ is the temperature source function; and
- j_ℓ is a spherical Bessel function.

The source function contains terms associated with:

- the photon monopole;
- the gravitational potential;
- the baryon velocity;
- the quadrupole;
- the early integrated Sachs–Wolfe effect;
- the late integrated Sachs–Wolfe effect; and
- reionization.

Schematically,

$$
\begin{aligned}
S_T \sim g\left(\Theta_0 + \Psi\right) + \frac{\mathrm{d}}{\mathrm{d}\eta}\left(g\frac{v_b}{k}\right) \\
+ e^{-\tau}\left(\Phi' + \Psi'\right) + \text{quadrupole terms.}
\end{aligned} \tag{953}
$$

The polarization transfer functions are

$$
\Delta_\ell^E(k), \qquad \Delta_\ell^B(k). \tag{954}
$$

Scalar perturbations generate E-mode polarization at linear order.

Tensor and vector perturbations may generate both E- and B-mode polarization.

167 Angular Power Spectra

For statistically isotropic perturbations, the observed spherical harmonic coefficients satisfy

$$
\left\langle a_{\ell m}^X a_{\ell' m'}^{Y*} \right\rangle = \delta_{\ell\ell'}\delta_{mm'}C_\ell^{XY}, \tag{955}
$$

where

$$
X, Y \in \{T, E, B, \phi\}, \tag{956}
$$

and ϕ denotes the lensing potential.

For one primordial scalar mode,

$$
C_\ell^{XY} = 4\pi \int \frac{\mathrm{d}k}{k}\mathcal{P}_{\mathcal{R}}(k)\Delta_\ell^X(k)\Delta_\ell^Y(k). \tag{957}
$$

For multiple primordial modes A and B,

$$
C_\ell^{XY} = 4\pi \sum_{A,B} \int \frac{\mathrm{d}k}{k}\mathcal{P}_{AB}(k)\Delta_{\ell,A}^X(k)\Delta_{\ell,B}^Y(k). \tag{958}
$$

The mode labels may include:

$$
A, B \in \{\mathcal{R}, S_1, \ldots, S_n, T, V\}. \tag{959}
$$

The derived central-output covariance matrix therefore enters the angular spectra directly.

The primary scalar observables are

$$C_\ell^{TT}, \qquad C_\ell^{TE}, \qquad C_\ell^{EE}. \tag{960}$$

Additional observables include

$$C_\ell^{BB}, \qquad C_\ell^{\phi\phi}, \qquad C_\ell^{T\phi}, \qquad C_\ell^{E\phi}. \tag{961}$$

A successful model must fit the correlated set rather than only the temperature spectrum.

168 Adiabatic and Isocurvature CMB Signatures

Different primordial modes generate different acoustic phases and peak patterns.

For a correlated adiabatic and isocurvature system, the angular spectrum is

$$C_\ell = C_\ell^{\mathcal{R}\mathcal{R}} + C_\ell^{SS} + 2C_\ell^{\mathcal{R}S}. \tag{962}$$

Parameterize the primordial isocurvature fraction by

$$\beta_{\rm iso}(k) = \frac{\mathcal{P}_S(k)}{\mathcal{P}_\mathcal{R}(k) + \mathcal{P}_S(k)}. \tag{963}$$

Define the cross-correlation coefficient

$$\cos \Delta_{\mathcal{R}S}(k) = \frac{\mathcal{P}_{\mathcal{R}S}(k)}{\sqrt{\mathcal{P}_\mathcal{R}(k)\mathcal{P}_S(k)}}. \tag{964}$$

Once a physical central-transition model is specified, it predicts both quantities. They may not be selected independently after the scalar spectrum is fitted.

A predominantly adiabatic Triple Bounce output requires

$$\beta_{\rm iso}(k) \ll 1 \tag{965}$$

for visible modes, unless a specific correlated isocurvature component remains observationally allowed.

169 Tensor and Vector CMB Signatures

Tensor perturbations contribute to:

$$TT, \qquad TE, \qquad EE, \qquad BB. \tag{966}$$

The tensor angular spectrum is

$$C_{\ell,\mathrm{T}}^{XY} = 4\pi \int \frac{\mathrm{d}k}{k} \mathcal{P}_T(k) \Delta_{\ell,\mathrm{T}}^X(k) \Delta_{\ell,\mathrm{T}}^Y(k). \tag{967}$$

Vector perturbations also source temperature and polarization, including B modes.

The total B-mode spectrum is schematically

$$C_\ell^{BB} = C_{\ell,\mathrm{tensor}}^{BB} + C_{\ell,\mathrm{vector}}^{BB} + C_{\ell,\mathrm{lensing}}^{BB} + C_{\ell,\mathrm{foreground}}^{BB}. \tag{968}$$

Candidate central-transition dynamics may generate tensor and vector spectra with shapes different from those of canonical slow-roll inflation.

A distinctive claim requires the model to predict:

$$\left\{\mathcal{P}_T(k), n_T(k), \mathcal{P}_V(k), \text{cross-correlations}\right\}. \tag{969}$$

The spectra must remain compatible with current observational limits and future searches.

170 Reionization and Large-Angle Polarization

Late reionization rescattered a fraction of the cosmic microwave background photons.

The integrated optical depth to reionization is

$$\tau_{\mathrm{re}} = \int_0^{z_{\mathrm{re}}} \frac{c\,\sigma_T n_e(z)}{(1+z)H(z)} \mathrm{d}z. \tag{970}$$

Reionization suppresses primary small-scale anisotropy approximately by

$$e^{-2\tau_{\mathrm{re}}}, \tag{971}$$

and generates large-angle E-mode polarization.

The inferred scalar amplitude is partly degenerate with τ_{re}.

Therefore, the Triple Bounce scalar normalization must be tested jointly against:

$$A_s, \qquad \tau_{\rm re}, \qquad A_s e^{-2\tau_{\rm re}}. \tag{972}$$

A low-multipole feature should not be attributed to the hidden history until reionization uncertainties are included.

171 Lensing and Late-Time Transfer

Large-scale structure deflects cosmic microwave background photons.

The lensing potential is

$$\phi(\hat{\boldsymbol{n}}) = -2\int_0^{\chi_*} \mathrm{d}\chi\, \frac{S_k(\chi_* - \chi)}{S_k(\chi_*)S_k(\chi)} \Phi_W\left(\chi\hat{\boldsymbol{n}}, \eta_0 - \chi\right), \tag{973}$$

where:

- $S_k(\chi)$ is the curvature-dependent transverse-distance function; and
- Φ_W is the Weyl potential.

Lensing:

- smooths acoustic peaks;
- converts E modes into B modes;
- probes matter growth;
- tests the gravitational potentials; and
- constrains dark-sector interactions.

The Triple Bounce model must therefore reproduce both the primary unlensed spectra and the lensing reconstruction.

A fit to the primary peaks with incorrect late-time structure growth is insufficient.

The lensing potential spectrum is

$$C_\ell^{\phi\phi}. \tag{974}$$

It links Paper II to the Gravity Triune dynamics developed in Paper III.

172 Inherited-Web Corrections to CMB Statistics

The inherited web may affect the cosmic microwave background through several channels.

172.1 Primordial scalar correction

The scalar spectrum may contain

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}^{(0)}(k)\left[1 + \Delta_{\text{mem}}(k)\right]. \tag{975}$$

A smooth correction may be parameterized provisionally as

$$\Delta_{\text{mem}}(k) = A_{\text{mem}} F_{\text{mem}}\left(\frac{k}{k_{\text{mem}}}\right), \tag{976}$$

where the shape function must ultimately be derived. This expression is a diagnostic template, not a Paper II prediction; its amplitude, scale, and shape may not be varied independently once the scaffold-retention dynamics are specified.

172.2 Oscillatory transition correction

A finite central transition may produce

$$\Delta_{\text{osc}}(k) = A_{\text{osc}}(k)\cos\left[2k\Delta\eta_{\text{C}} + \varphi_{\text{C}}\right]. \tag{977}$$

This could shift or modulate acoustic power. The phase, envelope, and frequency must be inherited from one finite transition model rather than introduced as separate feature-fitting parameters.

172.3 Directional correction

A residual preferred direction may produce

$$\mathcal{P}_{\mathcal{R}}(\boldsymbol{k}) = \mathcal{P}_{\mathcal{R}}^{(0)}(k)\left[1 + g_*(k)\left(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}}\right)^2\right]. \tag{978}$$

This would break diagonal statistical isotropy if a directional component survives the complete transfer:

$$\langle a_{\ell m} a^*_{\ell' m'}\rangle \not\propto \delta_{\ell\ell'}\delta_{mm'}. \tag{979}$$

The resulting bipolar spherical harmonic coefficients or equivalent anisotropy statistics must remain within observational limits.

172.4 Topology and phase correlations

The inherited web may also create higher-order or phase-sensitive statistics not visible in the ordinary power spectrum.

Candidate observables include:

$$\{B_{\ell_1\ell_2\ell_3}, T_{\ell_1\ell_2\ell_3\ell_4}, \text{phase correlations}, \text{alignment statistics}, \text{topological summaries}\}. \tag{980}$$

These quantities become meaningful tests only after the same primordial memory model predicts them jointly.

173 CMB Recovery Vector

Define the Triple Bounce cosmic microwave background prediction vector

$$\boldsymbol{\mathcal{C}}_{\mathrm{TB}} = \left(C_\ell^{TT}, C_\ell^{TE}, C_\ell^{EE}, C_\ell^{BB}, C_\ell^{\phi\phi}, \boldsymbol{\mathcal{B}}_\ell, \boldsymbol{\mathcal{T}}_\ell, \mathsf{A}_{\ell m,\ell' m'}\right), \tag{981}$$

where:

- $\boldsymbol{\mathcal{B}}_\ell$ represents angular bispectra;
- $\boldsymbol{\mathcal{T}}_\ell$ represents trispectra; and
- $\mathsf{A}_{\ell m,\ell' m'}$ represents anisotropic covariance.

The observationally acceptable region is

$$\mathcal{V}_{\mathrm{CMB}}. \tag{982}$$

CMB recovery requires

$$\boxed{\boldsymbol{\mathcal{C}}_{\mathrm{TB}} \in \mathcal{V}_{\mathrm{CMB}}.} \tag{983}$$

The region $\mathcal{V}_{\mathrm{CMB}}$ is a joint likelihood-defined acceptance region after nuisance parameters and covariance are included (Planck Collaboration, 2020a). It is not a collection of independent per-spectrum tolerances.

The input parameter vector should be finite:

$$\boldsymbol{\theta}_{\mathrm{CMB}} = (\boldsymbol{\theta}_{\mathrm{bg}}, \boldsymbol{\theta}_{\mathrm{C}}, \boldsymbol{\theta}_{\mathrm{mem}}, \boldsymbol{\theta}_{\mathrm{late}}) \,. \tag{984}$$

Here:

- $\boldsymbol{\theta}_{\mathrm{bg}}$ controls the third-leg background;
- $\boldsymbol{\theta}_{\mathrm{C}}$ controls the central transition;
- $\boldsymbol{\theta}_{\mathrm{mem}}$ controls inherited-memory transfer; and
- $\boldsymbol{\theta}_{\mathrm{late}}$ controls reionization and late structure.

The same parameters must be used in the horizon, flatness, primordial, and cosmic microwave background calculations.

174 Comparison with a Standard Inflationary Baseline

Let

$$\mathcal{M}_{\mathrm{base}} \tag{985}$$

denote a specified inflationary ΛCDM baseline.

Let

$$\mathcal{M}_{\mathrm{TB}} \tag{986}$$

denote the Triple Bounce model.

The comparison must use the same:

- cosmic microwave background likelihoods;
- foreground treatment;
- multipole ranges;
- recombination calculation;
- lensing treatment;
- late-time data where included; and
- statistical evidence standards.

The baseline primordial spectrum may be

$$\mathcal{P}_{\mathcal{R}}^{\mathrm{base}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1}. \tag{987}$$

The Triple Bounce spectrum is

$$\mathcal{P}_{\mathcal{R}}^{\mathrm{TB}}(k) = \mathcal{P}_{\mathcal{R}}^{\mathrm{base}}(k)\left[1 + \Delta_{\mathrm{C}}(k) + \Delta_{\mathrm{mem}}(k)\right] \tag{988}$$

only as a phenomenological residual decomposition.

In the completed theory, the central-transition and inherited-scaffold contributions need not be additive, and every term must be derived from the same ordered transfer history. The decomposition may be used for diagnostics, but not as a collection of independent fitting functions.

The likelihood is

$$\mathcal{L}_{\mathrm{CMB}} = P\left(\mathcal{D}_{\mathrm{CMB}} \mid \boldsymbol{\theta}, \mathcal{M}\right). \tag{989}$$

A better raw fit is insufficient if obtained through many weakly constrained parameters.

The comparison must include parameter complexity and predictive restriction.

175 Numerical Einstein–Boltzmann Implementation

The first complete cosmic microwave background calculation should use a modified Einstein–Boltzmann solver.

175.1 Step 1: import the derived central state

Provide:

$$\{\mathcal{P}_{\mathcal{R}}, \mathsf{P}_{SS}, \boldsymbol{\mathcal{P}}_{\mathcal{R}S}, \mathcal{P}_T, \mathcal{P}_V, \text{higher-order kernels}\}. \tag{990}$$

No standard primordial spectrum should be silently substituted.

175.2 Step 2: implement the third-leg background

Solve

$$H(a) \tag{991}$$

from the third-leg sector equations, including the chasing and scaffold contributions.

175.3 Step 3: evolve all perturbation modes

Evolve the adiabatic, entropy, tensor, and vector basis modes through the radiation and matter eras.

175.4 Step 4: calculate recombination

Compute:

$$x_e(z), \qquad \tau(z), \qquad g(z), \qquad k_D(z), \tag{992}$$

where $x_e(z)$ is the ionization fraction.

175.5 Step 5: calculate transfer functions

Produce:

$$\Delta^T_{\ell,A}(k), \qquad \Delta^E_{\ell,A}(k), \qquad \Delta^B_{\ell,A}(k), \qquad \Delta^\phi_{\ell,A}(k). \tag{993}$$

175.6 Step 6: construct angular spectra

Integrate the complete primordial covariance against the transfer functions.

175.7 Step 7: apply foreground and instrumental models

The theoretical comparison must include the same nuisance treatment used for the baseline model.

175.8 Step 8: evaluate likelihoods

Calculate posterior distributions, profile likelihoods, and model evidence or appropriate approximations.

175.9 Step 9: perform residual analysis

Define residuals

$$\Delta C_\ell^{XY} = C_{\ell,\mathrm{TB}}^{XY} - C_{\ell,\mathrm{data}}^{XY}. \tag{994}$$

Inspect:

- low-multipole residuals;

- peak-position residuals;
- odd–even peak residuals;
- damping-tail residuals;
- polarization residuals;
- lensing residuals; and
- cross-spectrum residuals.

176 Nested CMB Test Cases

The cosmic microwave background program should proceed through six nested cases.

176.1 Case A: standard adiabatic transfer

Use the Triple Bounce third-leg background with a conventional adiabatic power-law spectrum.

This isolates the effect of the background.

It is a diagnostic case, not the final theory.

176.2 Case B: derived scalar central output

Replace the conventional spectrum with the derived

$$\mathcal{P}_{\mathcal{R}}^{\mathrm{C}}(k). \tag{995}$$

Keep isocurvature, tensor, and vector modes zero.

176.3 Case C: scalar plus isocurvature

Include

$$\mathsf{P}_{SS}(k), \qquad \boldsymbol{\mathcal{P}}_{\mathcal{R}S}(k). \tag{996}$$

176.4 Case D: inherited-memory features

Include

$$\Delta_{\mathrm{mem}}(k), \qquad g_*(k), \qquad \text{phase correlations.} \tag{997}$$

176.5 Case E: tensor and vector output

Include

$$\mathcal{P}_T(k), \qquad \mathcal{P}_V(k). \tag{998}$$

176.6 Case F: full joint model

Use the complete central covariance, nonlinear statistics, third-leg background, reionization model, and lensing evolution.

Only Case F constitutes a full joint observational evaluation. Even then, historical identification requires comparison with alternative models under the same data, priors, nuisance treatment, and complexity penalty.

177 CMB-Recovery Rejection Criteria

The Triple Bounce primordial and third-leg model fails or requires revision if any of the following conditions is unavoidable.

1. The central-output perturbations do not satisfy the Einstein constraint equations.
2. The derived background produces an unacceptable sound horizon.
3. The same background produces an unacceptable distance to last scattering.
4. The acoustic angular scale cannot be reproduced.
5. The central output contains an excessive sine or decaying scalar mode.
6. The acoustic peak sequence loses phase coherence.
7. The odd–even peak-height pattern is incompatible with one consistent baryon density.
8. Matter–radiation equality occurs at an unacceptable epoch.
9. The recombination or visibility function is significantly inconsistent with the observed damping and peak structure.
10. The primordial scalar tilt or running is unacceptable.
11. Inherited-memory features exceed allowed residuals in the temperature or polarization spectra.
12. The scaffold imprint produces excessive statistical anisotropy.
13. Visible isocurvature exceeds observational limits.
14. Tensor power exceeds the allowed range.

15. Vector or vorticity power produces excessive temperature or polarization anisotropy.
16. The predicted TE correlation has the wrong sign or phase over key multipole ranges.
17. The EE spectrum fails even when the TT spectrum is fitted.
18. The model fits primary anisotropy but fails the cosmic microwave background lensing spectrum.
19. The required reionization history is astrophysically unacceptable.
20. A successful fit requires separate primordial functions for TT, TE, and EE.
21. The transition parameters are independently adjusted in separate multipole ranges.
22. The model reproduces the cosmic microwave background only after replacing its derived central output with standard inflationary initial conditions.

A claim of observational preference is unsupported if an improved raw fit disappears after accounting for additional parameter freedom, look-elsewhere effects, or prior volume.

The absence of a nondegenerate historical residue has a different meaning. It does not by itself invalidate a model that successfully recovers the observables; it leaves the proposed hidden history observationally unidentified:

$$\boldsymbol{\mathcal{C}}_{\mathrm{TB}} \in \mathcal{V}_{\mathrm{CMB}}, \qquad \Delta\boldsymbol{\mathcal{O}}_{\mathrm{hist}} \simeq \mathbf{0}. \tag{999}$$

178 Formal Summary of the CMB Test

The central transition supplies the primordial covariance,

$$\boxed{\mathsf{P}_Z^{+}(k).} \tag{1000}$$

The third-leg Einstein–Boltzmann system propagates that covariance into transfer functions,

$$\boxed{\Delta^X_{\ell,A}(k), \qquad X \in \{T, E, B, \phi\}.} \tag{1001}$$

The angular spectra are

$$\boxed{C_\ell^{XY} = 4\pi \sum_{A,B} \int \frac{\mathrm{d}k}{k} \mathcal{P}_{AB}(k) \Delta^X_{\ell,A}(k) \Delta^Y_{\ell,B}(k).} \tag{1002}$$

The sound horizon is

$$\boxed{r_{s*} = \int_{a_{\rm th}}^{a_*} \frac{c_s(a)}{a^2 H(a)} \mathrm{d}a.} \tag{1003}$$

The acoustic angular scale is

$$\boxed{\theta_* = \frac{r_{s*}}{D_M(z_*)}.} \tag{1004}$$

Phase coherence requires the derived phase and decaying-mode diagnostics to lie in their joint allowed region,

$$\boxed{\left(\Delta\varphi_{\rm ac}(k), \frac{C_{\rm dec}(k)}{C_{\rm grow}(k)}\right) \in \mathcal{V}_{\rm phase}.} \tag{1005}$$

Adiabatic dominance requires

$$\boxed{\beta_{\rm iso}(k) < \beta_{\rm iso,max}(k).} \tag{1006}$$

The complete recovery condition is

$$\boxed{\left\{C_\ell^{TT}, C_\ell^{TE}, C_\ell^{EE}, C_\ell^{BB}, C_\ell^{\phi\phi}, \text{higher-order statistics}\right\} \in \mathcal{V}_{\rm CMB}.} \tag{1007}$$

The decisive question is

$$\boxed{\begin{array}{l}\text{Can the perturbation state derived from the central reversal,}\\ \text{without replacement by standard inflationary initial data,}\\ \text{propagate through one consistent third-leg background and}\\ \text{reproduce the measured temperature, polarization, damping,}\\ \text{lensing, and higher-order microwave-background observables?}\end{array}} \tag{1008}$$

179 CMB Transfer Status

The cosmic microwave background test is now reduced to a complete computational sequence.

1. Derive the central-output background and perturbation covariance.
2. Verify all initial Einstein constraints.
3. Implement the third-leg expansion history.
4. Implement all additional sector perturbations.

5. Evolve adiabatic, entropy, tensor, and vector basis modes.
6. Calculate the photon–baryon tight-coupling regime.
7. Calculate the sound horizon.
8. Calculate matter–radiation equality.
9. Calculate recombination, the visibility function, and photon diffusion.
10. Calculate temperature and polarization line-of-sight sources.
11. Construct TT, TE, EE, BB, and lensing spectra.
12. Propagate inherited-memory features through the transfer functions.
13. Calculate anisotropic and higher-order statistics.
14. Compare all spectra with one common likelihood framework.
15. Inspect peak positions, phases, heights, and damping residuals.
16. Reject any model requiring separate adjustments for separate spectra.
17. Determine whether a surviving memory feature is statistically distinguishable from inflationary or late-time alternatives.

The Triple Bounce framework has therefore advanced from a proposed central perturbation state to a complete observational transfer problem.

The cosmic microwave background remains one of the strongest conditions the theory must meet.

180 Distinctive Historical Signatures

Reproducing the observational successes of inflationary cosmology is a necessary condition for the Triple Bounce framework.

It is not sufficient to establish a longer three-leg history.

A model that reproduces the same background and perturbation observables while leaving no measurable residue of the hidden first and second legs remains observationally degenerate with a suitably chosen third-leg initial state.

The distinctive-signature problem is therefore:

$$\boxed{\begin{array}{l}\text{Which observables retain information that specifically requires}\\ \text{the ordered first–second–third-leg history?}\end{array}} \tag{1009}$$

A genuine historical signature must satisfy four conditions.

1. It must be derived from the same dynamics used to address the horizon, flatness, and perturbation candidate resolutions.
2. It must survive the outer reversal, second leg, central reversal, and third-leg transfer.
3. It must not be freely adjustable independently of the rest of the model.
4. It must differ observably from the predictions of competing inflationary, bouncing, cyclic, topological, and late-time models.

The desired observable is not merely an anomaly.

It is a constrained consequence of the ordered history,

$$\mathcal{H}_{123} = \mathcal{U}_3 \circ \mathcal{T}_{\mathrm{C}} \circ \mathcal{U}_2 \circ \mathcal{T}_{\mathrm{O}} \circ \mathcal{U}_1. \tag{1010}$$

Because these operators generally do not commute,

$$\mathcal{H}_{123} \neq \mathcal{U}_3 \circ \mathcal{T}_{\mathrm{O}} \circ \mathcal{U}_2 \circ \mathcal{T}_{\mathrm{C}} \circ \mathcal{U}_1, \tag{1011}$$

the sequence may leave an order-dependent residue.

That residue is the principal observational target of this analysis.

181 Recovery, Residue, and Identification

Let the observable prediction vector be

$$\boldsymbol{\mathcal{O}} = \left(\boldsymbol{\mathcal{O}}_{\mathrm{bg}}, \boldsymbol{\mathcal{O}}_{\mathrm{CMB}}, \boldsymbol{\mathcal{O}}_{\mathrm{LSS}}, \boldsymbol{\mathcal{O}}_{\mathrm{GW}}, \boldsymbol{\mathcal{O}}_{\mathrm{mag}}, \ldots\right), \tag{1012}$$

where:

- $\boldsymbol{\mathcal{O}}_{\mathrm{bg}}$ contains background-distance and expansion observables;
- $\boldsymbol{\mathcal{O}}_{\mathrm{CMB}}$ contains microwave-background observables;
- $\boldsymbol{\mathcal{O}}_{\mathrm{LSS}}$ contains large-scale-structure observables;
- $\boldsymbol{\mathcal{O}}_{\mathrm{GW}}$ contains gravitational-wave observables; and
- $\boldsymbol{\mathcal{O}}_{\mathrm{mag}}$ contains magnetic and helicity observables.

The Triple Bounce prediction may be decomposed as

$$\boldsymbol{\mathcal{O}}_{\mathrm{TB}} = \boldsymbol{\mathcal{O}}_{\mathrm{rec}} + \Delta\boldsymbol{\mathcal{O}}_{\mathrm{hist}}, \tag{1013}$$

where:

- $\mathcal{O}_{\text{rec}}$ reproduces the successful baseline cosmology; and
- $\Delta\mathcal{O}_{\text{hist}}$ is the historical residue.

A viable model requires

$$\mathcal{O}_{\text{rec}} + \Delta\mathcal{O}_{\text{hist}} \in \mathcal{V}_{\text{obs}}. \tag{1014}$$

A distinguishable historical interpretation requires

$$\Delta\mathcal{O}_{\text{hist}} \neq \mathbf{0} \tag{1015}$$

at a measurable, cross-validated level. This is an identification requirement, not an additional condition for basic observational recovery.

A specifically identified three-leg history requires the stronger condition

$$P\left(\mathcal{H}_{123} \mid \mathcal{D}\right) > P\left(\mathcal{H}_{\text{alt}} \mid \mathcal{D}\right) \tag{1016}$$

for relevant alternative histories $\mathcal{H}_{\text{alt}}$ under a fair model comparison.

Definition 181.1 (Historical residue)**.** A historical residue is a calculable correlation, feature, or cross-observable relation that survives into the third leg and depends on the ordered hidden history rather than only on freely selectable third-leg boundary data.

182 Classes of Candidate Signatures

Candidate historical signatures may be grouped into six classes.

I. **Large-scale transfer features,** including low-wavenumber suppression, enhancement, or turnover.

II. **Finite-transition oscillations,** including periodic or quasiperiodic features generated by the reversal durations.

III. **Inherited-web correlations,** including phase, topology, morphology, and cross-scale ancestry.

IV. **Directional and parity structure,** including weak statistical anisotropy, chirality, or helicity.

V. **Cross-sector relations,** including constrained scalar–tensor, density–magnetic, or curvature–vorticity correlations.

VI. **Joint background–perturbation relations,** in which the same transition parameters control both the expansion history and primordial features.

A single isolated feature is unlikely to identify the full history.

The strongest evidence would be a linked signature vector,

$$\boldsymbol{\mathcal{S}}_{\rm hist} = \left(S_{\text{low-}k}, S_{\rm osc}, S_{\rm phase}, S_{\rm aniso}, S_{\rm cross}, S_{\rm bg}\right), \tag{1017}$$

derived from one finite parameter set.

183 Low-Wavenumber Transfer Features

The largest observable scales correspond to modes with the longest wavelengths.

These modes may be most sensitive to the earliest resolved post-singularity boundary, the outer reversal, the second-leg return, and the central transition.

The outgoing scalar spectrum may be written as

$$\mathcal{P}_{\mathcal{R}}^{\rm TB}(k) = \mathcal{P}_{\mathcal{R}}^{(0)}(k) T_{\rm hist}^2(k), \tag{1018}$$

where

$$T_{\rm hist}(k) = T_{\rm C}(k) T_2(k) T_{\rm O}(k) T_1(k). \tag{1019}$$

A regular low-wavenumber expansion may take the form

$$T_{\rm hist}(k) = T_0 + T_2\left(\frac{k}{k_{\rm L}}\right)^2 + T_4\left(\frac{k}{k_{\rm L}}\right)^4 + \cdots . \tag{1020}$$

A turnover model may instead be written as

$$T_{\rm hist}^2(k) = 1 - A_{\rm L} F_{\rm L}\left(\frac{k}{k_{\rm L}}\right), \tag{1021}$$

with

$$0 \le A_{\rm L} < 1. \tag{1022}$$

The shape function $F_{\rm L}$ must follow from the transfer dynamics.

It may not be chosen separately for each low-multipole residual.

A genuine first-boundary signature should connect

$$k_{\mathrm{L}} \tag{1023}$$

to a physical scale such as:

- the first resolved post-singularity boundary;
- the conformal duration of the first leg;
- the outer-transition width;
- the total retained causal interval; or
- the largest scaffold correlation scale.

184 Finite-Reversal Oscillations

A finite transition can mix positive- and negative-frequency mode components.

For one canonical mode,

$$v_k^+ = \alpha_k v_k^- + \beta_k v_k^{-*}. \tag{1024}$$

The normalization condition is

$$|\alpha_k|^2 - |\beta_k|^2 = 1 \tag{1025}$$

for a unitary canonical transformation.

The resulting spectrum contains interference:

$$\mathcal{P}_{\mathcal{R}}^+(k) = \mathcal{P}_{\mathcal{R}}^-(k)\,|\alpha_k + \beta_k|^2\,. \tag{1026}$$

For a transition with conformal width $\Delta\eta_j$, the interference may contain a phase

$$\varphi_j(k) \sim 2k\Delta\eta_j + \varphi_{j,0}. \tag{1027}$$

With two reversals, a schematic transfer is

$$\begin{aligned} T_{\mathrm{osc}}(k) = 1 &+ A_{\mathrm{O}}(k)\cos\left[2k\Delta\eta_{\mathrm{O}} + \varphi_{\mathrm{O}}\right] \\ &+ A_{\mathrm{C}}(k)\cos\left[2k\Delta\eta_{\mathrm{C}} + \varphi_{\mathrm{C}}\right] \\ &+ A_{\mathrm{OC}}(k)\cos\left[2k\Delta\eta_{\mathrm{OC}} + \varphi_{\mathrm{OC}}\right]. \end{aligned} \tag{1028}$$

Here, $\Delta\eta_{\mathrm{OC}}$ may contain the conformal separation between the two reversals.

The ordered three-leg history could therefore generate:

- one outer-transition frequency;
- one central-transition frequency;
- an interference or beat scale; and
- scale-dependent damping envelopes.

This multi-frequency relation would be more distinctive than one unconstrained oscillatory feature.

185 Two-Reversal Interference

Let the outer and central transition matrices for one mode be

$$\mathsf{M}_{\mathrm{O}}(k), \qquad \mathsf{M}_{\mathrm{C}}(k). \tag{1029}$$

The second-leg propagator is

$$\mathsf{U}_2(k) = \begin{pmatrix} e^{-\mathrm{i}\Theta_2(k)} & 0 \\ 0 & e^{+\mathrm{i}\Theta_2(k)} \end{pmatrix} \tag{1030}$$

in a simplified adiabatic basis, where

$$\Theta_2(k) = \int_{\eta_{\mathrm{O}}^{+}}^{\eta_{\mathrm{C}}^{-}} \omega_k(\eta)\mathrm{d}\eta. \tag{1031}$$

The total mode transfer is

$$\mathsf{M}_{123}(k) = \mathsf{M}_{\mathrm{C}}(k)\mathsf{U}_2(k)\mathsf{M}_{\mathrm{O}}(k). \tag{1032}$$

The output power may therefore contain terms proportional to

$$\cos\left[2\Theta_2(k) + \Delta\varphi_{\mathrm{OC}}(k)\right]. \tag{1033}$$

A specifically three-leg signature would require the oscillatory phase to depend on all three components:

$$\varphi_{\mathrm{hist}} = \varphi\left(\mathcal{T}_{\mathrm{O}}, \mathcal{U}_2, \mathcal{T}_{\mathrm{C}}\right). \tag{1034}$$

If the same feature can be reproduced by one arbitrary transition, it does not uniquely identify two reversals.

186 Inherited-Web Phase Statistics

A Gaussian statistically homogeneous field is fully characterized at the two-point level by its power spectrum.

Its Fourier phases are otherwise random subject to the reality condition.

An inherited structural web may introduce phase correlations not captured by $\mathcal{P}_{\mathcal{R}}(k)$ alone.

Write the scalar mode as

$$\mathcal{R}_{\boldsymbol{k}} = A_{\boldsymbol{k}} e^{\mathrm{i}\phi_{\boldsymbol{k}}}. \tag{1035}$$

A phase-coupling statistic may be defined as

$$\mathcal{P}_{\phi}\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3\right) = \left\langle e^{\mathrm{i}\left(\phi_{\boldsymbol{k}_1} + \phi_{\boldsymbol{k}_2} - \phi_{\boldsymbol{k}_3}\right)} \right\rangle, \tag{1036}$$

with

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{k}_3. \tag{1037}$$

For random independent phases,

$$\mathcal{P}_{\phi} \approx 0. \tag{1038}$$

A nonzero value indicates phase coupling.

The ordinary bispectrum also probes such coupling, but phase-only statistics may isolate morphology from amplitude.

A three-pass memory hypothesis may predict enhanced correlations among modes associated with:

- common first-pass ancestry;
- second-pass retracing;
- central-transition branching;
- shared junction origin; or
- harmonically related pathway scales.

The challenge is to derive the allowed mode triangles from the scaffold dynamics.

187 Topological and Morphological Memory

The inherited web may leave statistical information in the morphology of the density field.

Let the smoothed density contrast be

$$\delta_R(\boldsymbol{x}). \tag{1039}$$

Define an excursion set

$$\mathcal{E}_\nu = \{\boldsymbol{x} : \delta_R(\boldsymbol{x}) > \nu\sigma_R\}\,, \tag{1040}$$

where ν is a threshold and σ_R is the smoothed variance.

Candidate morphological observables include:

$$\{V_0(\nu), V_1(\nu), V_2(\nu), V_3(\nu), \beta_p(\nu), \mathcal{P}_{\rm pers}\}\,, \tag{1041}$$

where:

- V_μ are Minkowski functionals;
- β_p are Betti numbers; and
- $\mathcal{P}_{\rm pers}$ represents persistent-homology summaries.

The Triple Bounce hypothesis would be strengthened if one derived memory parameter controlled both:

$$\Delta\mathcal{P}_{\mathcal{R}}(k) \tag{1042}$$

and

$$\Delta V_\mu(\nu), \qquad \Delta\beta_p(\nu). \tag{1043}$$

An independently fitted topological correction would not provide strong historical evidence.

188 Scale-Dependent Structural Retention

The inherited web is not expected to preserve all scales equally.

Let

$$\mathcal{E}_{\rm mem}(k) \tag{1044}$$

be the cumulative memory-retention efficiency.

The third-leg inherited contribution is

$$\mathcal{W}_{12}^{(3)}(k) = \mathcal{E}_{\rm mem}(k)\mathcal{W}_{12}^{(\rm pre)}(k). \tag{1045}$$

A provisional smooth retention law may be written as

$$\mathcal{E}_{\rm mem}(k) = \left[1 + \left(\frac{k}{k_{\rm mem}}\right)^{p_{\rm mem}}\right]^{-1}, \tag{1046}$$

where:

- $k_{\rm mem}$ is the characteristic retention scale; and
- $p_{\rm mem}$ controls the transition sharpness.

This form is illustrative.

The physical theory must derive the shape.

Large-scale preservation with small-scale erasure would correspond to

$$\mathcal{E}_{\rm mem}(k) \to 1 \quad \text{as} \quad k \to 0, \tag{1047}$$

and

$$\mathcal{E}_{\rm mem}(k) \to 0 \quad \text{as} \quad k \to \infty. \tag{1048}$$

Such selective retention could preserve horizon-scale ancestry while allowing ordinary small-scale stochastic structure formation.

189 Weak Statistical Anisotropy

A retraced directional history may leave a weak anisotropic component.

Write

$$\mathcal{P}_{\mathcal{R}}(\boldsymbol{k}) = \mathcal{P}_{\mathcal{R}}^{(0)}(k) \left[1 + \sum_{LM} g_{LM}(k) Y_{LM}(\hat{\boldsymbol{k}})\right]. \tag{1049}$$

The leading quadrupolar form is

$$\mathcal{P}_{\mathcal{R}}(\boldsymbol{k}) = \mathcal{P}_{\mathcal{R}}^{(0)}(k) \left[1 + g_*(k) \left(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}}\right)^2\right]. \tag{1050}$$

The resulting angular covariance is

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \delta_{\ell\ell'} \delta_{mm'} + \Delta C_{\ell m, \ell' m'}. \tag{1051}$$

A viable historical residue requires

$$0 < |g_*(k)| < g_{\max}(k) \tag{1052}$$

over at least one measurable scale range.

If

$$g_*(k) = 0 \tag{1053}$$

identically, the directional history may still survive in isotropic higher-order statistics.

If

$$|g_*(k)| \geq g_{\max}(k), \tag{1054}$$

the model fails observationally.

190 Parity, Chirality, and Helicity

The emergence of electromagnetic and rotational structure near the central reversal may generate parity-sensitive observables.

For tensor modes, define right- and left-handed spectra,

$$\mathcal{P}_T^R(k), \qquad \mathcal{P}_T^L(k). \tag{1055}$$

The tensor chirality parameter is

$$\chi_T(k) = \frac{\mathcal{P}_T^R(k) - \mathcal{P}_T^L(k)}{\mathcal{P}_T^R(k) + \mathcal{P}_T^L(k)}. \tag{1056}$$

A parity-symmetric tensor background has

$$\chi_T(k) = 0. \tag{1057}$$

A chiral tensor background may generate parity-odd angular spectra such as

$$C_\ell^{TB}, \qquad C_\ell^{EB}. \tag{1058}$$

For magnetic fields, write

$$\left\langle B_i(\boldsymbol{k}) B_j^*(\boldsymbol{k}') \right\rangle = (2\pi)^3 \delta_D \left(\boldsymbol{k} - \boldsymbol{k}'\right) \left[P_B(k) P_{ij} + \mathrm{i} P_H(k) \epsilon_{ijl} \hat{k}_l\right], \tag{1059}$$

where:

- $P_B(k)$ is the parity-even magnetic power; and
- $P_H(k)$ is the helical magnetic power.

A central dynamo-junction hypothesis could predict a relation between

$$P_H(k) \tag{1060}$$

and the ordered reversal geometry.

That relation must be derived rather than inferred from any observed parity anomaly.

191 Curvature–Magnetic Cross-Correlation

If ordinary magnetic structure emerges while matter occupies the inherited scaffold, scalar curvature and magnetic observables may share a common central origin.

Define a magnetic scalar proxy

$$\mathcal{B}_{\mathrm{s}}(\boldsymbol{k}) \tag{1061}$$

constructed from magnetic energy density, helicity, or another statistically defined scalar.

The cross-spectrum is

$$\langle \mathcal{R}_{\boldsymbol{k}} \mathcal{B}_{\mathrm{s}}^{*}(\boldsymbol{k}') \rangle = \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}B}(k) \delta_D \left(\boldsymbol{k} - \boldsymbol{k}' \right). \tag{1062}$$

Define the correlation coefficient

$$r_{\mathcal{R}B}(k) = \frac{\mathcal{P}_{\mathcal{R}B}(k)}{\sqrt{\mathcal{P}_{\mathcal{R}}(k) \mathcal{P}_{B}(k)}}. \tag{1063}$$

A shared transition origin may produce

$$r_{\mathcal{R}B}(k) \neq 0. \tag{1064}$$

A distinctive prediction would require the same parameter combination to control:

$$\left\{ \mathcal{P}_{\mathcal{R}}(k), \mathcal{P}_{B}(k), \mathcal{P}_{\mathcal{R}B}(k), P_{H}(k) \right\}. \tag{1065}$$

This would be stronger than independently postulating primordial curvature and magnetic spectra.

192 Scalar–Tensor Consistency Relations

Canonical single-field slow-roll inflation predicts approximate relations among scalar and tensor observables.

A commonly discussed relation is

$$r \approx -8 n_T \tag{1066}$$

under the standard assumptions.

The Triple Bounce central transition need not satisfy this relation.

Its scalar and tensor spectra are generated through different blocks of the full transfer matrix:

$$\mathcal{R}^+ = \mathsf{M}_{\mathrm{s}}\boldsymbol{X}_{\mathrm{s}}^- + N_{\mathcal{R}}, \tag{1067}$$

$$h_\lambda^+ = \mathsf{M}_{\mathrm{T}}\boldsymbol{X}_{\mathrm{T}}^- + N_{h,\lambda}. \tag{1068}$$

Define a Triple Bounce consistency residual,

$$\Delta_{\mathrm{ST}}(k) = r(k) + 8n_T(k). \tag{1069}$$

A nonzero value,

$$\Delta_{\mathrm{ST}}(k) \neq 0, \tag{1070}$$

would not by itself identify the Triple Bounce history because many noncanonical and multifield inflationary models also violate the canonical relation.

A stronger signature would be a linked relation such as

$$\Delta_{\mathrm{ST}}(k) = \mathcal{F}_{\mathrm{ST}}\left[\Delta_{\mathrm{osc}}(k), \Delta_{\mathrm{mem}}(k), \chi_T(k)\right], \tag{1071}$$

derived from the common reversal parameters.

193 Scalar–Vector and Density–Vorticity Relations

The multipass and junction structure may create small vector or vorticity components correlated with scalar structure.

Define

$$\langle \mathcal{R}_{\boldsymbol{k}} V_i^*(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D\left(\boldsymbol{k} - \boldsymbol{k}'\right) \mathcal{P}_{\mathcal{R}V,i}(\boldsymbol{k}). \tag{1072}$$

Statistical isotropy strongly constrains such a cross-spectrum unless a preferred direction or helical structure is present.

A parity-allowed scalar–helicity relation may instead involve a pseudoscalar observable,

$$\mathcal{H}_\omega = \boldsymbol{v} \cdot \boldsymbol{\omega}. \tag{1073}$$

A candidate cross-correlation is

$$\mathcal{P}_{\delta\mathcal{H}_\omega}(k). \tag{1074}$$

Any such signature must remain small enough to preserve the successful scalar-dominated early universe.

A viable realization therefore requires, at most,

$$\mathcal{P}_V(k) \ll \mathcal{P}_\mathcal{R}(k) \tag{1075}$$

over the primary cosmological scales.

194 Background–Feature Correlations

A major source of predictive power would arise if the reversal parameters controlled both background and perturbation observables.

Let

$$\boldsymbol{\theta}_{\rm rev} = \left(\tau_{\rm O}, \tau_{\rm C}, \Delta\eta_{\rm OC}, v_{\rm O}^{\pm}, v_{\rm C}^{\pm}, \Gamma_{\rm O}, \Gamma_{\rm C}, \ldots\right). \tag{1076}$$

These parameters may determine:

$$H(z) = H\left(z; \boldsymbol{\theta}_{\rm rev}, \boldsymbol{\theta}_{\rm late}\right), \tag{1077}$$

$$\mathcal{P}_\mathcal{R}(k) = \mathcal{P}_\mathcal{R}\left(k; \boldsymbol{\theta}_{\rm rev}, \boldsymbol{\theta}_{\rm mem}\right), \tag{1078}$$

$$\mathcal{P}_T(k) = \mathcal{P}_T\left(k; \boldsymbol{\theta}_{\rm rev}\right). \tag{1079}$$

A background–feature consistency relation may then be written as

$$\mathcal{C}_{\rm BF}\left[H(z), \Delta_{\rm osc}(k), \mathcal{P}_T(k)\right] = 0. \tag{1080}$$

This relation should hold across independent datasets.

A model that fits the background with one parameter set and the primordial features with another has reduced historical credibility.

195 Degeneracy with Inflationary Feature Models

Inflationary models can also generate:

- low-wavenumber suppression;
- oscillations;
- steps and localized features;
- non-Gaussianity;
- statistical anisotropy;
- isocurvature;
- tensor chirality; and
- primordial magnetic correlations.

Therefore, no single feature class uniquely identifies the Triple Bounce history.

Let

$$\mathcal{M}_{\rm feat} \tag{1081}$$

denote an inflationary feature model.

The two models may be observationally degenerate when their difference is small in the full data covariance,

$$\Delta\boldsymbol{\mathcal{O}}^{\mathsf{T}}\mathsf{C}_{\rm obs}^{-1}\Delta\boldsymbol{\mathcal{O}} \lesssim \Delta\chi^2_{\rm crit}, \qquad \Delta\boldsymbol{\mathcal{O}} = \boldsymbol{\mathcal{O}}_{\rm TB} - \boldsymbol{\mathcal{O}}_{\rm feat}, \tag{1082}$$

after nuisance parameters and shared uncertainties are marginalized.

Degeneracy may be broken by requiring simultaneous agreement in:

$$\{TT, TE, EE, BB, \phi\phi, P_m(k), B_{\mathcal{R}}, \mathcal{P}_T, \mathcal{P}_B, \text{anisotropic covariance}\}\,. \tag{1083}$$

The Triple Bounce model is favored only if its shared parameters explain a broader linked pattern with comparable or lower effective complexity.

196 Degeneracy with Alternative Bounce and Cyclic Models

Other nonsingular, bouncing, emergent, and cyclic cosmologies may also produce pre-hot-history signatures.

The principal Triple Bounce distinction is not the generic existence of a prior phase.

It is the combination:

$$\boxed{\begin{array}{l}\text{outward } \textit{Electro} \text{ motion,}\\ \text{outer sectoral reversal,}\\ \text{inward } \textit{Electro} \text{ return,}\\ \text{central sectoral reversal,}\\ \text{candidate Electro–magnetism coupling and third-leg matter organization,}\\ \text{while the global background may remain expanding.}\end{array}} \tag{1084}$$

A comparison must therefore ask whether the candidate signature depends on:

- global contraction and a scale-factor bounce;
- one pre-hot transition;
- multiple cycles;
- an emergent static state;
- a matter-contraction phase;
- a two-reversal sectoral history; or
- a generic arbitrary primordial transfer function.

A proposed signature is specifically supportive of the Triple Bounce framework only if the two sectoral reversals and intermediate inward leg play a necessary calculable role.

197 Degeneracy with Late-Time Physics

Features in observed cosmological data may arise from late-time effects rather than primordial history.

Potential degeneracies include:

- reionization;
- foreground contamination;

- gravitational lensing;
- nonlinear structure growth;
- baryonic feedback;
- galaxy bias;
- redshift-space distortions;
- dark-sector interactions;
- modified gravity;
- survey geometry; and
- instrumental systematics.

Let the observed residual be

$$\Delta\mathcal{D} = \Delta\mathcal{D}_{\rm prim} + \Delta\mathcal{D}_{\rm late} + \Delta\mathcal{D}_{\rm sys}. \tag{1085}$$

A historical interpretation requires evidence that

$$\Delta\mathcal{D}_{\rm prim} \tag{1086}$$

is nonzero after marginalizing over late-time and systematic alternatives.

Cross-redshift consistency is especially useful.

A primordial feature should propagate predictably into multiple epochs,

$$\Delta\mathcal{P}_{\mathcal{R}}(k) \longrightarrow \{\Delta C_\ell, \Delta P_m(k, z_1), \Delta P_m(k, z_2), \ldots\}. \tag{1087}$$

A late-time artifact generally does not reproduce the same primordial transfer relation at all redshifts.

198 Null Tests

A strong research program should define tests whose null result directly constrains the historical residue.

198.1 Null test for oscillatory coherence

The outer and central transitions predict linked phase scales.

A null statistic may test whether the best-fit frequencies satisfy

$$\omega_{\mathrm{OC}} = \mathcal{F}\left(\omega_{\mathrm{O}}, \omega_{\mathrm{C}}, \Delta\eta_{\mathrm{OC}}\right). \tag{1088}$$

Failure of the required relation disfavors the two-reversal interpretation.

198.2 Null test for memory cross-correlation

If scaffold-correlated retention contributes to both scalar and magnetic structure, test

$$\mathcal{P}_{\mathcal{R}B}(k) = 0. \tag{1089}$$

A result consistent with zero constrains the coupling $T_{\mathcal{R}\mathcal{W}}T_{B\mathcal{W}}$.

198.3 Null test for phase coupling

Test

$$\mathcal{P}_{\phi}\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3\right) = 0 \tag{1090}$$

for the predicted mode configurations.

198.4 Null test for directional memory

Test

$$g_{LM}(k) = 0. \tag{1091}$$

A null result may still permit an isotropized memory model but excludes variants requiring observable directional residue.

198.5 Null test for parity residue

Test

$$C_\ell^{TB} = 0, \qquad C_\ell^{EB} = 0, \qquad \chi_T(k) = 0. \tag{1092}$$

A null result constrains chiral central-transition models.

199 Cross-Observable Consistency Tests

The most useful signatures are not independent anomalies but linked predictions.

Define a parameter vector

$$\boldsymbol{\theta}_{\text{hist}} = (\Delta\eta_{\text{O}}, \Delta\eta_{\text{C}}, \Delta\eta_{\text{OC}}, \lambda_{\text{mem}}, p_{\text{mem}}, \omega_I, \Gamma_j, \ldots) . \tag{1093}$$

The same vector predicts

$$\boldsymbol{\theta}_{\text{hist}} \longrightarrow \left\{\Delta C_\ell^{XY}, \Delta P_m(k,z), B_{\mathcal{R}}, \mathcal{P}_T, \mathcal{P}_B, g_{LM}, V_\mu\right\} . \tag{1094}$$

Let the parameter estimate from dataset A be

$$\hat{\boldsymbol{\theta}}_A, \tag{1095}$$

and from independent dataset B be

$$\hat{\boldsymbol{\theta}}_B. \tag{1096}$$

A consistency statistic is

$$\mathcal{Q}_{AB} = \left(\hat{\boldsymbol{\theta}}_A - \hat{\boldsymbol{\theta}}_B\right)^{\mathsf{T}} (\mathsf{C}_A + \mathsf{C}_B)^{-1} \left(\hat{\boldsymbol{\theta}}_A - \hat{\boldsymbol{\theta}}_B\right), \tag{1097}$$

where C_A and C_B are parameter covariance matrices.

A historically coherent model requires acceptable consistency among independent datasets.

200 Identifiability

A parameter is identifiable when distinct values produce observationally distinguishable predictions.

Let the model prediction be

$$\boldsymbol{\mu}(\boldsymbol{\theta}) . \tag{1098}$$

The Fisher information matrix is

$$F_{ab} = \frac{\partial \boldsymbol{\mu}^{\mathsf{T}}}{\partial \theta_a} \mathsf{C}^{-1} \frac{\partial \boldsymbol{\mu}}{\partial \theta_b}. \tag{1099}$$

A nearly singular Fisher matrix indicates parameter degeneracy.

If

$$\lambda_{\min}(\mathsf{F}) \approx 0, \tag{1100}$$

one or more parameter combinations are poorly identifiable.

A historical model may contain mathematically distinct first- and second-leg parameters that collapse observationally into one effective transfer function.

In that case, the data may support

$$\text{an unspecified prehistory} \tag{1101}$$

without identifying

$$\text{the specific Triple Bounce sequence.} \tag{1102}$$

The manuscript must distinguish these evidential levels.

201 Historical Signature Hierarchy

Evidence may be organized into five levels.

201.1 Level 0: no residue

$$\Delta\boldsymbol{\mathcal{O}}_{\text{hist}} = \mathbf{0}. \tag{1103}$$

The model remains observationally equivalent to a standard third-leg initial-condition model.

201.2 Level 1: generic prehistory residue

A feature suggests some nontrivial earlier history but does not identify its form.

201.3 Level 2: finite-transition residue

The data favor one or more finite pre-hot transitions.

201.4 Level 3: two-reversal residue

A linked pattern requires two separated transition events and an intermediate propagating interval.

201.5 Level 4: Triple Bounce identification

The combined observables favor:

$$\boxed{\text{outward} \to \text{inward} \to \text{outward sectoral history}} \tag{1104}$$

within a globally expanding or otherwise derived background.

The theory should not claim Level 4 evidence when only Level 1 evidence is available.

202 Calibration, Validation, and Prediction Discipline

Flexible historical models are vulnerable to turning every numerical coincidence or residual feature into a fitted explanation. Paper II therefore distinguishes three evidential roles:

$$\mathcal{D}_{\mathrm{cal}}, \tag{1105}$$

$$\mathcal{D}_{\mathrm{val}}, \tag{1106}$$

and

$$\mathcal{D}_{\mathrm{hold}}. \tag{1107}$$

The calibration data may constrain a predeclared finite parameterization and shared nuisance quantities. The validation data test the unchanged model on observables not used to choose its feature family. The held-out set contains genuinely withheld or later measurements used for prospective prediction.

A disciplined workflow is:

1. specify the dynamical laws, transition parameterization, priors, and candidate signature family before inspecting validation residuals;
2. record which numerical relations were suggested by already known data;
3. use only the calibration set to estimate permitted free parameters;
4. freeze the model version before opening the validation set;

5. propagate the resulting posterior into validation and held-out observables without changing the architecture;
6. publish predicted ranges, null tests, and failure conditions; and
7. count every post-validation change of feature shape, parameter range, or transition law as a new model version.

A data-anchored structural relation is a calibration clue, not an independent prediction. It acquires predictive force only when the same locked relation determines an additional observable that was not used to construct it. This distinction applies to the packet correspondences as well as to proposed spectral, curvature, magnetic, and morphological signatures.

203 Look-Elsewhere and Feature-Search Penalties

Searching many frequencies, phases, locations, and feature shapes increases the probability of finding a spurious improvement.

Let

$$p_{\text{local}} \tag{1108}$$

be the probability associated with one fixed feature test.

After scanning many effective trials, the global probability is

$$p_{\text{global}} \geq p_{\text{local}}. \tag{1109}$$

A simplified independent-trial approximation is

$$p_{\text{global}} \approx 1 - (1 - p_{\text{local}})^{N_{\text{eff}}}, \tag{1110}$$

where N_{eff} is the effective number of searched configurations.

The Triple Bounce feature search must report:

- the scanned frequency range;
- the scanned phase range;
- the number of feature families;
- the prior parameter volume;
- the effective trial factor; and
- the global rather than only local significance.

A feature predicted before the data search carries greater evidential weight than one selected afterward.

204 Model Comparison for Historical Signatures

Let

$$\{\mathcal{M}_{\mathrm{base}}, \mathcal{M}_{\mathrm{inf,feat}}, \mathcal{M}_{\mathrm{bounce}}, \mathcal{M}_{\mathrm{TB}}\} \tag{1111}$$

denote the baseline, inflationary-feature, alternative-bounce, and Triple Bounce models.

The maximum likelihood is

$$\mathcal{L}_{\max} = \max_{\boldsymbol{\theta}} P\left(\mathcal{D} \mid \boldsymbol{\theta}, \mathcal{M}\right). \tag{1112}$$

The Akaike information criterion (Akaike, 1974) is

$$\mathrm{AIC} = 2p - 2\ln \mathcal{L}_{\max}, \tag{1113}$$

where p is the number of fitted parameters.

The Bayesian information criterion (Schwarz, 1978) is

$$\mathrm{BIC} = p \ln N - 2\ln \mathcal{L}_{\max}, \tag{1114}$$

where N is the effective number of data points under the stated approximation.

The Bayesian evidence follows the cosmological model-comparison framework summarized by Trotta (2008):

$$Z_{\mathcal{M}} = \int \mathcal{L}\left(\boldsymbol{\theta}\right) \pi\left(\boldsymbol{\theta} \mid \mathcal{M}\right) \mathrm{d}\boldsymbol{\theta}. \tag{1115}$$

The Bayes factor comparing the Triple Bounce model with model $\mathcal{M}_X$ is

$$B_{\mathrm{TB},X} = \frac{Z_{\mathrm{TB}}}{Z_X}. \tag{1116}$$

No single statistic should be treated as universally decisive.

The comparison should report likelihood improvement, parameter complexity, prior dependence, posterior predictive performance, and physical coherence.

205 Signature Matrix

Table 4: Candidate historical signatures, principal degeneracies, and required cross-checks.

Signature	**Triple Bounce origin**	**Major degeneracy**	**Required cross-check**
Low-k turnover	Finite geometric beginning or largest retained scaffold scale.	Inflationary cutoff, cosmic variance, foregrounds.	Joint temperature, polarization, and large-scale-structure transfer.
Oscillatory spectrum	Outer and central transition interference.	Inflationary steps, resonant features, systematics.	Linked frequencies and phases across TT, TE, EE, and matter power.
Phase coupling	Common pathway ancestry and central branching.	Generic nonlinear evolution and foreground morphology.	Primordial bispectrum and redshift-dependent structure tests.
Weak anisotropy	Residual directional memory of retraced pathways.	Survey geometry, scanning strategy, anisotropic inflation.	Independent sky maps, polarization, and large-scale structure.
Tensor chirality	Parity-sensitive central junction dynamics.	Chiral inflation or propagation effects.	Joint TB, EB, tensor spectrum, and magnetic helicity.
Curvature–magnetic correlation	Shared scaffold and electromagnetic emergence.	Astrophysical magnetogenesis and late feedback.	Redshift evolution and cross-correlation with primordial tracers.
Topology correction	Three-pass inherited web morphology.	Nonlinear gravitational clustering and bias.	Scale and redshift evolution predicted from one memory parameter.
Scalar–tensor relation	Common reversal transfer matrices.	Multifield or noncanonical inflation.	Correlation with oscillatory or memory features.
Background–feature relation	Shared reversal parameters control $H(z)$ and primordial transfer.	Flexible dark energy plus independent primordial features.	One joint posterior across background and perturbation data.

206 Minimum Distinctive Prediction Set

A minimum useful prediction set should contain at least three linked outputs.

One candidate set is

$$\boxed{\{\Delta_{\mathrm{osc}}(k), \Delta_{\mathrm{mem}}(k), \mathcal{P}_{\mathcal{R}B}(k)\}}\,. \tag{1117}$$

The transition theory would then predict:

1. the oscillatory frequencies from $\Delta\eta_{\mathrm{O}}$, $\Delta\eta_{\mathrm{C}}$, and $\Delta\eta_{\mathrm{OC}}$;
2. the memory-retention envelope from $\mathcal{E}_{\mathrm{mem}}(k)$; and
3. the curvature–magnetic correlation from the common scaffold conversion.

Another candidate set is

$$\boxed{\{\Delta_{\mathrm{low}\text{-}k}, g_*(k), \Delta_{\mathrm{ST}}(k)\}}\,. \tag{1118}$$

The final choice should be made after the central transition theory reveals which outputs are genuinely linked and robust.

The theory should not promise every possible anomaly simultaneously.

207 Historical-Identification Rejection Criteria

The distinctive-signature program fails or requires revision if any of the following conditions is unavoidable.

1. Every proposed signature is inserted phenomenologically rather than derived from the reversal dynamics.
2. The model requires a separate free function for every observable.
3. The outer- and central-transition features have unrelated fitted frequencies despite being claimed as one two-reversal pattern.
4. The inferred transition parameters are inconsistent across TT, TE, EE, large-scale structure, and lensing.
5. A proposed low-wavenumber feature is indistinguishable from cosmic variance under realistic model comparison.
6. The oscillatory improvement disappears after applying the look-elsewhere penalty.

7. The inherited-web morphology appears only after nonlinear evolution and has no primordial cross-observable counterpart.
8. The scaffold-memory model predicts anisotropy beyond observational limits.
9. The central model predicts parity violation that is not observed at the required amplitude.
10. The predicted curvature–magnetic correlation is absent where the model requires it to be nonzero.
11. The scalar and tensor spectra cannot be generated by one common transition parameter set.
12. The claimed historical residue is better explained by a simpler inflationary feature model.
13. The claimed residue is better explained by foregrounds, survey geometry, nonlinear evolution, or another late-time systematic.
14. The successful parameter region is disconnected from the region that meets the horizon and flatness requirements.
15. The model predicts a feature in withheld data that is not observed.
16. A feature is repeatedly redefined after each null result.
17. The data support only an unspecified prehistory while the manuscript claims identification of the complete Triple Bounce sequence.
18. No finite observable set can distinguish the model from arbitrary third-leg initial conditions.
19. The model's apparent improvement is entirely due to increased parameter volume.
20. The full joint evidence does not improve over the standard baseline or physically comparable alternatives.

208 Formal Summary of Distinctive Signatures

The complete observable prediction is

$$\boxed{\boldsymbol{\mathcal{O}}_{\rm TB} = \boldsymbol{\mathcal{O}}_{\rm rec} + \Delta\boldsymbol{\mathcal{O}}_{\rm hist}.} \tag{1119}$$

A measurable residue requires

$$\boxed{\Delta\boldsymbol{\mathcal{O}}_{\rm hist} \neq \mathbf{0}} \tag{1120}$$

within observational sensitivity.

The ordered transfer is

$$\boxed{\mathsf{M}_{123}(k) = \mathsf{M}_{\mathrm{C}}(k)\mathsf{U}_2(k)\mathsf{M}_{\mathrm{O}}(k).} \tag{1121}$$

A two-reversal interference phase may be

$$\boxed{\varphi_{\mathrm{hist}}(k) = 2\Theta_2(k) + \Delta\varphi_{\mathrm{OC}}(k).} \tag{1122}$$

The retained memory field is

$$\boxed{\mathcal{W}_{12}^{(3)}(k) = \mathcal{E}_{\mathrm{mem}}(k)\mathcal{W}_{12}^{(\mathrm{pre})}(k).} \tag{1123}$$

A linked historical signature vector is

$$\boxed{\boldsymbol{\mathcal{S}}_{\mathrm{hist}} = \left(S_{\mathrm{low}\text{-}k}, S_{\mathrm{osc}}, S_{\mathrm{phase}}, S_{\mathrm{aniso}}, S_{\mathrm{cross}}, S_{\mathrm{bg}}\right).} \tag{1124}$$

Historical identification requires more than one fitted anomaly:

$$\boxed{\begin{aligned}&\text{common parameters} + \text{multiple observables}\\ &+ \text{withheld predictions} + \text{fair alternative comparison.}\end{aligned}} \tag{1125}$$

The decisive question is

$$\boxed{\begin{array}{l}\text{Does one constrained two-reversal transfer theory predict a}\\ \text{linked set of background, scalar, tensor, magnetic, phase, and}\\ \text{morphological observables that survives independent validation}\\ \text{and cannot be reproduced as easily by a simpler alternative?}\end{array}} \tag{1126}$$

209 Historical-Signature Status

The distinctive-signature problem is now reduced to a controlled research sequence.

1. Derive the outer-transition transfer matrix.
2. Derive the second-leg mode propagator.
3. Derive the central-transition transfer matrix.
4. Calculate the ordered two-reversal interference structure.
5. Derive the scale-dependent memory-retention function.
6. Propagate the resulting scalar, tensor, vector, and magnetic covariance.

7. Identify which features share the same physical parameters.
8. Reject features that require independent phenomenological functions.
9. Construct low-wavenumber, oscillatory, phase, anisotropy, parity, and topology observables.
10. Propagate each primordial signature into the cosmic microwave background and large-scale structure.
11. Include late-time, foreground, and instrumental alternatives.
12. Apply look-elsewhere and parameter-volume penalties.
13. Fit only the declared calibration dataset.
14. publish validation results and withheld predictions.
15. Test cross-observable parameter consistency.
16. compare the model with inflationary-feature and other specified prehistory models.
17. state the evidential level actually supported by the data.
18. reject the historical interpretation if the linked predictions fail.

The Triple Bounce framework therefore does not treat unexplained cosmological anomalies as automatic evidence.

It requires a constrained, ordered, and independently testable historical residue.

210 Joint Model Comparison and Reciprocal Falsification

The preceding sections define separate tests for full-history causal sufficiency, curvature evolution, transition regularity, primordial conversion, cosmic microwave background recovery, and possible historical residues. These tests cannot be passed with mutually inconsistent backgrounds or with a new parameter choice for each observable.

A fixed Triple Bounce realization must generate one derived solution,

$$\mathcal{S}_{\mathrm{TB}}\left[\boldsymbol{\Theta}_{\mathrm{TB}}\right] = \Big\{a(t), H(t), R_E(t), \rho_i(t), p_i(t), Q_i(t), \mathsf{M}_{\mathrm{O}}, \mathsf{U}_2, \mathsf{M}_{\mathrm{C}}, \mathcal{H}_{\mathrm{th}}, \mathsf{P}_Z^{+}, \ldots\Big\}, \tag{1127}$$

from one finite, declared parameter vector.

Observational viability is the first joint burden:

$$\boxed{\begin{array}{cc} \mathcal{Q}_{\rm hor} \geq 1, & |\Omega_{k,0}| < \Omega_{k,\rm obs}^{\rm max}, \\ \mathcal{C}_{\rm TB} \in \mathcal{V}_{\rm CMB}, & \mathcal{O}_{\rm late} \in \mathcal{V}_{\rm late}, \\ \multicolumn{2}{c}{\mathcal{S}_{\rm stable} = 1.} \end{array}} \tag{1128}$$

Here,

$$\mathcal{S}_{\rm stable} = 1 \tag{1129}$$

denotes satisfaction of the adopted background, conservation, transition, and perturbative stability requirements on the resolved history.

Historical identification is a second and stronger burden. It requires that at least one derived residue be measurable, linked to the same parameters that produced observational viability, and more difficult to reproduce with relevant alternatives:

$$\boxed{\text{viability} + \text{two-reversal necessity} + \text{linked held-out evidence} + \text{fair alternative comparison.}} \tag{1130}$$

A model may therefore be observationally viable yet historically unidentified. Passing one component test supports neither the full incomplete-history hypothesis nor the specific Triple Bounce sequence.

211 Competing Model Classes

The comparison set must be broad enough to prevent distinctiveness by construction. At minimum, Paper II distinguishes five model classes.

211.1 Baseline inflationary cosmology

Let

$$\mathcal{M}_0 \tag{1131}$$

denote a specified inflationary ΛCDM baseline, with parameter vector

$$\boldsymbol{\theta}_0 = (\omega_b, \omega_c, \theta_*, \tau_{\rm re}, A_s, n_s, \ldots)\,. \tag{1132}$$

211.2 Inflation with additional features

Let

$$\mathcal{M}_{\mathrm{IF}} \tag{1133}$$

denote inflationary models with primordial features, multiple fields, nonstandard sound speed, isocurvature, or additional transition structure. These models are essential comparators whenever Triple Bounce uses similar observational flexibility.

211.3 Generic noninflationary prehistory

Let

$$\mathcal{M}_{\mathrm{P}} \tag{1134}$$

denote a physically specified bounce, emergent, contracting, or other pre-hot-history model that does not require the ordered Triple Bounce sequence. This class tests whether the data favor a longer history at all, rather than this particular history.

211.4 Triple Bounce complement model

Let

$$\mathcal{M}_{\mathrm{TB+I}} \tag{1135}$$

denote a model in which the first two legs prepare boundary conditions for a later inflationary stage:

$$\mathcal{H}_{\mathrm{TB}} \longrightarrow \mathcal{H}_{\mathrm{inf}} \longrightarrow \mathcal{H}_3. \tag{1136}$$

211.5 Triple Bounce replacement model

Let

$$\mathcal{M}_{\mathrm{TB}} \tag{1137}$$

denote a model in which the resolved hidden history and early third-leg handoff perform the causal, curvature, perturbative, and hot-state functions ordinarily assigned to inflation. Replacement requires

$$\mathcal{H}_{\mathrm{inf}} \notin \mathcal{M}_{\mathrm{TB}} \tag{1138}$$

as an independently necessary stage.

Every comparison must use compatible datasets, nuisance treatment, accuracy requirements, and prior disclosure. Alternative models should be granted the same opportunity for physically motivated features, but not arbitrary functions unavailable to the Triple Bounce model.

212 Master Parameter Hierarchy

Paper II separates the finite fitted parameter vector from the state functions and transfer operators derived from it. The master vector is

$$\boldsymbol{\Theta}_{\mathrm{TB}} = (\boldsymbol{\theta}_{\mathrm{res}}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_{\mathrm{O}}, \boldsymbol{\theta}_2, \boldsymbol{\theta}_{\mathrm{C}}, \boldsymbol{\theta}_3, \boldsymbol{\theta}_{\mathrm{late}}, \boldsymbol{\theta}_{\mathrm{nuis}})\,. \tag{1139}$$

The dimensions of all blocks must be fixed before validation. Whole functions such as $a(t)$, $H(t)$, or $H_3(a)$ and whole transfer matrices such as $\mathsf{M}_{\mathrm{O}}(k)$ are not automatically single parameters; they must be derived from stated dynamics or represented by a finite declared basis whose coefficients are counted.

212.1 Resolved-boundary input parameters

$$\boldsymbol{\theta}_{\mathrm{res}} = (\lambda_{\mathrm{res},1}, \ldots, \lambda_{\mathrm{res},n_{\mathrm{res}}})\,. \tag{1140}$$

This block parameterizes only the first resolved post-singularity boundary prescription $\mathcal{B}_{\mathrm{res}}$. It does not claim a complete parameterization of the pre-singularity state or of the singular opening.

212.2 First-leg parameters

$$\boldsymbol{\theta}_1 = (\Delta t_1, \lambda_{1,1}, \ldots, \lambda_{1,n_1})\,. \tag{1141}$$

212.3 Outer-transition parameters

$$\boldsymbol{\theta}_{\mathrm{O}} = (\tau_{\mathrm{O}}, \lambda_{\mathrm{O},1}, \ldots, \lambda_{\mathrm{O},n_{\mathrm{O}}})\,. \tag{1142}$$

212.4 Second-leg parameters

$$\boldsymbol{\theta}_2 = (\Delta t_2, \lambda_{2,1}, \ldots, \lambda_{2,n_2}) . \tag{1143}$$

212.5 Central-transition parameters

$$\boldsymbol{\theta}_{\mathrm{C}} = (\tau_{\mathrm{C}}, \Gamma_{\mathrm{C}}, \omega_I, \lambda_{\mathrm{mem}}, \lambda_{\mathrm{C},1}, \ldots, \lambda_{\mathrm{C},n_{\mathrm{C}}}) . \tag{1144}$$

These symbols remain effective until a central-transition law derives them. The reversal cause itself remains unresolved.

212.6 Third-leg parameters

$$\boldsymbol{\theta}_3 = (\boldsymbol{\theta}_{\mathrm{th}}, \boldsymbol{\theta}_{\mathrm{std}}, \boldsymbol{\lambda}_3) , \tag{1145}$$

where $\boldsymbol{\theta}_{\mathrm{th}}$ describes the finite thermal handoff, $\boldsymbol{\theta}_{\mathrm{std}}$ contains standard third-leg cosmological parameters, and $\boldsymbol{\lambda}_3$ contains only genuinely new Triple-Bounce-specific quantities.

212.7 Late-time and nuisance parameters

$$\boldsymbol{\theta}_{\mathrm{late}} = (\tau_{\mathrm{re}}, \boldsymbol{\lambda}_{\mathrm{late}}) , \tag{1146}$$

$$\boldsymbol{\theta}_{\mathrm{nuis}} = (\boldsymbol{\lambda}_{\mathrm{fg}}, \boldsymbol{\lambda}_{\mathrm{cal}}, \boldsymbol{\lambda}_{\mathrm{sys}}) . \tag{1147}$$

The hierarchy prevents one physical freedom from being silently reintroduced under several names. The Paper III dynamical realization is expected to reduce this hierarchy by deriving background and interaction quantities from one action or other explicit dynamical law.

213 Fundamental, Derived, and Effective Parameters

Every quantity in the model ledger must be assigned both a physical stage and an epistemic status.

213.1 Fundamental parameters

A fundamental parameter appears in a specified action, constitutive law, or resolved-boundary law. Paper II does not declare a parameter fundamental merely because it is given a physical name. Until the underlying dynamics are fixed, the set may be written schematically as

$$\boldsymbol{\theta}_{\mathrm{fund}} = (\boldsymbol{\theta}_{\mathrm{action}}, \boldsymbol{\theta}_{\mathrm{boundary}}) . \tag{1148}$$

213.2 Derived parameters and functions

A derived quantity is calculated from the fixed dynamical specification and fundamental inputs. The derived set includes

$$\left\{a(t), H(t), \Omega_k(t), \mathsf{M}_{\mathrm{O}}(k), \mathsf{M}_{\mathrm{C}}(k), \mathcal{H}_{\mathrm{th}}, \mathcal{P}_{\mathcal{R}}(k), C_\ell^{XY}\right\}. \tag{1149}$$

Quantities such as A_s, n_s, r, or a memory-retention scale count as derived predictions only when the model calculates them rather than fits them independently.

213.3 Effective parameters

An effective parameter summarizes unresolved microphysics over a stated regime. Examples may include

$$\left\{\bar{w}_j, c_{\mathrm{eff}}^{(j)}, \Gamma_j, T_{\mathcal{RW}}\right\}. \tag{1150}$$

Each effective quantity requires a domain of validity, a prior, the observables that constrain it, and a retirement criterion describing how later dynamics would replace it. A collection of independently fitted effective functions is not a predictive theory merely because each function has a plausible label.

214 Shared-Background Enforcement

The same background functions must be used throughout Paper II:

$$\mathcal{B}_{\mathrm{shared}} = \left\{a(t), H(t), \dot{H}(t), \rho_i(t), p_i(t), Q_i(t)\right\}. \tag{1151}$$

The horizon calculation uses

$$\chi_{\mathrm{causal}} = \chi_{\mathrm{causal}}\left[\mathcal{B}_{\mathrm{shared}}, c_{\mathrm{eff}}\right]. \tag{1152}$$

The curvature calculation uses

$$\Omega_k = \Omega_k\left[\mathcal{B}_{\mathrm{shared}}\right]. \tag{1153}$$

The transition calculation uses

$$\mathsf{M}_j = \mathsf{M}_j\left[\mathcal{B}_{\mathrm{shared}}, \boldsymbol{\theta}_j\right]. \tag{1154}$$

The cosmic microwave background calculation uses

$$C_\ell^{XY} = C_\ell^{XY}\left[\mathcal{B}_{\text{shared}}, \mathsf{P}_Z^{+}\right]. \tag{1155}$$

A shared-background consistency condition is

$$\boxed{\mathcal{B}_{\text{hor}} = \mathcal{B}_{\text{flat}} = \mathcal{B}_{\text{trans}} = \mathcal{B}_{\text{CMB}}.} \tag{1156}$$

Numerical implementations should enforce this equality at the software architecture level rather than rely only on editorial consistency.

215 Physical Prior Structure

For a frozen model version v, the prior is

$$\pi\left(\boldsymbol{\Theta}_{\text{TB}} \mid \mathcal{M}_{\text{TB}}^{(v)}\right). \tag{1157}$$

The model version includes the dynamical equations, finite parameterization, prior family, likelihood construction, and calibration–validation split. These choices must be recorded before the validation or held-out data are examined.

The prior may be factorized conditionally as

$$\begin{aligned}\pi\left(\boldsymbol{\Theta}_{\text{TB}}\right) = {} & \pi\left(\boldsymbol{\theta}_{\text{res}}\right)\pi\left(\boldsymbol{\theta}_1 \mid \boldsymbol{\theta}_{\text{res}}\right)\pi\left(\boldsymbol{\theta}_{\text{O}} \mid \boldsymbol{\theta}_1\right) \\ & \times \pi\left(\boldsymbol{\theta}_2 \mid \boldsymbol{\theta}_{\text{O}}\right)\pi\left(\boldsymbol{\theta}_{\text{C}} \mid \boldsymbol{\theta}_2\right) \\ & \times \pi\left(\boldsymbol{\theta}_3 \mid \boldsymbol{\theta}_{\text{C}}\right)\pi\left(\boldsymbol{\theta}_{\text{late}} \mid \boldsymbol{\theta}_3\right)\pi\left(\boldsymbol{\theta}_{\text{nuis}}\right).\end{aligned} \tag{1158}$$

The conditional structure represents physical continuity, not a license to tune each stage after seeing the next dataset. Central output densities, for example, cannot be assigned independently of the central input state when the transition law links them.

Unacceptable practices include arbitrarily broad priors that hide unconstrained flexibility, narrow priors centered on the observed answer, prior changes made after evidence inspection, independent priors for theoretically linked quantities, and removal of unstable regions only after they fit the data poorly. Any such change defines a new model version and must restart validation.

216 Hard Theoretical Priors

Some conditions are exclusions from a specified realization rather than soft likelihood penalties. Define

$$\mathbb{I}_{\mathrm{th}}(\boldsymbol{\Theta}) = \begin{cases} 1, & \boldsymbol{\Theta} \in \mathcal{V}_{\mathrm{th}}, \\ 0, & \boldsymbol{\Theta} \notin \mathcal{V}_{\mathrm{th}}. \end{cases} \tag{1159}$$

The domain $\mathcal{V}_{\mathrm{th}}$ applies to the resolved history beginning at t_{res}; it does not assert classical regularity through the singular opening. On that resolved domain, the scale factor must satisfy

$$a(t) > 0, \tag{1160}$$

and the adopted background must remain finite:

$$|H(t)| < \infty. \tag{1161}$$

For the globally expanding candidate branch specifically tested in this paper,

$$H(t) > 0 \tag{1162}$$

is a branch condition, not a universal theorem of the broader framework.

Once a dynamical realization is supplied, kinetic matrices must be free of ghostlike directions,

$$\lambda_a(\mathsf{K}) > 0, \tag{1163}$$

and squared propagation speeds must avoid uncontrolled gradient instabilities,

$$c_{s,a}^2 \geq 0. \tag{1164}$$

Total sector exchange must preserve the adopted conservation law,

$$\sum_i Q_i^{\nu} = 0, \tag{1165}$$

and both internal reversals must satisfy the derived finite-transition or controlled matching equations. The effective prior is

$$\pi_{\mathrm{eff}}(\boldsymbol{\Theta}) = \mathbb{I}_{\mathrm{th}}(\boldsymbol{\Theta})\pi(\boldsymbol{\Theta}). \tag{1166}$$

Until Papers III and IV provide the relevant dynamics, these are admissibility requirements to be tested, not conditions already shown to hold.

217 Likelihood Construction

Let the complete dataset be

$$\mathcal{D} = \{\mathcal{D}_{\rm CMB}, \mathcal{D}_{\rm BAO}, \mathcal{D}_{\rm SN}, \mathcal{D}_{H}, \mathcal{D}_{\rm LSS}, \mathcal{D}_{\rm GW}, \mathcal{D}_{\rm mag}\}\,. \tag{1167}$$

The joint likelihood is

$$\mathcal{L}_{\rm joint} = P\left(\mathcal{D} \mid \mathbf{\Theta}, \mathcal{M}\right). \tag{1168}$$

When conditional independence is justified,

$$\begin{aligned}\mathcal{L}_{\rm joint} = \mathcal{L}_{\rm CMB}\mathcal{L}_{\rm BAO}\mathcal{L}_{\rm SN}\mathcal{L}_{H} \\ \times \mathcal{L}_{\rm LSS}\mathcal{L}_{\rm GW}\mathcal{L}_{\rm mag}.\end{aligned} \tag{1169}$$

Correlated datasets require a joint covariance treatment rather than a simple product.

For a Gaussian data vector,

$$-2\ln\mathcal{L} = \left(\boldsymbol{d} - \boldsymbol{\mu}(\mathbf{\Theta})\right)^{\mathsf{T}} \mathsf{C}^{-1} \left(\boldsymbol{d} - \boldsymbol{\mu}(\mathbf{\Theta})\right) + \ln\det\mathsf{C} + \text{constant}. \tag{1170}$$

The covariance matrix may itself depend on the model.

That dependence must be retained when it is significant.

218 Benchmark Dataset Roles

Different datasets constrain different parts of the model.

218.1 Cosmic microwave background

The cosmic microwave background constrains:

$$\{\theta_*, \omega_b, \omega_c, A_s, n_s, \tau_{\rm re}, \Omega_k, \beta_{\rm iso}, r, g_*, \Delta_{\rm osc}\}\,. \tag{1171}$$

218.2 Baryon acoustic oscillations

baryon acoustic oscillation measurements constrain combinations such as

$$\frac{D_M(z)}{r_d}, \qquad H(z)r_d, \qquad \frac{D_V(z)}{r_d}, \tag{1172}$$

where r_d is the sound horizon at the baryon drag epoch.

218.3 Supernova distances

Type Ia supernova observations constrain the luminosity-distance relation,

$$D_L(z). \tag{1173}$$

218.4 Expansion-rate measurements

Direct or differential-age expansion measurements constrain

$$H(z). \tag{1174}$$

218.5 Large-scale structure

Large-scale structure constrains:

$$\{P_m(k,z), f\sigma_8(z), \mathrm{galaxy\ bias}, \mathrm{lensing}, \mathrm{topology}\}\,. \tag{1175}$$

218.6 Gravitational waves and magnetic observables

These datasets constrain candidate tensor, vector, parity, and curvature–magnetic historical residues.

Not every dataset must be included in the first implementation.

The minimum dataset must nevertheless test the specific claim being made.

219 Posterior Distribution

Bayes' theorem gives

$$P(\mathbf{\Theta} \mid \mathcal{D}, \mathcal{M}) = \frac{\mathcal{L}(\mathcal{D} \mid \mathbf{\Theta}, \mathcal{M}) \, \pi(\mathbf{\Theta} \mid \mathcal{M})}{Z_{\mathcal{M}}}, \tag{1176}$$

where

$$Z_{\mathcal{M}} = \int \mathcal{L}(\mathcal{D} \mid \mathbf{\Theta}, \mathcal{M}) \, \pi(\mathbf{\Theta} \mid \mathcal{M}) \, \mathrm{d}\mathbf{\Theta} \tag{1177}$$

is the Bayesian evidence.

Marginal posteriors are obtained by integrating over the other parameters.

For parameter θ_a,

$$P(\theta_a \mid \mathcal{D}) = \int P(\mathbf{\Theta} \mid \mathcal{D}) \prod_{b \neq a} \mathrm{d}\theta_b. \tag{1178}$$

Posterior concentration does not guarantee physical identifiability.

A narrow posterior may result from an overly restrictive prior or a misspecified model.

220 Evidence and Information Criteria

The Bayes factor comparing a fixed Triple Bounce model version with a specified baseline is

$$B_{\mathrm{TB},0} = \frac{Z_{\mathrm{TB}}}{Z_0}, \tag{1179}$$

with

$$\ln B_{\mathrm{TB},0} = \ln Z_{\mathrm{TB}} - \ln Z_0. \tag{1180}$$

The evidence comparison must report its dependence on prior volume, model version, dataset combination, nuisance treatment, and numerical accuracy. A favorable Bayes factor is not a substitute for physical admissibility or out-of-sample prediction.

Information criteria may be used as approximate complementary diagnostics (Akaike, 1974; Schwarz, 1978). The Akaike information criterion is

$$\mathrm{AIC} = 2p - 2 \ln \mathcal{L}_{\max}, \tag{1181}$$

and the Bayesian information criterion is

$$\mathrm{BIC} = p \ln N - 2 \ln \mathcal{L}_{\max}. \tag{1182}$$

When its assumptions are appropriate, the corrected Akaike criterion is

$$\mathrm{AIC}_c = \mathrm{AIC} + \frac{2p(p+1)}{N-p-1}. \tag{1183}$$

For correlated cosmological data, the effective sample size N and the regularity assumptions behind an information criterion may be nontrivial. No single evidence threshold, information criterion, or likelihood improvement is treated as decisive by itself. The final classification combines theoretical admissibility, parameter recovery, posterior prediction, complexity, and fair alternative comparison.

221 Effective Parameter Counting

The number of printed symbols is not the same as the model's effective flexibility. Shared baseline and nuisance parameters must be identified separately from genuinely new Triple-Bounce-specific freedom:

$$p_{\mathrm{fit}} = p_{\mathrm{shared}} + p_{\mathrm{TB}} + p_{\mathrm{nuis}}. \tag{1184}$$

All fitted quantities enter inference, while the incremental complexity of the proposed history is controlled primarily by p_{TB} and by the flexibility of any effective functions. A free function must be counted through its basis dimension, smoothing prior, regularization, or other effective degrees of freedom; it does not count as one parameter merely because it has one name.

Under one common diagnostic, Bayesian model complexity may be estimated as

$$p_{\mathrm{eff}} = \left\langle \chi^2 \right\rangle_{\mathrm{post}} - \chi^2(\hat{\mathbf{\Theta}}). \tag{1185}$$

The analysis should report the nominal and effective parameter counts, prior-to-posterior volume reduction, parameter correlations, unconstrained directions, the observables constraining each quantity, and whether the same parameter controls more than one prediction.

A public parameter ledger should record, for every fitted quantity, its symbol, physical stage, status as fundamental/derived/effective, prior, calibration data, validation data, affected observables, and whether it was introduced before or after a null result. A model with many unidentifiable directions or independently adjustable effective functions should not be described as highly predictive.

222 Simulation-Based Calibration

Before a frozen model version is applied to validation data, its inference pipeline must be tested on synthetic realizations. Let

$$\boldsymbol{\Theta}^{(s)} \sim \pi_{\text{eff}}(\boldsymbol{\Theta}) \tag{1186}$$

be an admissible simulated draw, and generate

$$\mathcal{D}^{(s)} \sim P\Big(\mathcal{D} \mid \boldsymbol{\Theta}^{(s)}, \mathcal{M}_{\text{TB}}^{(v)}\Big) . \tag{1187}$$

The pipeline then infers

$$P\Big(\boldsymbol{\Theta} \mid \mathcal{D}^{(s)}, \mathcal{M}_{\text{TB}}^{(v)}\Big) . \tag{1188}$$

Repeated simulations test sampler correctness, likelihood and prior implementation, parameter transformations, degeneracy handling, boundary behavior, numerical stability, and coverage. Calibration on synthetic data validates the inference machinery; it does not validate the cosmological hypothesis. A complex model should not be interpreted scientifically until the locked pipeline can recover controlled injections and reject deliberately misspecified cases.

223 Parameter-Recovery Tests

Parameter recovery asks whether known injected parameters can be reconstructed from simulated data.

For parameter θ_a, define the normalized recovery residual

$$z_a = \frac{\hat{\theta}_a - \theta_a^{\text{true}}}{\sigma_{\theta_a}}, \tag{1189}$$

where:

- $\hat{\theta}_a$ is a posterior point estimate; and
- σ_{θ_a} is the posterior uncertainty.

Over repeated simulations, a well-calibrated estimator should not show a persistent bias,

$$\langle z_a \rangle \approx 0. \tag{1190}$$

Parameter recovery should be performed for:

$$\{\tau_{\mathrm{O}}, \tau_{\mathrm{C}}, \Delta\eta_{\mathrm{OC}}, \lambda_{\mathrm{mem}}, \epsilon_{\mathrm{asym}}, \beta_{\mathrm{iso}}, A_{\mathrm{osc}}\}\,. \tag{1191}$$

If two-reversal parameters cannot be separately recovered even under high-quality synthetic data generated by the model, the observational claim should be reduced to an effective prehistory rather than a specific identified sequence.

224 Posterior Predictive Checks

The posterior predictive distribution is

$$P\left(\widetilde{\mathcal{D}} \mid \mathcal{D}, \mathcal{M}\right) = \int P\left(\widetilde{\mathcal{D}} \mid \boldsymbol{\Theta}, \mathcal{M}\right) P\left(\boldsymbol{\Theta} \mid \mathcal{D}, \mathcal{M}\right) \mathrm{d}\boldsymbol{\Theta}. \tag{1192}$$

Here, $\widetilde{\mathcal{D}}$ is a replicated dataset.

Define a discrepancy statistic

$$T\left(\mathcal{D}, \boldsymbol{\Theta}\right). \tag{1193}$$

A posterior predictive probability is

$$p_{\mathrm{ppc}} = P\left[T\left(\widetilde{\mathcal{D}}, \boldsymbol{\Theta}\right) \geq T\left(\mathcal{D}, \boldsymbol{\Theta}\right) \mid \mathcal{D}\right]. \tag{1194}$$

Useful discrepancy statistics include:

- maximum cosmic microwave background peak-position residual;
- integrated low-multipole residual;
- damping-tail residual power;
- largest cross-dataset parameter inconsistency;
- largest anisotropic-covariance residual;
- maximum large-scale-structure transfer residual; and
- largest stability-margin violation.

Posterior predictive checks identify ways in which a model may fit the overall likelihood while systematically failing a specific data structure.

225 Residual Structure

For observable vector $\boldsymbol{d}$, define the residual

$$\boldsymbol{r} = \boldsymbol{d} - \boldsymbol{\mu}\left(\hat{\boldsymbol{\Theta}}\right). \tag{1195}$$

The whitened residual is

$$\boldsymbol{r}_{\mathrm{w}} = \mathsf{C}^{-1/2}\boldsymbol{r}. \tag{1196}$$

A statistically adequate model should not leave strong structured patterns in $\boldsymbol{r}_{\mathrm{w}}$.

Residual analysis should test for:

- frequency-dependent oscillations;
- long runs of one residual sign;
- multipole-localized failures;
- redshift-dependent drift;
- direction-dependent residuals;
- foreground-correlated residuals; and
- mode-specific failures.

A model may improve the total likelihood while shifting the failure from one observable to another.

Joint residual analysis prevents such compensation from being hidden.

226 Ablation Studies

An ablation study removes one model component while preserving the rest.

The purpose is to identify which components are required by the data and which merely increase flexibility.

Define the full model prediction as

$$\boldsymbol{\mu}_{\mathrm{full}}. \tag{1197}$$

For component A, define the ablated prediction

$$\boldsymbol{\mu}_{-A}. \tag{1198}$$

The likelihood contribution of component A is

$$\Delta \ln \mathcal{L}_A = \ln \mathcal{L}_{\text{full}} - \ln \mathcal{L}_{-A}. \tag{1199}$$

Required ablations include:

1. remove the first-leg causal contribution;
2. remove the outer transition;
3. remove second-leg memory;
4. remove the central entropy-to-curvature conversion;
5. remove scaffold-memory conversion;
6. remove finite-transition oscillations;
7. remove statistical anisotropy;
8. remove tensor and vector outputs;
9. replace the derived central spectrum with a power law; and
10. replace the Triple Bounce background with the baseline background.

A specifically three-leg interpretation requires that the ordered combination perform better than a simpler one-transition or generic feature model after complexity penalties.

227 Causal and Curvature Ablations

The horizon statistic in the full model is

$$\mathcal{Q}_{\text{hor}}^{\text{full}} = \frac{2\left(\chi_1^{\text{ret}} + \chi_{\text{O}}^{\text{ret}} + \chi_2^{\text{ret}} + \chi_{\text{C}}^{\text{ret}} + \chi_3\right)}{\Delta\chi_{\max}}. \tag{1200}$$

The first-leg ablation is

$$\mathcal{Q}_{\text{hor}}^{-1} = \frac{2\left(\chi_{\text{O}}^{\text{ret}} + \chi_2^{\text{ret}} + \chi_{\text{C}}^{\text{ret}} + \chi_3\right)}{\Delta\chi_{\max}}. \tag{1201}$$

The second-leg ablation is

$$\mathcal{Q}_{\text{hor}}^{-2} = \frac{2\left(\chi_1^{\text{ret}} + \chi_{\text{O}}^{\text{ret}} + \chi_{\text{C}}^{\text{ret}} + \chi_3\right)}{\Delta\chi_{\max}}. \tag{1202}$$

If

$$\mathcal{Q}_{\rm hor}^{\rm full} \geq 1 \tag{1203}$$

but

$$\mathcal{Q}_{\rm hor}^{-1} < 1 \quad \text{and} \quad \mathcal{Q}_{\rm hor}^{-2} < 1, \tag{1204}$$

then both hidden legs are required for the causal candidate resolution under the adopted model.

A similar ablation should be performed for curvature transfer.

Define

$$\Xi_k = \ln T_k^{(\rm res \to 0)}. \tag{1205}$$

Remove each interval contribution to determine which stage suppresses, preserves, or amplifies curvature.

228 Historical-Signature Ablations

The full historical transfer is

$$\mathsf{M}_{123} = \mathsf{M}_{\rm C}\mathsf{U}_2\mathsf{M}_{\rm O}. \tag{1206}$$

A one-transition reduction is

$$\mathsf{M}_{\rm one} = \mathsf{M}_{\rm C}. \tag{1207}$$

A no-memory reduction is

$$T_{\mathcal{RW}} = 0, \qquad \mathcal{E}_{\rm mem} = 0. \tag{1208}$$

A no-interference reduction removes the phase-coherent cross terms:

$$A_{\rm OC} = 0. \tag{1209}$$

A signature supports the ordered history only if the full model predicts a linked feature that is lost or substantially degraded under these ablations.

Define the signature necessity statistic

$$\mathcal{N}_{{\rm sig},A} = 2\left[\ln \mathcal{L}_{\rm full} - \ln \mathcal{L}_{-A}\right]. \tag{1210}$$

This statistic must be interpreted with the appropriate parameter and boundary corrections.

229 Replacement Thesis and Empirical Outcome Decisions

Paper II separates the declared replacement thesis from the outcome that completed calculations and data may support. It also separates functional success from historical identification.

229.1 Function-specific replacement counterexample

For a specified function X, a Triple Bounce realization is a replacement counterexample when its frozen shared model passes the relevant theoretical and observational gate without an independent inflationary stage performing that function. For the joint horizon–flatness–perturbation program, representative gates include

$$\mathcal{Q}_{\rm hor} \geq 1, \tag{1211}$$

$$|\Omega_{k,0}| < \Omega_{k,\rm obs}^{\max}, \tag{1212}$$

$$\boldsymbol{\mathcal{C}}_{\rm TB} \in \mathcal{V}_{\rm CMB}, \tag{1213}$$

and

$$\mathbb{I}_{\rm th} = 1. \tag{1214}$$

Each gate decides only its own claim. Joint passage supports the maximal replacement package; no single failure is allowed to erase independently successful claims or the triple directional history.

229.2 Forced complement or restricted package

A completed model may retain a derived Triple Bounce contribution while requiring an additional stage for one or more remaining functions. This is a restricted or forced-complement outcome, not the paper's starting thesis. The surviving and rejected functions must be named separately.

229.3 Rejected realization, non-identification, and degeneracy

A fixed realization is rejected when no stable parameter region satisfies its predeclared conditions. A background-only success remains incomplete; an observationally viable but degenerate model remains historically unidentified; and absence of a distinctive residue is a failure of identification rather than proof that the hidden history was impossible. Failure of one replacement counterexample neither proves inflation necessary nor rejects another independent Triple Bounce claim.

230 Decision Matrix

Table 5: Decision matrix for the incomplete-history hypothesis.

Outcome	**Causal/geometric status**	**Perturbative/data status**	**Permitted interpretation**
Replacement candidate viable	Named horizon and flatness counterclaims pass without an independent inflationary stage performing those functions.	Derived primordial and thermal outputs recover the accepted third-leg observables.	Functional replacement remains viable; comparative support is a separate test.
Complement candidate viable	Hidden history contributes a derived function, while the completed joint model still requires an additional stage for another named function.	Inflation supplies part of the accepted primordial evolution.	The replacement package is restricted; independently successful Triple Bounce claims remain standing.
Background-only viability	Causal and curvature conditions can be met.	No acceptable derived perturbation or thermal handoff exists.	Incomplete background scenario only.
Observational recovery without identification	One admissible history exists.	Observables are recovered, but specific reversal parameters are not required or distinguishable.	Viable phenomenology; ordered Triple Bounce history not identified.
Generic prehistory evidence	Data favor a nontrivial prior stage.	A simpler noninflationary or feature model explains the same residue.	Evidence supports unspecified prehistory, not the complete sequence.
Fixed realization rejected	No admissible joint causal/geometric region exists, or data exclude it.	Derived perturbations or predictions fail locked validation.	The tested realization or corresponding claim is rejected.

231 Decisive Claim-Specific Rejection Conditions

The following conditions reject only the named counterclaim after the dynamical laws, finite parameter domain, priors, transfer operators, and numerical tolerances of a model version have been frozen. They do not declare every future realization impossible, do not propagate to independent claim families, and do not prove a competing necessity claim.

231.1 Horizon-counterexample rejection

The tested horizon counterexample is rejected for the frozen model class if

$$\sup_{\boldsymbol{\Theta}\in\mathcal{V}_{\rm th}} \mathcal{Q}_{\rm hor}(\boldsymbol{\Theta}) < 1. \tag{1215}$$

No theoretically admissible point in that class supplies sufficient retained causal preparation. This does not reject the triple directional history or prove inflation necessary.

231.2 Flatness-mechanism rejection

The tested near-flatness mechanism is rejected if

$$\inf_{\boldsymbol{\Theta}\in\mathcal{V}_{\rm th}} |\Omega_{k,0}(\boldsymbol{\Theta})| > \Omega_{k,\rm obs}^{\rm max}. \tag{1216}$$

This result does not decide the horizon mechanism or the existence of the three legs.

231.3 Central-conversion rejection

The tested central-conversion mechanism is rejected if every stable solution gives

$$\mathsf{P}_Z^+ \notin \mathcal{V}_{\rm prim}. \tag{1217}$$

231.4 CMB-recovery rejection

The tested CMB-recovery realization is rejected if

$$\boldsymbol{\mathcal{C}}_{\rm TB} \notin \mathcal{V}_{\rm CMB} \tag{1218}$$

for all admissible central outputs, thermal handoffs, and third-leg backgrounds in the frozen model version.

231.5 Maximal replacement-package status

The maximal replacement package is not established by a frozen joint realization if every admissible solution requires an independent accelerated stage satisfying

$$\epsilon_H < 1 \tag{1219}$$

for sufficient duration while performing one or more of the functions assigned to the Triple Bounce counterclaims. This verdict restricts the maximal package. It does not erase any independently successful Triple Bounce claim, reject the ordered trajectory, or prove that inflation is uniquely necessary.

231.6 Historical-identification failure

Specific identification of the ordered three-leg history fails when its predictions remain observationally equivalent, within the accessible covariance and resolution, to a relevant alternative:

$$\boldsymbol{\mu}_{\mathrm{TB}}(\boldsymbol{\Theta}_{\mathrm{TB}}) \simeq \boldsymbol{\mu}_{\mathrm{alt}}(\boldsymbol{\Theta}_{\mathrm{alt}}), \tag{1220}$$

with no locked, derived, degeneracy-breaking prediction. This is non-identification, not proof that the hidden history is physically impossible.

232 Partial Failure and Model Revision

Not every negative result rejects every layer of the framework. The revision order should move from implementation errors toward deeper architectural assumptions:

1. test numerical implementation and convergence;
2. test likelihood, prior, and covariance construction;
3. test parameter identifiability;
4. test the effective approximation and transition profile;
5. test the sector interaction law;
6. test the resolved-boundary prescription;
7. test the globally expanding candidate branch;
8. test the replacement claim;
9. test the complement claim; and
10. test the complete ordered three-leg architecture.

Revision is legitimate when an independently identifiable physical or computational deficiency motivates it. Adding free functions solely to restore agreement converts the model into unconstrained phenomenology. Every revision must receive a new version identifier and state which assumption and equation changed, which previous results no longer apply, which parameter or function was introduced or removed, which new prediction follows, and what observation could reject the revision. Previously opened validation data cannot remain "held out" for the new version.

233 Joint Validation Workflow

The full Paper II program should proceed in the following order.

Stage 1: dynamical specification

Fix one finite model version, including the resolved-boundary law, background equations, transition laws, thermal handoff, parameter ledger, and data split.

Stage 2: theoretical admissibility

Reject resolved histories with conservation failure, additional singularities, ghostlike modes, uncontrolled instabilities, or invalid matching.

Stage 3: background construction

Derive one shared background across all resolved legs and transitions.

Stage 4: horizon calculation

Calculate the raw and retained causal intervals.

Stage 5: curvature calculation

Propagate the resolved-boundary curvature distribution to the present.

Stage 6: transition perturbations

Derive the outer and central transfer operators.

Stage 7: primordial covariance and thermal handoff

Calculate the third-leg scalar, entropy, tensor, and vector covariance and establish a viable radiation-rich handoff.

Stage 8: Einstein–Boltzmann evolution

Propagate the accepted third-leg initial state into cosmic microwave background and matter observables.

Stage 9: synthetic-data calibration

Test inference, parameter recovery, identifiability, and deliberate misspecification before opening validation data.

Stage 10: calibration inference

Fit only the declared calibration data with predeclared priors and likelihoods.

Stage 11: locked validation and prediction

Propagate the frozen posterior into validation and held-out observables.

Stage 12: posterior predictive and residual tests

Inspect structured residuals, cross-observable consistency, and null tests.

Stage 13: ablation and model comparison

Determine whether both reversals and the intermediate leg are required, then compare replacement, complement, feature-inflation, and generic prehistory models.

Stage 14: scientific classification

Classify the result as rejected realization, restricted viability, complement candidate, replacement candidate, comparative support, or historical identification.

234 Minimum Publishable Numerical Result

A first publishable numerical result need not solve the complete model.

It should nevertheless contain one closed test.

A minimum horizon–flatness study should provide:

$$\{a(t), H(t), R_E(t), c_{\text{eff}}(t), \Omega_k(t)\}\,, \tag{1221}$$

together with:

$$\left\{\mathcal{Q}_{\text{hor}}, T_k^{(\text{res}\to 0)}, f_{\text{flat}}\right\}. \tag{1222}$$

A minimum perturbation study should provide:

$$\{\mathsf{M}_{\mathrm{C}}(k), \mathcal{P}_{\mathcal{R}}(k), \beta_{\text{iso}}(k), \mathcal{P}_T(k)\}\,, \tag{1223}$$

and at least one propagated observational spectrum.

A minimum distinctive-signature study should provide:

$$\{\text{one derived feature}, \text{one linked cross-check}, \text{one null test}\}\,. \tag{1224}$$

The result should report failure regions as clearly as successful regions.

235 Paper II Claim Classification

Paper II inherits Paper I's two-axis claim discipline.

235.1 Claim role

Each statement is identified as a core architectural postulate, candidate mechanism, candidate quantitative consequence, or open extension. A role describes what the claim does in the framework; it does not establish that the claim is true.

235.2 Development status

The maturity scale is:

1. conceptually defined;
2. analytically illustrated in a restricted toy model;

3. realized in a stable numerical solution;
4. shown to recover principal observations;
5. competitively supported against specified alternatives; and
6. historically identified by linked, held-out observables.

235.3 Current status

The present manuscript primarily defines architecture, test equations, and failure conditions. It does not yet establish stable numerical existence, observational recovery, comparative support, or historical identification.

235.4 Reserved maturity labels

The terms “observationally recovered,” “competitively supported,” and “historically identified” are reserved for the corresponding completed tests. They must not be inferred from conceptual coherence or from one numerical correspondence.

235.5 Cross-paper development

Paper III supplies candidate dynamics that may reduce effective freedoms. Paper IV carries the strongest reconstruction, numerical existence, sensitivity, and comparison work.

235.6 Historical identification

Specific identification requires independent linked observables to favor the ordered three-leg history over relevant alternatives. Observational recovery alone is not historical identification.

236 Formal Summary of Joint Model Comparison

The finite master parameter vector is

$$\boxed{\boldsymbol{\Theta}_{\mathrm{TB}} = (\boldsymbol{\theta}_{\mathrm{res}}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_{\mathrm{O}}, \boldsymbol{\theta}_2, \boldsymbol{\theta}_{\mathrm{C}}, \boldsymbol{\theta}_3, \boldsymbol{\theta}_{\mathrm{late}}, \boldsymbol{\theta}_{\mathrm{nuis}})\,.} \tag{1225}$$

One derived background must serve all component calculations:

$$\boxed{\mathcal{B}_{\mathrm{hor}} = \mathcal{B}_{\mathrm{flat}} = \mathcal{B}_{\mathrm{trans}} = \mathcal{B}_{\mathrm{CMB}}.} \tag{1226}$$

The theoretically admissible prior is

$$\boxed{\pi_{\mathrm{eff}}(\boldsymbol{\Theta}) = \mathbb{I}_{\mathrm{th}}(\boldsymbol{\Theta})\pi(\boldsymbol{\Theta}).} \tag{1227}$$

The posterior is

$$\boxed{P(\boldsymbol{\Theta} \mid \mathcal{D}, \mathcal{M}) = \frac{\mathcal{L}(\mathcal{D} \mid \boldsymbol{\Theta}, \mathcal{M})\pi(\boldsymbol{\Theta} \mid \mathcal{M})}{Z_{\mathcal{M}}}.} \tag{1228}$$

Observational viability requires a nonempty admissible region satisfying

$$\boxed{\begin{gathered} \mathcal{Q}_{\mathrm{hor}} \geq 1, \qquad |\Omega_{k,0}| < \Omega_{k,\mathrm{obs}}^{\max}, \\ \boldsymbol{\mathcal{C}}_{\mathrm{TB}} \in \mathcal{V}_{\mathrm{CMB}}, \qquad \mathbb{I}_{\mathrm{th}} = 1. \end{gathered}} \tag{1229}$$

Replacement requires

$$\boxed{\text{joint recovery without an independently necessary inflationary stage.}} \tag{1230}$$

Complement requires

$$\boxed{\text{a derived hidden-history contribution while inflation remains necessary.}} \tag{1231}$$

Specific historical identification requires

$$\boxed{\begin{aligned} &\text{two-reversal necessity} + \text{parameter recovery} \\ &+ \text{linked held-out observables} + \text{fair alternative comparison.} \end{aligned}} \tag{1232}$$

The decisive scientific question is

$$\boxed{\begin{aligned} &\text{Can one frozen, theoretically admissible model recover the} \\ &\text{causal, geometric, perturbative, thermal, and observational} \\ &\text{successes of inflationary cosmology while the ordered three-leg} \\ &\text{history improves constrained prediction rather than flexibility?} \end{aligned}} \tag{1233}$$

237 Integrated Scientific Synthesis

Paper II converts the incomplete-history proposal into a joint causal, geometric, perturbative, and observational test. The central claim is not that a longer history automatically replaces inflation. It is that the historical inference must remain open until the full ordered history has been calculated and compared fairly with alternatives.

237.1 Incomplete-history claim

Let $\mathcal{H}_{\rm inf}$ denote an inflationary pre-hot history and $\mathcal{H}_{123}$ the ordered Triple Bounce history. A history-compression degeneracy exists when

$$\mathcal{H}_{\rm inf} \neq \mathcal{H}_{123}, \qquad \mathcal{O}[\mathcal{H}_{\rm inf}] \approx \mathcal{O}[\mathcal{H}_{123}] \tag{1234}$$

within observational precision. Such equivalence establishes neither a Triple Bounce preference nor inflationary uniqueness. Historical identification requires at least one independently derived observable whose linked structure depends on the ordered two-reversal history.

237.2 Status of the first outward leg

The first leg is defined sectorally by $\dot{R}_E^{(1)} > 0$, whereas inflation is defined geometrically by $\ddot{a} > 0$, or $\epsilon_H < 1$. Therefore

$$\dot{R}_E^{(1)} > 0 \quad \not\Rightarrow \quad \epsilon_H^{(1)} < 1. \tag{1235}$$

A limited first-leg or second-leg interval may nevertheless be inflationary in the geometric sense. If such an interval performs the standard causal, curvature, and perturbation functions, it must be reported as inflationary rather than relabeled away.

237.3 Joint causal and curvature burden

The retained full-history causal domain must satisfy

$$\mathcal{Q}_{\rm hor}(k,\theta) = \frac{2\chi_{\rm ret}^{(123)}(k)}{d_k(\chi_*,\theta)} \geq 1 \tag{1236}$$

on the required scales and angular separations. The same background must also propagate the resolved curvature into the observed range,

$$|\Omega_{k,0}| = T_k^{(\rm res \to 0)} |\Omega_{k,\rm res}| < \Omega_{k,\rm obs}^{\rm max}. \tag{1237}$$

Within standard fixed-k Friedmann–Lemaître–Robertson–Walker evolution, $\mathrm{d}\ln|\Omega_k|/\mathrm{d}N = 2(\epsilon_H - 1)$. An everywhere-decelerating expanding history therefore does not suppress the standard curvature fraction. A viable near-flatness candidate must derive a sufficiently concentrated singular-opening boundary distribution, preserve it, include controlled accelerated subintervals, or derive modified dynamics. Causal reach and near-flatness must occupy the same finite-measure parameter region rather than separate toy backgrounds.

237.4 Transition, perturbation, and thermal burden

Neither internal reversal is a singularity, and neither reversal cause is specified in Paper II. Their effects must instead be represented by regular finite-width dynamics or a controlled thin-limit law. The central perturbation output has the schematic form

$$\boldsymbol{X}_{\mathrm{C}}^{+} = \mathsf{M}_{\mathrm{C}}\mathsf{U}_{2}\mathsf{M}_{\mathrm{O}}\boldsymbol{X}_{\mathrm{O}}^{-} + \boldsymbol{N}_{\mathrm{tot}}. \tag{1238}$$

This output must be stable, predominantly adiabatic, nearly scale-invariant, and phase coherent without using unconstrained hidden sectors to absorb unwanted power. The central boundary state is not identified with an immediately complete hot plasma. A derived thermal handoff must connect it to a radiation-rich third-leg state compatible with the standard downstream thermal chronology.

237.5 Observational recovery and historical identification

Observational viability requires the jointly predicted temperature, polarization, lensing, matter, and higher-order observables to lie within the accepted likelihood region after nuisance and late-time parameters are treated consistently. Historical identification is a stronger and separate requirement:

$$\boldsymbol{\mathcal{C}}_{\mathrm{TB}} \in \mathcal{V}_{\mathrm{CMB}}, \qquad \Delta\boldsymbol{\mathcal{O}}_{\mathrm{hist}} \neq \mathbf{0}. \tag{1239}$$

The first condition establishes recovery. The second, when measurable, independently validated, and favored against specified alternatives, would support the ordered prehistory. A model may satisfy the first without satisfying the second.

238 Comparison Classes and Outcome Boundaries

Inflation remains the principal benchmark because it provides a mature account of causal preparation, curvature suppression, coherent primordial perturbations, and thermal handoff. Triple Bounce must be compared with viable inflationary models rather than with a caricature of inflation.

Other comparison classes include global bounce cosmologies, cyclic histories, emergent-universe models, and generic noninflationary prehistories. Triple Bounce differs architecturally by containing one singular opening and two later nonsingular *Electro* reversals; it does not require global contraction, a second singularity, or indefinite recurrence. Those distinctions do not confer empirical preference. They only determine which equations, matching laws, and historical residues must be tested.

The permitted scientific outcomes remain:

$$\{\text{replacement candidate}, \text{complement}, \text{partial viability}, \text{generic prehistory support}, \text{failure}\}\,. \tag{1240}$$

Replacement requires one noninflationary history to recover the full benchmark set. Complement permits prior Triple Bounce preparation while an inflationary interval remains necessary. Partial viability records claim-specific success without promoting it to a complete replacement. Failure rejects the tested realization or mechanism, not every possible nonstandard prehistory.

239 Paper II Results and Limits

239.1 What the paper establishes

Paper II establishes a mathematical test architecture that:

1. separates sectoral *Electro* motion from metric inflation;
2. defines raw and retained full-history causal reach;
3. defines the angular horizon statistic $\mathcal{Q}_{\mathrm{hor}}$;
4. derives the conditional curvature no-suppression result within standard fixed-k Friedmann–Lemaître–Robertson–Walker evolution;
5. specifies singular-opening, outer-reversal, and central-reversal transfer requirements without treating the internal reversals as singularities;
6. defines the central primordial covariance and its thermal handoff;
7. specifies joint cosmic microwave background and structure tests;
8. distinguishes observational recovery from historical identification; and
9. defines finite-parameter comparison, complexity control, and claim-specific failure conditions.

239.2 What the paper does not establish

Paper II does not supply the microscopic identity of *Electro*, the force causing either reversal, a quantum theory of the singular opening, exact first- and second-leg backgrounds, a derived Gravity Triune action, a stable central-transition action, numerical horizon or curvature success, the measured primordial spectrum, a recovered cosmic microwave background likelihood, or observational evidence identifying the three-leg history. These remain explicit dependencies rather than hidden assumptions.

240 Series Boundary and Dependency Map

The four-paper series assigns different tasks to different levels of the framework. No paper may borrow an unresolved result from another as though it were already derived.

Table 6: Primary ownership and dependency boundaries in the first four papers.

Paper	**Primary ownership**	**Boundary carried forward**
Paper I	Frozen architecture, terminology, one-singularity history, Gravity Triune roles, candidate mechanisms, and claim discipline.	Does not provide the full causal, curvature, dynamical, or numerical solution.
Paper II	Inflation benchmark, full-history causal test, curvature transfer, transition requirements, primordial covariance, observational recovery, and falsification architecture.	Uses effective sector functions but does not derive their underlying covariant microphysics.
Paper III	Candidate binding–scaffolding–chasing dynamics, total conservation, interaction currents, stability, and background/perturbation equations.	Must supply or constrain the effective functions used here without assuming Paper II has already established viability.
Paper IV	Packet reconstruction, global mapped geometry, cumulative growth representation, numerical sensitivity, and comparison with Triple Bounce and standard Big Bang timing.	Must not use the packet correspondences as substitutes for the continuous causal, curvature, and perturbation tests defined in Paper II.

The provisional 13.824-gigayear, 46.08-gigalight-year, 10/3, and 1.0283 correspondences belong to the calibrated packet program. Paper II does not use them to establish causal sufficiency, curvature suppression, or an inflation replacement. Their reconstruction and physical interpretation remain Paper IV responsibilities.

241 Completion Dependencies Across Papers II–IV

The remaining calculations are ordered by dependency rather than by paper length.

241.1 Shared background and conservation

Paper III must supply a covariant or otherwise explicit dynamical realization from which Paper II can obtain one background $a(t), H(t), R_E(t), \rho_i(t), p_i(t), Q_i(t)$ across all legs and finite transitions. Global expansion throughout the hidden legs is a candidate branch to be solved, not an assumption that every realization must obey.

241.2 Causal and curvature evaluation

Using the same background, Paper II must calculate the effective propagation laws, retained causal domain, singular-opening curvature distribution, and complete curvature transfer. A viable model requires a nonempty finite-measure region satisfying both the horizon and flatness criteria.

241.3 Finite transition dynamics

Paper III must constrain the outer and central transition dynamics, including total conservation, stability, anisotropic stress, and any sector exchange. Paper II then derives the corresponding background and perturbation transfer operators rather than fitting them independently.

241.4 Primordial covariance and thermal handoff

Paper II must propagate the inherited covariance through the central transition, derive the thermal handoff, and establish a viable third-leg scalar, tensor, vector, and isocurvature state.

241.5 Observational propagation and comparison

The derived covariance must be implemented in an Einstein–Boltzmann solver and evaluated with one frozen parameter ledger, declared priors, calibration and validation data, complexity control, and at least one held-out linked prediction.

241.6 Global packet reconstruction

Paper IV reconstructs the packet hierarchy and units, audits the proposed 2.304-billion-year candidate, tests sensitivity to approximate inputs, and compares the declared third-leg endpoint structure with relevant cosmological benchmarks. Its final provenance audit excludes the 2.304-billion-year candidate from the active hierarchy. Only after that reconstruction may numerical correspondences be compared with the dynamical quantities used in Papers II and III.

242 Minimum Mathematical Construction

A minimum closed Paper II model requires the following system.

242.1 Background equations

$$H^2 = \frac{8\pi G}{3}\rho_{\mathrm{tot}} - \frac{kc^2}{a^2}, \tag{1241}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{\mathrm{tot}} + \frac{3p_{\mathrm{tot}}}{c^2}\right), \tag{1242}$$

and

$$\dot{\rho}_i + 3H\left(\rho_i + \frac{p_i}{c^2}\right) = Q_i. \tag{1243}$$

242.2 Sector trajectory equation

$$\ddot{R}_E + \Gamma_E \dot{R}_E + \frac{\partial V_{\mathrm{eff}}}{\partial R_E} = \mathcal{F}_E. \tag{1244}$$

This schematic equation must eventually be replaced by a covariant field law.

242.3 Causal propagation

$$\chi_j(k) = \int_{\mathcal{I}_j} \frac{c_{\mathrm{eff}}^{(j)}(t,k)}{a(t)}\mathrm{d}t. \tag{1245}$$

242.4 Curvature transfer

$$\frac{\mathrm{d}\ln|\Omega_k|}{\mathrm{d}N} = 2\left(\epsilon_H - 1\right) + \mathcal{S}_k. \tag{1246}$$

The standard model has

$$\mathcal{S}_k = 0. \tag{1247}$$

242.5 Perturbation transfer

$$\boldsymbol{X}_{\mathrm{C}}^{+} = \mathsf{M}_{\mathrm{C}}(k)\mathsf{U}_2(k)\mathsf{M}_{\mathrm{O}}(k)\boldsymbol{X}_{\mathrm{O}}^{-} + \boldsymbol{N}_{\mathrm{tot}}(k). \tag{1248}$$

242.6 Observational propagation

$$C_\ell^{XY} = 4\pi \sum_{A,B} \int \frac{\mathrm{d}k}{k} \mathcal{P}_{AB}(k) \Delta_{\ell,A}^X(k) \Delta_{\ell,B}^Y(k). \tag{1249}$$

These equations define the minimum closed calculation required to move beyond conceptual architecture.

243 Core Testable Hypotheses After Paper II

The completed Paper II architecture yields seven central hypotheses.

Hypothesis 243.1 (Full-history causal extension)**.** *The first and second legs provide a non-negligible retained causal extension beyond the third-leg-only history.*

Hypothesis 243.2 (Causal sufficiency)**.** *The retained full-history causal domain is large enough to prepare the required last-scattering separations.*

Hypothesis 243.3 (Balanced boundary-curvature concentration)**.** *A singular-opening boundary prescription constrained by the balanced pre-singularity state produces a distribution of resolved classical curvature concentrated near zero.*

Hypothesis 243.4 (Near-flat curvature survival)**.** *The complete hidden and observable history propagates the resolved boundary curvature through both nonsingular reversals into the observationally allowed present range.*

Hypothesis 243.5 (Stable central conversion)**.** *A finite stable central transition converts the inherited two-pass state into an acceptable third-leg perturbation covariance.*

Hypothesis 243.6 (CMB recovery)**.** *The derived central state reproduces the measured microwave-background temperature, polarization, damping, and lensing observables.*

Hypothesis 243.7 (Historical residue)**.** *At least one linked observable retains information that specifically depends on the ordered two-reversal history.*

Each hypothesis may fail independently. No conjunction of these hypotheses is called "the Triple Bounce claim." A maximal all-function replacement package requires the relevant counterclaims to pass jointly, but failure of one does not reject the triple directional history or the other independent claim families.

244 Final Decision Logic

The final scientific classification follows the sequence below.

1. Does a regular global background exist?
2. Does it preserve total energy–momentum conservation?
3. Does it remain free of fatal instabilities?
4. Does the full retained causal history satisfy

$$\mathcal{Q}_{\rm hor} \geq 1? \tag{1250}$$

5. Does the same background satisfy

$$|\Omega_{k,0}| < \Omega_{k,\rm obs}^{\max}? \tag{1251}$$

6. Does the central transition generate an acceptable primordial covariance?
7. Does the derived covariance reproduce the cosmic microwave background?
8. Are the required parameters identifiable?
9. Does the model remain competitive after complexity penalties?
10. Is an independent inflationary stage still necessary?
11. Does a distinctive historical residue survive?

The outcome is then classified as:

$$\boxed{\{\text{replacement}, \text{complement}, \text{partial viability}, \text{generic prehistory}, \text{failure}\}.} \tag{1252}$$

245 Conclusion and Compact Test Set

Paper II asks whether inflationary inference may be partly an incomplete-history effect. It does not answer that question by invoking a longer duration, an outward *Electro* trajectory, or inherited structure alone. The proposal survives only if one derived physical history satisfies the complete causal, curvature, transition, perturbation, thermal, and observational burden.

The minimum causal condition is

$$\mathcal{Q}_{\rm hor}(k,\theta) = \frac{2\chi_{\rm ret}^{(123)}(k)}{d_k(\chi_*,\theta)} \geq 1. \tag{1253}$$

The same background must satisfy

$$|\Omega_{k,0}| = T_k^{(\mathrm{res}\to 0)} |\Omega_{k,\mathrm{res}}| < \Omega_{k,\mathrm{obs}}^{\max}. \tag{1254}$$

The central output must be derived from the ordered transfer,

$$\boldsymbol{X}_{\mathrm{C}}^{+} = \mathsf{M}_{\mathrm{C}}\mathsf{U}_2\mathsf{M}_{\mathrm{O}}\boldsymbol{X}_{\mathrm{O}}^{-} + \boldsymbol{N}_{\mathrm{tot}}, \tag{1255}$$

and must yield an observationally viable third-leg covariance,

$$\boldsymbol{\mathcal{C}}_{\mathrm{TB}} \in \mathcal{V}_{\mathrm{CMB}}. \tag{1256}$$

A positive historical identification carries the additional burden

$$\Delta\boldsymbol{\mathcal{O}}_{\mathrm{hist}} \neq \mathbf{0} \tag{1257}$$

at a measurable, independently validated level after comparison with inflationary feature models and other prehistories.

The present result is therefore a reciprocal test architecture, not a completed validation. Its declared thesis is that the full three-leg history replaces the separate inflationary stage and the single-direction historical reconstruction. Replacement, restricted package, non-identification, degeneracy, and claim-specific rejection remain empirical outcomes. Each outcome must land on the exact claim tested, and neither side receives automatic validation from the local failure of the other.

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