The Gravity Triune

Binding, Scaffolding, and Chasing as Coupled Gravitational Functions in the Triple Bounce Cosmological Framework

Paper III of the Triple Bounce Cosmology Series

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Status statement. This Version 1.1.0 manuscript inherits the primary replacement thesis and independent-claim protocol of Papers I and II (Fine, 2026a,b). It selects GT1 as the minimum third-leg reference realization: binding is the metric response of general relativity, scaffolding is a cold non-luminous sector coupled through $A^2(\phi)g_{\mu\nu}$, and chasing is represented provisionally by a canonical scalar field. GT1, the broader Gravity Triune role architecture, the full Triple Bounce history, the pre-singularity oscillation, and the inflation-replacement counterclaims are not logically identical. Failure of GT1 does not automatically reject those broader claims; success of GT1 does not validate them. The integrated audit establishes internal series consistency, not physical validation. Every observable must still descend from one shared background, one finite parameter ledger, and the Paper II covariance and handoff. Version 1.1.0 supersedes Version 1.0.1 by adding explicit non-propagation, reciprocal falsification, and revision-lock rules while preserving the action, equations, and numerical results of the selected GT1 realization.

Abstract

Paper I defined the Triple Bounce Cosmological Framework as a speculative outward–inward–outward history of a provisional effective sector called *Electro*, while Paper II established the causal, curvature, transition-matching, perturbative, and observational burdens that any realization must satisfy. Paper III develops the candidate dynamics of the Gravity Triune: *binding*, *scaffolding*, and *chasing*.

The three terms denote connected gravitational functions rather than three automatically independent forces. Binding remains the metric response of general relativity and is not inserted a second time as an additional material stress–energy sector. Scaffolding denotes the non-luminous clustering and structural-support function conventionally assigned to dark matter. Chasing denotes the expansion-driving function conventionally represented by a cosmological constant or another dark-energy-like component. The paper imposes a non-double-counting rule, total covariant conservation, a single physical metric for visible matter, and explicit recovery of general relativity and ΛCDM in the appropriate control limit.

A minimal hybrid reference realization, designated GT1, is developed in which the Einstein metric supplies binding, a cold scaffold sector couples conformally to a canonical scalar field, and the scalar supplies chasing behavior. The common action fixes the sector-exchange currents and permits one background, local-gravity, perturbation, lensing, structure-growth, microwave-background, and gravitational-wave test program. The manuscript also separates minimum third-leg viability from historical identification of the hidden first and second legs. A viable third-leg interacting model would not by itself demonstrate inherited scaffold memory or the complete Triple Bounce history.

The resulting construction is a covariantly formulable and falsifiable candidate model, not a completed validation. The homogeneous GT1 system admits an exact general-relativity plus ΛCDM background control, radiation and matter-like saddle regimes, and scalar-dominated accelerated solutions. For the positive-slope, positive constant-coupling exponential branch, an accelerated scaffold–chasing scaling attractor requires a coupling large enough to prevent recovery of a near-standard pressureless-matter epoch within the observable third-leg benchmark; this is a conditional obstruction to that simplest branch rather than a no-go theorem for GT1 as a whole. The local, scaffolding, chasing, interaction, and perturbation audits define one sign-consistent third-leg theory and export the same central covariance defined in frozen Paper II. The integrated closure audit requires all background-distance, microwave-background, baryon-acoustic-oscillation, growth, lensing, cluster, local-gravity, compact-object, and gravitational-wave calculations to use that same background, covariance, thermal handoff, and parameter ledger. It separates calibration from locked validation, includes nuisance parameters and cross-covariances, controls functional freedom, and distinguishes ordinary third-leg viability from the stronger burden of historical identification. A historical claim requires a linked, action-derived residue that survives independent validation and comparison with relevant alternatives; one fitted anomaly is insufficient. Ordinary species and Einstein–Boltzmann evolution begin only after a separately derived thermal handoff, and nothing in the downstream radiation and pressureless-matter benchmarks fixes when matter, antimatter, or particle-like excitations first originated. Numerical inference, finite reversals, nonlinear structure, local and compact-system solutions, and historical identification remain required. Paper III therefore supplies the internally reconciled dynamical

and computational bridge between the architecture of Paper I, the causal and geometric tests of Paper II, and the constrained reconstruction reserved for Paper IV.

Series Replacement Thesis and Paper III Claim Boundary

The four-paper series advances the full outward–inward–outward history as a replacement for the standard single-direction historical reconstruction and for the necessity of a separate primordial inflationary stage. Paper III does not withdraw that thesis. Its narrower task is to determine whether one Gravity Triune realization can supply a covariant, conserving, and stable third-leg gravitational sector and can later be extended across the hidden legs.

The following objects are distinct:

$$\boxed{\begin{aligned}\text{GT1} &\neq \text{Gravity Triune role architecture}\\ &\neq \text{Triple Bounce directional history}\\ &\neq \text{inflation replacement package.}\end{aligned}} \tag{1}$$

Accordingly,

$$\text{Fail(GT1)} \not\Rightarrow \text{Fail(Gravity Triune)} \not\Rightarrow \text{Fail(Triple Bounce)}, \tag{2}$$

while

$$\text{Pass(GT1)} \not\Rightarrow \text{Pass(historical identification)}. \tag{3}$$

Within GT1 itself, every result must name its scope: parameter point, parameter region, mechanism, selected realization, or model class. A failure of historical memory does not reject minimum third-leg viability. A failure of minimum third-leg GT1 rejects the selected realization but does not automatically decide the independent horizon, flatness, pre-singularity, packet, or triple-directional claims.

Paper III also participates in reciprocal testing. A completed noninflationary three-leg realization that uses one shared Gravity Triune model and performs a specified inflationary function would falsify the claim that inflation is necessary for that function. Conversely, failure of GT1 does not prove inflation necessary. The action, coupling signs, parameter domain, observable pipeline, and validation data of GT1 must be frozen before decisive testing; changing them defines a new realization rather than an ad hoc rescue.

Keywords: gravity triune; binding; scaffolding; chasing; dark matter; dark energy; general relativity; cosmology; Triple Bounce; stress–energy exchange; cosmological perturbations.

Contents

1 Introduction

Literature basis. The observational and theoretical baseline summarized here follows the general-relativistic, dark-matter, and accelerating-expansion literature (Einstein, 1915; Wald, 1984; Zwicky, 1933; Rubin and Ford, 1970; Riess et al., 1998; Perlmutter et al., 1999; Planck Collaboration et al., 2020). The series-specific architecture is defined in frozen Papers I and II (Fine, 2026a,b).

Modern cosmology describes gravitational phenomena across an enormous range of scales.

At local and astrophysical scales, gravitation binds planets, stars, galaxies, and larger systems.

At galactic and cosmological scales, observations indicate substantially more gravitating structure than visible matter alone supplies.

At the largest scales, the expansion history is consistent with a late-time acceleration conventionally attributed to a cosmological constant or another dark-energy-like component.

The standard ΛCDM framework organizes these phenomena through:

1. general relativity as the gravitational theory;
2. ordinary baryonic and radiative stress–energy;
3. cold dark matter as a non-luminous clustering component; and
4. a cosmological constant or dark-energy sector as the source of accelerated expansion.

The Gravity Triune does not begin by denying these successful effective descriptions.

It asks whether the observable functions conventionally divided among ordinary gravitation, dark matter, and dark energy may be parts of a deeper coupled architecture.

The proposed functional triune is

$$\boxed{\textit{binding} + \textit{scaffolding} + \textit{chasing}.} \tag{4}$$

The terms identify proposed dynamical roles.

They do not yet identify unique fundamental substances.

The paper therefore distinguishes:

$$\boxed{\text{functional decomposition} \neq \text{established ontological decomposition.}} \tag{5}$$

This distinction is necessary because general relativity does not represent gravitational binding as an ordinary local material stress–energy tensor.

Geometry responds to stress–energy through the field equations.

Accordingly, a binding function may be encoded by the standard metric response, by an additional field, by modified gravitational dynamics, or by some combination of these possibilities.

The scaffolding and chasing functions are similarly provisional.

Their relation to conventional dark matter and dark energy must be derived and tested.

2 Purpose of Paper III

Paper III has six principal purposes.

1. Define the Gravity Triune without prematurely declaring three new fundamental forces.
2. Distinguish the functional roles of binding, scaffolding, and chasing.
3. Construct a covariant stress–energy and interaction framework capable of representing those roles.
4. Determine how the three functions may operate across the directional legs and reversals of the Triple Bounce history.
5. Derive the background and perturbative requirements needed to recover known gravitational and cosmological observations.
6. Identify observations and calculations capable of supporting, restricting, or falsifying the proposed triune.

The central theoretical question is

$$\boxed{\begin{array}{l}\text{Can one covariant, conserving, and stable gravitational theory}\\ \text{produce attractive binding, persistent non-luminous}\\ \text{scaffolding, and large-scale chasing behavior while recovering}\\ \text{the tested limits of general relativity and cosmology?}\end{array}} \tag{6}$$

3 Scientific Status

The Gravity Triune is a speculative research framework.

It is not presently an established physical theory.

This paper does not claim that:

- dark matter has been identified;
- dark energy has been identified;
- general relativity has been experimentally falsified;

- three new fundamental gravitational forces have been discovered;
- the selected GT1 action has been uniquely derived from a deeper theory or empirically established;
- the Triple Bounce background has already been solved;
- the galaxy and cluster data are already reproduced;
- the observed expansion history has already been fitted;
- the gravitational-wave constraints have already been satisfied; or
- the triune is observationally preferred to ΛCDM.

The framework is instead evaluated through the sequence

$$\boxed{\text{definition} \longrightarrow \text{equations} \longrightarrow \text{stability} \longrightarrow \text{solutions} \longrightarrow \text{observables} \longrightarrow \text{model comparison.}} \tag{7}$$

Citation and derivation convention. Citations identify established equations, benchmark models, observational datasets, and prior theoretical constructions. Triple Bounce-specific definitions, GT1 choices, conditional obstructions, transition templates, memory variables, and historical-identification criteria remain this manuscript's own proposals or derivation targets. A cited precedent does not validate the Triple Bounce interpretation, and an uncited model-specific equation is not presented as an established external result.

Until this sequence is completed, the terms *binding*, *scaffolding*, and *chasing* should be understood as provisional functional categories.

4 Scope and Exclusions

Paper III addresses the gravitational-sector architecture of the Triple Bounce framework.

Its scope includes:

- covariant field equations;
- stress–energy decomposition;
- sector interaction currents;
- homogeneous cosmological backgrounds;
- local and cosmological limits;
- linear perturbations;

- structure growth;
- gravitational lensing;
- accelerated expansion;
- stability;
- conservation laws;
- the relation to the three *Electro* legs; and
- observational falsification.

The following are not derived in this foundational formulation:

- the microscopic identity of dark matter;
- the microscopic identity of dark energy;
- a quantum theory of gravity;
- a complete theory of the pre-singularity state;
- the physical reversal force of *Electro*;
- a detailed galaxy-halo fitting program;
- a complete nonlinear simulation suite;
- the baryon asymmetry;
- the origin of the magnetic field;
- the complete thermodynamic history of either reversal;
- the global 10/3 mapped-geometry interpretation; or
- the packet-scale construction reserved for Paper IV.

The detailed mapped geometry and packet reconstruction are reserved for Paper IV and may not be used to repair an unresolved Paper III equation or interaction law.

5 Definition of the Gravity Triune

Definition 5.1 (Gravity Triune)**.** *The Gravity Triune is a proposed decomposition of gravitational behavior into three coupled functional roles: binding, scaffolding, and chasing.*

The three roles may be represented schematically as

$$\mathfrak{G} = \{\mathfrak{G}_{\text{bind}}, \mathfrak{G}_{\text{scaf}}, \mathfrak{G}_{\text{chase}}\}. \tag{8}$$

Here:

- $\mathfrak{G}_{\text{bind}}$ denotes the binding function;
- $\mathfrak{G}_{\text{scaf}}$ denotes the scaffolding function; and
- $\mathfrak{G}_{\text{chase}}$ denotes the chasing function.

The word *function* is used deliberately.

The initial framework permits:

$$\boxed{\begin{array}{l}\text{three distinct fields,}\\ \text{three regimes of one field,}\\ \text{three projections of a coupled system,}\\ \text{or a hybrid of geometric and material effects.}\end{array}} \tag{9}$$

The framework therefore does not assume at the outset that

$$\mathfrak{G}_{\text{bind}}, \quad \mathfrak{G}_{\text{scaf}}, \quad \mathfrak{G}_{\text{chase}} \tag{10}$$

are three independent fundamental forces.

6 Binding

Standard benchmark. Metric binding, geodesic motion, and tested weak-field limits are used in their standard general-relativistic sense (Einstein, 1915; Wald, 1984; Will, 2014).

6.1 Functional definition

Definition 6.1 (Binding function)**.** *Binding is the gravitational function associated with attractive response, dynamical consolidation, orbital organization, collapse, and the formation or maintenance of gravitationally bound systems.*

Binding is expected to dominate phenomena such as:

- planetary and stellar orbits;
- hydrostatic equilibrium;
- gravitational collapse;
- binary dynamics;
- the binding of galaxies and clusters;
- gravitational redshift;
- time delay;
- light deflection; and
- the local weak-field limit.

6.2 Binding in general relativity

In standard general relativity, gravitational binding is not represented by a unique local tensor

$$T_{\mu\nu}^{\text{gravity}} \tag{11}$$

with the same invariant status as material stress–energy.

The attractive response is encoded in the relation between geometry and stress–energy:

$$G_{\mu\nu} + \Lambda_{\text{bare}} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{\text{tot}}. \tag{12}$$

Accordingly, the most conservative realization of the binding function is

$$\boxed{\mathfrak{G}_{\text{bind}} = \text{the attractive metric response of general relativity.}} \tag{13}$$

In that realization, no independent binding stress–energy tensor is introduced.

6.3 Independent binding-sector possibility

A more general model may introduce an additional binding field ψ_{bind}.

It would require:

$$\left\{S_{\text{bind}}, T_{\mu\nu}^{\text{bind}}, Q_{\text{bind}}^{\mu}, \text{local coupling law}, \text{screening or recovery limit}\right\}. \tag{14}$$

Such a field must not double count the attractive response already produced by the metric and ordinary stress–energy.

In the selected GT1 reference realization, this alternative is not active:

$$\boxed{S_{\text{bind}}^{\text{extra}} = 0, \qquad T_{\mu\nu}^{\text{bind}} = 0.} \tag{15}$$

Independent binding fields remain comparison models and may not be used to rescue the selected GT1 realization after its predictions are fixed.

7 Scaffolding

Standard benchmark. The scaffolding target is anchored to the historical and modern evidence for non-luminous gravitating structure and cold-dark-matter phenomenology (Zwicky, 1933; Rubin and Ford, 1970; Peebles, 1980; Bertone et al., 2005; Planck Collaboration et al., 2020).

7.1 Functional definition

Definition 7.1 (Scaffolding function)**.** *Scaffolding is the non-luminous clustering and structural-support function benchmarked against the gravitational behavior conventionally assigned to dark matter. Historical memory is an additional Triple Bounce hypothesis and is not included in this functional definition.*

At the phenomenological level, scaffolding is benchmarked against the functions conventionally assigned to dark matter.

These include:

- additional gravitational potentials in galaxies and clusters;
- gravitational lensing not accounted for by visible matter alone;
- enhanced structure growth;
- the matter contribution to acoustic and equality scales;
- the formation of a cosmic web;

- the persistence of large-scale gravitational wells; and
- non-radiative clustering.

7.2 Persistent structural memory

The Triple Bounce framework assigns scaffolding an additional proposed role.

It may preserve an imprint of first-leg *Electro* organization through the second inward leg and into the third outward leg.

Represent the inherited scaffold state by

$$\mathcal{W}_{12} = \mathcal{W}\left[\mathcal{I}_1, \mathcal{I}_{\mathrm{O}}, \mathcal{I}_2\right]. \tag{16}$$

The third-leg scaffold may then be written as

$$\mathcal{W}_3(k) = \mathcal{E}_{\mathrm{mem}}(k)\mathcal{W}_{12}(k) + N_{\mathcal{W}}(k), \tag{17}$$

where:

- $\mathcal{E}_{\mathrm{mem}}(k)$ is a scale-dependent retention factor; and
- $N_{\mathcal{W}}(k)$ represents newly generated scaffold structure.

The existence of such memory is a hypothesis.

It is not assumed to follow automatically from dark matter.

Criterion 7.2 (Covariant memory requirement)**.** *A pressureless dark component counts as a carrier of Triple Bounce history only if the theory supplies a covariant memory variable, its transport equation, its transfer through both finite transitions, and a derived contribution to third-leg observables. Ordinary clustering or phase-space persistence alone does not establish historical memory.*

7.3 Scaffolding burden

A viable scaffolding theory must determine:

$$\left\{\rho_{\mathrm{scaf}}, p_{\mathrm{scaf}}, c^2_{s,\mathrm{scaf}}, \sigma_{\mathrm{scaf}}, Q^{\mu}_{\mathrm{scaf}}, \mathcal{P}_{\mathrm{scaf}}(k), \text{lensing response}\right\}. \tag{18}$$

It must recover the successful clustering behavior of cold dark matter or provide a quantitatively superior alternative.

8 Chasing

Standard benchmark. The chasing target is anchored to late-time acceleration and the cosmological-constant and dynamical-dark-energy literature (Riess et al., 1998; Perlmutter et al., 1999; Weinberg, 1989; Peebles and Ratra, 2003; Copeland et al., 2006).

8.1 Functional definition

Definition 8.1 (Chasing function)**.** *Chasing is the proposed large-scale expansion-driving function represented at background level by a sufficiently negative effective pressure or an equivalent covariant contribution. Its persistence across the three legs is a Triple Bounce hypothesis, not a local velocity field or a material shell.*

The term *chasing* refers to a dynamical role.

It does not by itself imply:

- a material shell moving through space;
- a physical outer boundary;
- superluminal transport;
- a literal fluid chasing the edge of the universe;
- identity with microwave-background photons; or
- identity with the E front.

In standard cosmology, a cosmological constant is represented by

$$T^{(\Lambda)}_{\mu\nu} = -\rho_\Lambda c^2 g_{\mu\nu}, \tag{19}$$

with

$$w_\Lambda = -1. \tag{20}$$

The Gravity Triune asks whether the chasing function is:

1. exactly cosmological-constant-like;
2. a dynamical dark-energy field;
3. an interaction-generated effective pressure;
4. a geometric modification;

5. a state-dependent component of a unified sector; or
6. a combination of these possibilities.

8.2 Acceleration requirement

For a homogeneous background, accelerated expansion requires

$$\rho_{\text{tot}} + \frac{3p_{\text{tot}}}{c^2} < 0. \tag{21}$$

If chasing dominates the acceleration, it must contribute sufficiently negative effective pressure:

$$\rho_{\text{chase}} + \frac{3p_{\text{chase}}}{c^2} < 0 \tag{22}$$

after interactions and the other sectors are included.

9 Ontological Realizations

The Gravity Triune permits three principal realization classes.

9.1 Distinct-sector realization

In the distinct-sector realization,

$$\{\psi_{\text{bind}}, \psi_{\text{scaf}}, \psi_{\text{chase}}\} \tag{23}$$

are physically distinct degrees of freedom.

Their stress–energy tensors are

$$\left\{T^{\text{bind}}_{\mu\nu}, T^{\text{scaf}}_{\mu\nu}, T^{\text{chase}}_{\mu\nu}\right\}. \tag{24}$$

This realization has the clearest additive decomposition.

It also introduces the largest number of independent degrees of freedom.

9.2 Unified-sector realization

In the unified-sector realization, one underlying field or medium

$$\Psi_{\text{G}} \tag{25}$$

exhibits different functional regimes.

Schematically,

$$\Psi_{\mathrm{G}} \longrightarrow \begin{cases} \mathfrak{G}_{\mathrm{bind}}, & \mathcal{R}_{\mathrm{bind}}, \\ \mathfrak{G}_{\mathrm{scaf}}, & \mathcal{R}_{\mathrm{scaf}}, \\ \mathfrak{G}_{\mathrm{chase}}, & \mathcal{R}_{\mathrm{chase}}, \end{cases} \tag{26}$$

where $\mathcal{R}_i$ denotes a state-space or scale-dependent regime.

The three functions are then not separately additive substances.

9.3 Hybrid geometric–sector realization

In the hybrid realization:

- binding is primarily the standard metric response;
- scaffolding is a non-luminous material or effective field sector; and
- chasing is a vacuum, field, interaction, or geometric contribution.

This is the most conservative initial architecture because it minimizes the risk of double counting ordinary gravity.

Remark 9.1. *The names of the three functions do not determine which realization is correct. The realization must be selected through mathematical consistency and observation.*

9.4 Selected GT1 reference realization

Paper III selects one realization for controlled development rather than carrying every ontology forward with equal freedom. The minimum third-leg reference theory is

$$\boxed{\mathcal{G}^{(3)}_{\mathrm{GT1}} = \langle a(t), g_{\mu\nu}, \phi, \psi_{\mathrm{scaf}}; V(\phi), A(\phi) \rangle}\,, \tag{27}$$

with the functional assignment

$$\boxed{\begin{aligned} \textit{binding} &= \text{general-relativistic metric response}, \\ \textit{scaffolding} &= \text{cold non-luminous source sector}, \\ \textit{chasing} &= \text{canonical scalar } \phi. \end{aligned}} \tag{28}$$

The complete historical extension is denoted

$$\mathcal{G}_{\mathrm{GT1}}^{(123)} = \mathcal{G}_{\mathrm{GT1}}^{(3)} + \{\mathrm{E}, \mathcal{W}, \mathcal{I}_1, \mathcal{I}_{\mathrm{O}}, \mathcal{I}_2, \mathcal{I}_{\mathrm{C}}\}\,, \tag{29}$$

where the additional variables encode the hidden legs, scaffold memory, and the two finite nonsingular transitions. The reference-model development order is

$$\boxed{\text{third-leg closure} \longrightarrow \text{third-leg viability} \longrightarrow \text{historical closure} \longrightarrow \text{historical identification.}} \tag{30}$$

Selecting GT1 does not claim that it is the unique microscopic ontology. It prevents later sections from changing the action, coupling, or source content independently whenever a phenomenological difficulty appears.

10 The Non-Double-Counting Principle

A gravitational model can become internally inconsistent if the same physical effect is counted once through geometry and again through an additional stress–energy component.

Definition 10.1 (Non-double-counting principle)**.** *Every gravitational contribution must appear exactly once in the fundamental field equations or in a mathematically specified effective reduction of those equations.*

For example, if binding is defined entirely as the ordinary general-relativistic response to

$$T_{\mu\nu}^{\mathrm{vis}} + T_{\mu\nu}^{\mathrm{rad}} + T_{\mu\nu}^{\mathrm{scaf}} + T_{\mu\nu}^{\mathrm{chase}}, \tag{31}$$

then one must set

$$T_{\mu\nu}^{\mathrm{bind}} = 0. \tag{32}$$

If an independent binding field is introduced, its effects must be distinguished from the response already generated by the Einstein tensor.

Similarly, if chasing is represented by a cosmological constant, one must avoid simultaneously inserting an identical vacuum component into

$$\Lambda_{\mathrm{bare}} \tag{33}$$

and

$$T_{\mu\nu}^{\mathrm{chase}}. \tag{34}$$

A useful bookkeeping choice is

$$\Lambda_{\mathrm{bare}} = 0 \tag{35}$$

when the complete effective acceleration is assigned to the chasing sector.

Alternatively, both may be retained as physically distinct quantities, but their degeneracy must be tested.

For the selected GT1 reference realization, the bookkeeping is fixed more strictly:

$$\boxed{T_{\mu\nu}^{\mathrm{bind}} = 0, \qquad S_{\mathrm{bind}}^{\mathrm{extra}} = 0, \qquad \Lambda_{\mathrm{bare}} = 0.} \tag{36}$$

The ΛCDM control limit is recovered by taking $A(\phi) \to 1$, $\dot{\phi} \to 0$, and $V(\phi) \to \rho_\Lambda$, rather than by adding a second identical vacuum term. Any later departure from this bookkeeping defines a different comparison model, not an adjustment of GT1.

11 Covariant Action-Level Framework

Formal basis. The action-level construction, metric variation, and covariant conservation requirements follow standard field-theoretic and general-relativistic practice (Noether, 1918; Wald, 1984; Weinberg, 1995).

A general action may be written as

$$S = S_{\mathrm{EH}} + S_{\mathrm{vis}} + S_{\mathrm{rad}} + S_{\mathrm{bind}} + S_{\mathrm{scaf}} + S_{\mathrm{chase}} + S_{\mathrm{int}}, \tag{37}$$

where:

- S_{EH} is the Einstein–Hilbert or generalized gravitational action;
- S_{vis} contains visible matter;
- S_{rad} contains radiation and relativistic species;
- S_{bind} is present only in independent binding-field realizations;
- S_{scaf} contains the scaffolding degrees of freedom;
- S_{chase} contains the chasing degrees of freedom; and
- S_{int} contains interactions among sectors.

For standard general relativity,

$$S_{\mathrm{EH}} = \frac{c^3}{16\pi G} \int \mathrm{d}^4x \, \sqrt{-g} \, (R - 2\Lambda_{\mathrm{bare}}) \, . \tag{38}$$

The stress–energy tensor of sector i is

$$T^{(i)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_i}{\delta g^{\mu\nu}}. \tag{39}$$

The complete field equations are obtained by varying the total action.

A candidate Gravity Triune theory should not specify the background equation of state independently from the covariant action if the latter is already known.

12 Stress–Energy Decomposition

In the distinct- or hybrid-sector realization, the total stress–energy tensor may be written as

$$T^{\mathrm{tot}}_{\mu\nu} = T^{\mathrm{vis}}_{\mu\nu} + T^{\mathrm{rad}}_{\mu\nu} + T^{\mathrm{bind}}_{\mu\nu} + T^{\mathrm{scaf}}_{\mu\nu} + T^{\mathrm{chase}}_{\mu\nu} + T^{\mathrm{other}}_{\mu\nu}. \tag{40}$$

When binding is purely metric,

$$T^{\mathrm{bind}}_{\mu\nu} \equiv 0. \tag{41}$$

The field equations are then

$$G_{\mu\nu} + \Lambda_{\mathrm{bare}} g_{\mu\nu} = \frac{8\pi G}{c^4} T^{\mathrm{tot}}_{\mu\nu}. \tag{42}$$

A unified-field realization need not admit the same additive decomposition.

Instead, one may have

$$T^{\mathrm{G}}_{\mu\nu} = T^{\mathrm{G}}_{\mu\nu} \left[\Psi_{\mathrm{G}}, \nabla\Psi_{\mathrm{G}}, g_{\mu\nu} \right], \tag{43}$$

with different observable functions arising from different limits.

The decomposition into binding, scaffolding, and chasing is then an effective projection:

$$\mathfrak{P}_{\mathrm{bind}} + \mathfrak{P}_{\mathrm{scaf}} + \mathfrak{P}_{\mathrm{chase}} = \mathfrak{I} \tag{44}$$

over the relevant response space.

The projection operators must be defined by the theory.

13 Covariant Conservation and Sector Exchange

Formal basis. The total-conservation bookkeeping used here follows from diffeomorphism invariance and the contracted Bianchi identity (Noether, 1918; Wald, 1984).

The Bianchi identity implies

$$\nabla_\mu G^{\mu\nu} = 0. \tag{45}$$

Therefore, the complete matter and effective-sector system must satisfy

$$\boxed{\nabla_\mu T^{\mu\nu}_{\text{tot}} = 0.} \tag{46}$$

Individual sectors may exchange energy and momentum:

$$\nabla_\mu T^{\mu\nu}_i = Q^\nu_i. \tag{47}$$

Total conservation requires

$$\boxed{\sum_i Q^\nu_i = 0.} \tag{48}$$

The exchange vector may be decomposed relative to a four-velocity u^μ:

$$Q^\mu_i = Q_i u^\mu + F^\mu_i, \tag{49}$$

where

$$u_\mu F^\mu_i = 0. \tag{50}$$

Here:

- Q_i is the energy-transfer rate in the selected frame; and
- F^μ_i is the momentum-transfer component.

The total transfer conditions are

$$\sum_i Q_i = 0, \tag{51}$$

and

$$\sum_i F_i^\mu = 0. \tag{52}$$

A proposed exchange law must specify its frame dependence and covariant origin.

14 Homogeneous Cosmological Background

Background basis. The homogeneous background equations retain the Friedmann–Lemaître framework and standard cosmological notation (Friedmann, 1922; Lemaître, 1927; Weinberg, 2008; Planck Collaboration et al., 2020).

Consider an FLRW background,

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right]. \tag{53}$$

Define the Hubble rate

$$H = \frac{\dot{a}}{a}. \tag{54}$$

In this paper, ρ_i denotes mass-equivalent energy density so that the pressure contribution appears as p_i/c^2.

The first Friedmann equation is

$$H^2 = \frac{8\pi G}{3}\rho_{\mathrm{tot}} - \frac{kc^2}{a^2}. \tag{55}$$

The second background equation may be written as

$$\dot{H} = -4\pi G\left(\rho_{\mathrm{tot}} + \frac{p_{\mathrm{tot}}}{c^2}\right) + \frac{kc^2}{a^2}. \tag{56}$$

The acceleration equation is

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{\mathrm{tot}} + \frac{3p_{\mathrm{tot}}}{c^2}\right). \tag{57}$$

The total density and pressure are

$$\rho_{\mathrm{tot}} = \rho_{\mathrm{vis}} + \rho_{\mathrm{rad}} + \rho_{\mathrm{bind}} + \rho_{\mathrm{scaf}} + \rho_{\mathrm{chase}} + \rho_{\mathrm{other}}, \tag{58}$$

$$p_{\mathrm{tot}} = p_{\mathrm{vis}} + p_{\mathrm{rad}} + p_{\mathrm{bind}} + p_{\mathrm{scaf}} + p_{\mathrm{chase}} + p_{\mathrm{other}}. \tag{59}$$

Again,

$$\rho_{\text{bind}} = p_{\text{bind}} = 0 \tag{60}$$

in the purely metric-binding realization.

15 Background Sector Continuity Equations

The homogeneous sector equations are

$$\dot{\rho}_i + 3H\left(\rho_i + \frac{p_i}{c^2}\right) = Q_i. \tag{61}$$

Total conservation requires

$$\sum_i Q_i = 0. \tag{62}$$

Define the equation-of-state parameter

$$w_i = \frac{p_i}{\rho_i c^2}. \tag{63}$$

Then

$$\dot{\rho}_i + 3H\left(1 + w_i\right)\rho_i = Q_i. \tag{64}$$

For an uncoupled constant-w_i sector,

$$\rho_i \propto a^{-3(1+w_i)}. \tag{65}$$

Thus:

$$w_i = 0 \quad \Longrightarrow \quad \rho_i \propto a^{-3}, \tag{66}$$

$$w_i = \frac{1}{3} \quad \Longrightarrow \quad \rho_i \propto a^{-4}, \tag{67}$$

$$w_i = -1 \quad \Longrightarrow \quad \rho_i = \text{constant}. \tag{68}$$

The Gravity Triune interactions may alter these simple scalings.

Any alteration must be derived from the exchange currents.

16 Effective Acceleration and the Chasing Function

Using

$$w_{\text{eff}} = \frac{p_{\text{tot}}}{\rho_{\text{tot}} c^2}, \tag{69}$$

the acceleration condition becomes

$$w_{\text{eff}} < -\frac{1}{3}. \tag{70}$$

If the chasing sector is primarily responsible, then

$$\rho_{\text{chase}} \left(1 + 3w_{\text{chase}}\right) + \sum_{i \neq \text{chase}} \rho_i \left(1 + 3w_i\right) < 0. \tag{71}$$

The chasing function may generate acceleration through:

1. an intrinsic equation of state with $w_{\text{chase}} < -1/3$;
2. interaction terms that produce an effective negative pressure;
3. a modified gravitational coupling;
4. a nonminimal geometric interaction; or
5. a unified-field state whose large-scale response resembles dark energy.

Define an effective chasing equation of state through the continuity equation:

$$w_{\text{chase}}^{\text{eff}} = w_{\text{chase}} - \frac{Q_{\text{chase}}}{3H\rho_{\text{chase}}}. \tag{72}$$

This definition shows that apparent negative-pressure behavior can arise from intrinsic pressure, energy exchange, or both.

17 Scaffolding Background Behavior

A cold scaffolding benchmark has

$$w_{\text{scaf}} \approx 0, \tag{73}$$

and small effective sound speed,

$$c_{s,\text{scaf}}^2 \approx 0. \tag{74}$$

If uncoupled,

$$\rho_{\mathrm{scaf}} \propto a^{-3}. \tag{75}$$

A coupled scaffold obeys

$$\dot{\rho}_{\mathrm{scaf}} + 3H\left(1 + w_{\mathrm{scaf}}\right)\rho_{\mathrm{scaf}} = Q_{\mathrm{scaf}}. \tag{76}$$

The background scaffold must provide the matter-like contribution required by the third-leg expansion history.

Its perturbations must also cluster appropriately.

Matching the background density alone is insufficient.

The scaffolding theory must simultaneously determine:

$$\boxed{\rho_{\mathrm{scaf}}(a), \quad \delta_{\mathrm{scaf}}(k,a), \quad \theta_{\mathrm{scaf}}(k,a), \quad \sigma_{\mathrm{scaf}}(k,a), \quad \Phi(k,a), \quad \Psi(k,a).} \tag{77}$$

18 Binding Across Scales

The binding function must recover the tested local limit.

In a weak-field, slowly moving regime, write

$$g_{00} \approx -\left(1 + \frac{2\Phi_N}{c^2}\right). \tag{78}$$

The standard Newtonian limit is

$$\nabla^2 \Phi_N = 4\pi G \rho_{\mathrm{source}}. \tag{79}$$

A generalized triune model may instead produce

$$\nabla^2 \Phi = 4\pi G_{\mathrm{eff}}\left(k, a, \mathcal{S}\right)\rho_{\mathrm{eff}} + \mathcal{S}_{\Phi}, \tag{80}$$

where:

- G_{eff} is an effective gravitational coupling;
- $\mathcal{S}$ denotes the sector state; and
- $\mathcal{S}_{\Phi}$ is an additional derived source.

Local recovery requires

$$G_{\text{eff}} \longrightarrow G \tag{81}$$

and

$$\mathcal{S}_\Phi \longrightarrow 0 \tag{82}$$

within the experimentally tested regime, unless a nonzero correction is already observationally permitted.

A cosmological model that fails the local binding limit is not viable.

19 Relationship to the Three Electro Legs

The Gravity Triune is intended to operate throughout the Triple Bounce history.

Its role in each leg remains provisional.

19.1 First outward leg

During the first leg,

$$\dot{R}_E^{(1)} > 0. \tag{83}$$

The provisional functional ordering is

$$\boxed{\begin{aligned} \textit{scaffolding} &: \text{initial structural organization,} \\ \textit{chasing} &: \text{persistent outward background contribution,} \\ \textit{binding} &: \text{weak, latent, or unresolved attractive response.} \end{aligned}} \tag{84}$$

This ordering is a hypothesis.

It must be derived from the sector equations.

19.2 Outer reversal

At the outer reversal,

$$\dot{R}_E : + \longrightarrow -. \tag{85}$$

The scaffold is hypothesized to retain part of the first-leg imprint.

The chasing function is hypothesized to remain outward or expansion-supporting.

The background scale factor may continue satisfying

$$H > 0. \tag{86}$$

19.3 Second inward leg

During the second leg,

$$\dot{R}_E^{(2)} < 0. \tag{87}$$

The provisional ordering is

$$\boxed{\begin{aligned} \textit{scaffolding} &: \text{persistent inherited structure,} \\ \textit{chasing} &: \text{continued background expansion support,} \\ \textit{binding} &: \text{increasing consolidation as interaction density rises.} \end{aligned}} \tag{88}$$

The inward *Electro* trajectory does not imply that chasing or the global scale factor reverses.

19.4 Central reversal

At the central reversal,

$$\dot{R}_E : - \longrightarrow +. \tag{89}$$

The Gravity Triune must participate in the conversion of the inherited state into:

- a radiation-rich hot universe;
- ordinary electromagnetic behavior;
- stable matter;
- third-leg gravitational potentials;
- a persistent scaffold; and
- an expansion-driving chasing component.

19.5 Third outward leg

During the observable third leg,

$$\dot{R}_E^{(3)} > 0. \tag{90}$$

The proposed functional roles become

$$\boxed{\begin{aligned} \textit{binding} &: \text{local and astrophysical attraction}, \\ \textit{scaffolding} &: \text{non-luminous structure and potential support}, \\ \textit{chasing} &: \text{large-scale expansion-driving behavior}. \end{aligned}} \tag{91}$$

This third-leg mapping is the portion most directly testable against observations.

20 Functional Persistence and Regime Change

The triune functions need not have constant amplitudes throughout the three legs.

Define dimensionless functional weights

$$f_{\text{bind}}(t), \qquad f_{\text{scaf}}(t), \qquad f_{\text{chase}}(t). \tag{92}$$

A normalized representation may satisfy

$$f_{\text{bind}} + f_{\text{scaf}} + f_{\text{chase}} = 1 \tag{93}$$

only when the three functions form a complete effective decomposition.

The individual weights may evolve as

$$\dot{f}_A = \mathcal{F}_A \left(f_{\text{bind}}, f_{\text{scaf}}, f_{\text{chase}}, \mathrm{E}, H, \rho_i, \Gamma_j \right). \tag{94}$$

The theory must determine whether the transitions involve:

- energy transfer among distinct sectors;
- a phase transition within one unified sector;
- a change in coupling strength;
- a change in the effective equation of state;
- a change in propagation speed;

- a change in clustering behavior; or
- a combination of these mechanisms.

The descriptive claim that one function becomes dominant does not replace the need for an evolution equation.

21 Benchmark Against General Relativity and LCDM

Benchmark basis. The control target is the tested general-relativistic plus cold-dark-matter plus dark-energy framework rather than a weakened surrogate (Wald, 1984; Peebles, 1980; Planck Collaboration et al., 2020).

The standard benchmark is not a straw man.

It has successfully organized a wide range of gravitational and cosmological observations.

A schematic standard decomposition is

$$T^{\mathrm{std}}_{\mu\nu} = T^{b}_{\mu\nu} + T^{\gamma}_{\mu\nu} + T^{\nu}_{\mu\nu} + T^{\mathrm{CDM}}_{\mu\nu} + T^{\Lambda}_{\mu\nu}. \tag{95}$$

The nearest functional comparison is

$$\begin{aligned} \mathfrak{G}_{\mathrm{bind}} &\longleftrightarrow \text{general-relativistic attractive response,} \\ \mathfrak{G}_{\mathrm{scaf}} &\longleftrightarrow \text{cold-dark-matter gravitational structure,} \\ \mathfrak{G}_{\mathrm{chase}} &\longleftrightarrow \text{cosmological-constant or dark-energy behavior.} \end{aligned} \tag{96}$$

The arrows denote benchmark correspondence.

They do not establish identity.

A Gravity Triune model must recover or improve upon the benchmark in:

$$\boxed{\begin{aligned}&\text{local gravity,}\\&\text{orbital and relativistic dynamics,}\\&\text{gravitational lensing,}\\&H(z),\\&\text{distance–redshift relations,}\\&\text{CMB anisotropies,}\\&\text{BAO,}\\&\text{structure growth,}\\&\text{cluster and galaxy-scale potentials,}\\&\text{and gravitational-wave propagation.}\end{aligned}} \tag{97}$$

22 Core Hypotheses

Hypothesis 22.1 (Functional triune)**.** *The observed gravitational roles conventionally assigned to ordinary gravity, dark matter, and dark energy arise from a coupled architecture that can be decomposed into binding, scaffolding, and chasing functions.*

Hypothesis 22.2 (Scaffold persistence)**.** *A non-luminous scaffolding state persists across the outer reversal and second leg, retaining part of the first-leg structural information.*

Hypothesis 22.3 (Chasing continuity)**.** *The chasing function remains active through all three directional Electro legs and contributes to continued global expansion.*

Hypothesis 22.4 (Binding emergence)**.** *The material third leg exhibits a binding regime that recovers the local and astrophysical gravitational behavior of general relativity.*

Hypothesis 22.5 (Conserving interaction)**.** *Energy and momentum may transfer among the triune sectors while the complete stress–energy tensor remains covariantly conserved.*

Hypothesis 22.6 (Unified observational recovery)**.** *One shared parameter set can recover local gravity, cosmological expansion, lensing, structure growth, microwave-background evolution, and gravitational-wave constraints.*

Hypothesis 22.7 (Triune residue)**.** *At least one observable relation distinguishes a coupled Gravity Triune from an uncoupled* ΛCDM *parameterization with the same background expansion.*

23 Claims Not Made

The foundational Gravity Triune framework does not presently claim that:

1. binding is a separate particle;
2. scaffolding is identical to one known dark-matter candidate;
3. chasing is a literal outward-moving material fluid;
4. the cosmological constant is disproven;
5. dark matter and dark energy are one substance;
6. the three functions always carry separately conserved stress–energy;
7. the functions possess equal strengths;
8. the functions share one propagation speed;
9. the functions operate identically in all three legs;
10. the scaffold is exactly the visible cosmic web;
11. the chasing sector is the cosmic microwave background;
12. the chasing sector defines a cosmic boundary;
13. ordinary gravity switches on discontinuously;
14. the triune automatically solves the horizon or flatness problems;
15. the triune automatically produces matter–antimatter asymmetry;
16. the minimum third-leg GT1 realization, by itself, establishes the series-level elimination of a separate inflationary stage;
17. the triune has already reproduced galaxy rotation or cluster lensing;
18. the global scale factor reverses with *Electro*;
19. the standard radiation- and pressureless-matter epochs used as third-leg benchmarks determine when matter, antimatter, or particle-like excitations first originated in the complete three-leg history; or
20. the terminology alone constitutes a physical theory.

24 Success Conditions

A complete Gravity Triune theory must satisfy the following conditions.

1. **Covariance:**

   The equations must be covariant or arise as a controlled limit of a covariant theory.

2. **Conservation:**

$$\nabla_\mu T^{\mu\nu}_{\text{tot}} = 0. \tag{98}$$

3. **No double counting:**

   Each physical contribution must enter the field equations exactly once.

4. **Stable background:**

   The homogeneous solutions must remain finite and dynamically controlled.

5. **No ghosts:**

   The physical kinetic matrix must be positive.

6. **No uncontrolled gradient instability:**

   The physical propagation-speed eigenvalues must be nonnegative.

7. **Local recovery:**

   The tested weak-field and relativistic limits of general relativity must be recovered.

8. **Scaffold recovery:**

   The theory must reproduce gravitational structure, lensing, and growth attributed to dark matter.

9. **Chasing recovery:**

   The theory must reproduce the observed expansion history attributed to dark energy.

10. **Third-leg consistency:**

    The same sectors must produce an acceptable hot-universe background and perturbation evolution.

11. **Cross-leg consistency:**

    The sector equations must remain meaningful across the first two legs and both reversals.

12. **Shared parameters:**

    The theory may not use unrelated parameter values for local, astrophysical, and cosmological calculations without a derived scale-dependence mechanism.

13. **Observational competitiveness:**

    The model must be compared fairly with ΛCDM and other relevant alternatives.

14. **Falsifiable residue:**

    At least one observable must distinguish the proposed coupling architecture from a flexible effective fit.

25 Claim-Specific Failure and Falsification Conditions

The following verdicts apply to the tested Gravity Triune claim level. A result must distinguish GT1, another specific realization, the model class, and the broader role architecture. No result in this list automatically rejects the Triple Bounce directional history or Paper II's independent replacement counterclaims.

1. No covariant or mathematically closed formulation exists.
2. The total stress–energy tensor is not conserved.
3. Binding is counted both as metric response and as an identical additional attractive source.
4. Chasing duplicates a cosmological constant without introducing a physically distinct effect.
5. Scaffolding reproduces the background density but fails gravitational lensing or structure growth.
6. The chasing sector reproduces acceleration but destabilizes perturbations.
7. The theory violates local gravitational tests.
8. The effective gravitational coupling varies beyond observational limits.
9. The scalar sector contains a ghost.
10. The perturbations contain an uncontrolled gradient instability.
11. The tensor propagation speed or damping is unacceptable.
12. The model produces excessive gravitational slip.
13. The scaffold erases the first-leg memory required by the Triple Bounce history.
14. The scaffold retains so much directional memory that statistical isotropy is violated.
15. The chasing sector cannot remain consistent through the three-leg history.
16. The interaction currents require energy creation without compensating sector loss.
17. The outer or central reversal requires a divergent sector density or pressure.

18. Different datasets require mutually incompatible triune parameters.

19. The model reproduces observations only through arbitrary scale-dependent functions.

20. No observable can distinguish the triune from a relabeled ΛCDM parameterization.

26 Notation and Conventions

The metric signature is

$$(-,+,+,+). \tag{99}$$

The principal stress–energy labels are:

$$\left\{T_{\mu\nu}^{\text{bind}}, T_{\mu\nu}^{\text{scaf}}, T_{\mu\nu}^{\text{chase}}\right\}. \tag{100}$$

The corresponding homogeneous densities are:

$$\{\rho_{\text{bind}}, \rho_{\text{scaf}}, \rho_{\text{chase}}\}. \tag{101}$$

The corresponding pressures are:

$$\{p_{\text{bind}}, p_{\text{scaf}}, p_{\text{chase}}\}. \tag{102}$$

The corresponding exchange currents are:

$$\{Q^{\mu}_{\text{bind}}, Q^{\mu}_{\text{scaf}}, Q^{\mu}_{\text{chase}}\}. \tag{103}$$

The magnetic field is written as

$$\boldsymbol{B} \tag{104}$$

and should not be confused with the binding label.

The symbol

$$\mathrm{E} \tag{105}$$

denotes the provisional *Electro* sector or front of the Triple Bounce architecture.

The symbol

$$\mathcal{W} \tag{106}$$

denotes inherited scaffold memory when required.

27 Organization of Paper III

Paper III proceeds in nine linked parts. The opening part defines the Gravity Triune and the non-double-counting rule. The second establishes the general-relativistic and standard-cosmology benchmarks. The third compares candidate covariant realizations and selects the minimal GT1 reference model. The fourth develops homogeneous background dynamics and the viable phase-space burden. The fifth treats local, weak-field, strong-field, and gravitational-wave limits. The sixth develops scaffolding, chasing, interaction, reversal, and memory sectors. The seventh constructs the linear perturbation system and its Einstein–Boltzmann implementation. The eighth defines the joint observational and historical-identification program. The final part states the closure conditions, claims, limitations, computational agenda, and dependency gate before Paper IV.

28 Opening Summary

The Gravity Triune is defined by the functional decomposition

$$\boxed{\mathfrak{G} = \{\mathfrak{G}_{\text{bind}}, \mathfrak{G}_{\text{scaf}}, \mathfrak{G}_{\text{chase}}\}\,.} \tag{107}$$

The three roles are

$$\boxed{\begin{aligned} \mathfrak{G}_{\text{bind}} &: \text{attractive binding and local gravitational organization,}\\ \mathfrak{G}_{\text{scaf}} &: \text{persistent non-luminous gravitational scaffolding,}\\ \mathfrak{G}_{\text{chase}} &: \text{large-scale expansion-driving chasing behavior.} \end{aligned}} \tag{108}$$

The generic field equation is

$$\boxed{G_{\mu\nu} + \Lambda_{\text{bare}} g_{\mu\nu} = \frac{8\pi G}{c^4} T^{\text{tot}}_{\mu\nu}.} \tag{109}$$

The total stress–energy decomposition is

$$\boxed{T^{\text{tot}}_{\mu\nu} = T^{\text{vis}}_{\mu\nu} + T^{\text{rad}}_{\mu\nu} + T^{\text{bind}}_{\mu\nu} + T^{\text{scaf}}_{\mu\nu} + T^{\text{chase}}_{\mu\nu} + T^{\text{other}}_{\mu\nu}.} \tag{110}$$

The total conservation law is

$$\boxed{\nabla_\mu T^{\mu\nu}_{\text{tot}} = 0.} \tag{111}$$

Sector exchange is permitted when

$$\boxed{\nabla_\mu T_i^{\mu\nu} = Q_i^\nu, \qquad \sum_i Q_i^\nu = 0.} \tag{112}$$

The homogeneous acceleration condition is

$$\boxed{\rho_{\text{tot}} + \frac{3p_{\text{tot}}}{c^2} < 0.} \tag{113}$$

The non-double-counting condition is

$$\boxed{\text{each physical gravitational contribution enters the fundamental equations exactly once.}} \tag{114}$$

The central Paper III burden is

$$\boxed{\begin{aligned} &\text{covariant dynamics} + \text{total conservation} + \text{stability} \\ &+ \text{local gravity} + \text{scaffold structure} + \text{cosmic acceleration} \\ &+ \text{cross-leg consistency} + \text{observational distinction.} \end{aligned}} \tag{115}$$

29 Benchmark Logic

The Gravity Triune is not evaluated against an empty theoretical background.

It must be compared with general relativity and the standard ΛCDM cosmological model, which together provide a quantitatively successful description of:

- local gravitational dynamics;
- relativistic orbital effects;
- gravitational redshift and time delay;
- light deflection and gravitational lensing;
- compact objects;
- gravitational waves;
- the homogeneous expansion history;

- the microwave-background acoustic structure;
- the growth of large-scale structure;
- galaxy and cluster gravitational potentials; and
- late-time accelerated expansion.

The benchmark question is not whether every element of the standard model is microscopically understood.

The benchmark question is whether the standard model supplies a successful effective description that a proposed replacement must recover.

Define the benchmark observable vector

$$\boldsymbol{\mathcal{O}}_{\mathrm{bench}} = \left(\boldsymbol{\mathcal{O}}_{\mathrm{local}}, \boldsymbol{\mathcal{O}}_{\mathrm{lens}}, \boldsymbol{\mathcal{O}}_{\mathrm{bg}}, \boldsymbol{\mathcal{O}}_{\mathrm{growth}}, \boldsymbol{\mathcal{O}}_{\mathrm{CMB}}, \boldsymbol{\mathcal{O}}_{\mathrm{GW}}\right). \tag{116}$$

A Gravity Triune realization predicts

$$\boldsymbol{\mathcal{O}}_{\mathrm{triune}}\left(\boldsymbol{\Theta}_{\mathrm{triune}}\right). \tag{117}$$

Minimum recovery requires

$$\boxed{\boldsymbol{\mathcal{O}}_{\mathrm{triune}} \in \mathcal{V}_{\mathrm{obs}},} \tag{118}$$

where $\mathcal{V}_{\mathrm{obs}}$ is the observationally allowed region.

A stronger result requires either:

$$\text{improved explanatory or statistical performance,} \tag{119}$$

or

$$\text{a successful independently testable prediction.} \tag{120}$$

Renaming the standard functions as binding, scaffolding, and chasing does not constitute a new theory.

30 The General-Relativistic Benchmark

Benchmark literature. The field equations and their standard interpretation are taken from general relativity (Einstein, 1915; Wald, 1984).

The gravitational benchmark is Einstein's field equation,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{121}$$

The Einstein tensor is

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{122}$$

The Bianchi identity gives

$$\nabla_\mu G^{\mu\nu} = 0. \tag{123}$$

For constant Λ,

$$\nabla_\mu \left(\Lambda g^{\mu\nu}\right) = 0. \tag{124}$$

Therefore,

$$\nabla_\mu T^{\mu\nu} = 0. \tag{125}$$

The metric determines:

- the causal structure;
- proper time and proper distance;
- freely falling trajectories;
- light propagation;
- gravitational redshift;
- tidal acceleration; and
- gravitational-wave propagation.

A Gravity Triune model may preserve the Einstein tensor on the left-hand side and modify the source.

Alternatively, it may modify the geometric field equations.

The two possibilities are schematically:

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu}^{\text{standard}} + T_{\mu\nu}^{\text{triune}}\right), \tag{126}$$

$$G_{\mu\nu} + \Delta G_{\mu\nu}^{\text{triune}} = \frac{8\pi G}{c^4} T_{\mu\nu}^{\text{standard}}. \tag{127}$$

A mixed realization may contain both.

The theory must state which side of the field equation carries each new effect.

31 Universal Metric Coupling and Geodesic Motion

Benchmark literature. Universal metric coupling and geodesic test-body motion are treated using the standard general-relativistic formulation (Wald, 1984; Will, 2014).

In the minimal metric description, freely falling test bodies follow geodesics,

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0. \tag{128}$$

The Christoffel symbols are

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}\left(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}\right). \tag{129}$$

Universal coupling to one metric implies that freely falling bodies with different internal compositions respond to the same local geometry.

A triune theory must determine whether visible matter follows:

$$\text{geodesics of one universal metric,} \tag{130}$$

or whether additional forces produce

$$u^\nu \nabla_\nu u^\mu = f^\mu_{\text{triune}}. \tag{131}$$

The force must satisfy

$$u_\mu f^\mu_{\text{triune}} = 0 \tag{132}$$

for a constant rest mass.

Composition-dependent forces are tightly constrained conceptually and observationally.

The most conservative binding realization therefore retains universal metric coupling for ordinary matter.

Criterion 31.1 (Metric universality)**.** *A viable binding sector must reproduce universal free fall in every regime where no departure from metric geodesic motion has been established.*

32 Weak-Field Binding Benchmark

Benchmark literature. The Newtonian and weak-field limits are evaluated against established tests of general relativity (Will, 2014).

In a weak and slowly varying gravitational field, write

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1. \tag{133}$$

For nonrelativistic motion,

$$g_{00} \approx -\left(1 + \frac{2\Phi_N}{c^2}\right). \tag{134}$$

The geodesic equation reduces to

$$\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = -\nabla\Phi_N. \tag{135}$$

The Newtonian potential satisfies

$$\nabla^2\Phi_N = 4\pi G\rho. \tag{136}$$

For a point mass,

$$\Phi_N(r) = -\frac{GM}{r}. \tag{137}$$

The corresponding acceleration is

$$\boldsymbol{a} = -\frac{GM}{r^3}\boldsymbol{r}. \tag{138}$$

A generalized binding equation may be written as

$$\nabla^2\Phi = 4\pi G\mu_{\mathrm{loc}}\left(r, \rho, \mathcal{S}\right)\rho + \mathcal{S}_\Phi. \tag{139}$$

Local recovery requires

$$\mu_{\text{loc}} \longrightarrow 1, \tag{140}$$

and

$$\mathcal{S}_{\Phi} \longrightarrow 0 \tag{141}$$

in the tested Newtonian regime.

A theory that explains cosmological effects through a large unscreened change in local attraction is not automatically viable.

33 Post-Newtonian Benchmark

Benchmark literature. The post-Newtonian parameters and local-gravity requirements follow the standard experimental framework (Will, 2014; Damour and Esposito-Farèse, 1992).

The post-Newtonian expansion tests relativistic corrections to weak-field gravity.

For a static source, a simplified metric may be written as

$$g_{00} = -1 + \frac{2U}{c^2} - 2\beta\frac{U^2}{c^4} + \mathcal{O}\left(c^{-6}\right), \tag{142}$$

$$g_{ij} = \left(1 + 2\gamma\frac{U}{c^2}\right)\delta_{ij} + \mathcal{O}\left(c^{-4}\right). \tag{143}$$

Here:

- γ measures the spatial curvature produced per unit rest mass; and
- β measures nonlinear gravitational superposition.

General relativity predicts

$$\gamma = 1, \qquad \beta = 1. \tag{144}$$

A general triune theory predicts

$$\gamma_{\text{triune}} = \gamma\left(\boldsymbol{\Theta}_{\text{triune}}, \mathcal{S}_{\text{local}}\right), \tag{145}$$

and

$$\beta_{\text{triune}} = \beta\left(\boldsymbol{\Theta}_{\text{triune}}, \mathcal{S}_{\text{local}}\right). \tag{146}$$

Recovery requires

$$\boxed{\gamma_{\text{triune}} \approx 1, \qquad \beta_{\text{triune}} \approx 1} \tag{147}$$

within the relevant experimental regime.

Additional post-Newtonian parameters test:

- preferred-frame effects;
- preferred-location effects;
- momentum nonconservation;
- violations of universal free fall;
- time variation of the gravitational coupling; and
- additional long-range fields.

The Gravity Triune must either suppress these effects locally or predict them below allowed levels.

34 Strong-Field and Compact-System Benchmark

Benchmark literature. The strong-field comparison includes compact-body and scalarization effects known from tensor–scalar gravity (Damour and Esposito-Farèse, 1992, 1993, 1996).

The local binding function must remain consistent beyond the post-Newtonian regime.

Define compactness

$$\mathcal{C} = \frac{GM}{Rc^2}. \tag{148}$$

Strong-field systems include:

- neutron stars;
- black holes;
- relativistic binaries;
- accretion environments;
- merging compact systems; and
- high-curvature wave-generation regions.

A triune field may respond differently inside strongly bound objects than in weak fields.

Possible consequences include:

$$\{\text{scalarization}, \text{additional radiation}, \text{modified tidal response}, \text{altered horizons}, \text{modified ringdown}\}. \tag{149}$$

The strong-field binding benchmark requires a consistent solution for:

$$\left\{g_{\mu\nu}, T_{\mu\nu}^{\text{matter}}, \psi_{\text{bind}}, \psi_{\text{scaf}}, \psi_{\text{chase}}\right\} \tag{150}$$

in the presence of compact sources.

A cosmological triune theory cannot assume that its additional fields vanish around compact objects without deriving a screening or decoupling mechanism.

35 Gravitational Lensing Benchmark

Benchmark literature. The lensing target uses the standard metric-potential and weak-lensing framework (Bartelmann and Schneider, 2001).

In conformal Newtonian gauge,

$$\mathrm{d}s^2 = a^2(\eta)\left[-\left(1+2\Psi\right)\mathrm{d}\eta^2 + \left(1-2\Phi\right)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]. \tag{151}$$

Nonrelativistic motion responds primarily to

$$\Psi. \tag{152}$$

Weak gravitational lensing responds to the Weyl combination

$$\Phi_W = \frac{\Phi+\Psi}{2}. \tag{153}$$

The deflection angle is schematically

$$\boldsymbol{\alpha} \propto \int \nabla_\perp \left(\Phi+\Psi\right)\mathrm{d}\chi. \tag{154}$$

The lensing convergence is related to the projected Weyl potential,

$$\kappa \propto \int_0^{\chi_s} \mathrm{d}\chi\, W_{\text{lens}}\left(\chi,\chi_s\right)\nabla_\perp^2\left(\Phi+\Psi\right). \tag{155}$$

A scaffolding model must therefore reproduce both:

$$\text{dynamical gravitational mass,} \tag{156}$$

and

$$\text{lensing gravitational mass.} \tag{157}$$

Matching orbital velocities while failing lensing is insufficient.

Define the lensing-to-dynamical consistency ratio

$$\mathcal{R}_{\mathrm{LD}} = \frac{\nabla^2 (\Phi + \Psi)}{2\nabla^2 \Psi}. \tag{158}$$

In general relativity with negligible anisotropic stress,

$$\Phi = \Psi, \qquad \mathcal{R}_{\mathrm{LD}} = 1. \tag{159}$$

A triune deviation must be computed and observationally tested.

36 Cold-Dark-Matter Background Benchmark

Benchmark literature. The cold-dark-matter background target follows standard large-scale-structure and cosmological-parameter treatments (Peebles, 1980; Bertone et al., 2005; Planck Collaboration et al., 2020).

Cold dark matter is modeled as a pressureless or nearly pressureless component,

$$p_c \approx 0, \tag{160}$$

with

$$w_c \approx 0. \tag{161}$$

When uncoupled,

$$\dot{\rho}_c + 3H\rho_c = 0. \tag{162}$$

Therefore,

$$\rho_c \propto a^{-3}. \tag{163}$$

Its homogeneous density parameter is

$$\Omega_c = \frac{8\pi G\rho_c}{3H^2}. \tag{164}$$

The minimum third-leg scaffolding benchmark is consequently

$$\boxed{w_{\rm scaf} \approx 0, \qquad \rho_{\rm scaf} \propto a^{-3}} \tag{165}$$

over the era in which cold-dark-matter-like behavior is required.

A scaffolding sector may depart from this limit at earlier or later times.

Such departures must not disrupt:

- matter–radiation equality;
- the acoustic peak pattern;
- structure growth;
- gravitational lensing;
- cluster abundance; or
- the late expansion history.

37 Cold-Dark-Matter Perturbation Benchmark

Benchmark literature. The linear cold-dark-matter perturbation equations and gauge conventions follow standard cosmological perturbation theory (Bardeen, 1980; Ma and Bertschinger, 1995; Malik and Wands, 2009).

For pressureless cold dark matter in conformal Newtonian gauge,

$$\delta_c' = -\theta_c + 3\Phi', \tag{166}$$

and

$$\theta_c' = -\mathcal{H}\theta_c + k^2\Psi. \tag{167}$$

Here:

- δ_c is the cold-dark-matter density contrast;
- θ_c is its velocity divergence; and

- $\mathcal{H} = a'/a$ is the conformal Hubble rate.

For a generalized scaffolding fluid,

$$\begin{aligned}\delta'_{\rm scaf} = & -(1+w_{\rm scaf})\left(\theta_{\rm scaf} - 3\Phi'\right) \\ & - 3\mathcal{H}\left(c^2_{s,{\rm scaf}} - w_{\rm scaf}\right)\delta_{\rm scaf} + \mathcal{Q}_{\delta,{\rm scaf}},\end{aligned} \tag{168}$$

and

$$\begin{aligned}\theta'_{\rm scaf} = & -\mathcal{H}\left(1-3w_{\rm scaf}\right)\theta_{\rm scaf} - \frac{w'_{\rm scaf}}{1+w_{\rm scaf}}\theta_{\rm scaf} \\ & + \frac{c^2_{s,{\rm scaf}}}{1+w_{\rm scaf}}k^2\delta_{\rm scaf} - k^2\sigma_{\rm scaf} + k^2\Psi + \mathcal{Q}_{\theta,{\rm scaf}}.\end{aligned} \tag{169}$$

Cold-dark-matter-like clustering requires approximately

$$\boxed{w_{\rm scaf} \approx 0, \qquad c^2_{s,{\rm scaf}} \approx 0, \qquad \sigma_{\rm scaf} \approx 0} \tag{170}$$

over the relevant scales and epochs.

A large sound speed suppresses scaffold clustering below its effective Jeans scale.

A large anisotropic stress modifies gravitational slip and lensing.

38 Matter Growth Benchmark

Benchmark literature. The matter-growth target follows the standard perturbative and large-scale-structure framework (Peebles, 1980; Ma and Bertschinger, 1995; Dodelson, 2003).

On subhorizon scales in a simple pressureless regime, the matter density contrast satisfies approximately

$$\delta''_m + \mathcal{H}\delta'_m - 4\pi G a^2 \rho_m \delta_m = 0. \tag{171}$$

Using derivatives with respect to $\ln a$, one may write

$$\frac{\mathrm{d}^2 D}{\mathrm{d}(\ln a)^2} + \left[2 + \frac{\mathrm{d}\ln H}{\mathrm{d}\ln a}\right]\frac{\mathrm{d}D}{\mathrm{d}\ln a} - \frac{3}{2}\Omega_m(a) D = 0, \tag{172}$$

where $D(a)$ is the linear growth factor in the standard limit.

The logarithmic growth rate is

$$f(a) = \frac{\mathrm{d}\ln D}{\mathrm{d}\ln a}. \tag{173}$$

A commonly measured combination is

$$f\sigma_8(z). \tag{174}$$

A generalized triune growth equation may be written as

$$\frac{\mathrm{d}^2 D}{\mathrm{d}(\ln a)^2} + \left[2 + \frac{\mathrm{d}\ln H}{\mathrm{d}\ln a} + \Gamma_{\mathrm{drag}}\right]\frac{\mathrm{d}D}{\mathrm{d}\ln a} - \frac{3}{2}\mu(k,a)\Omega_m(a)D = \mathcal{S}_D. \tag{175}$$

Here:

- Γ_{drag} represents interaction-induced drag;
- $\mu(k, a)$ represents a modified effective gravitational coupling; and
- $\mathcal{S}_D$ represents additional derived sources.

The scaffolding and chasing sectors affect growth through both

$$H(a), \tag{176}$$

and

$$\{\mu, \Gamma_{\mathrm{drag}}, \mathcal{S}_D\}. \tag{177}$$

Matching the expansion history does not guarantee matching structure growth.

39 Transfer Function and Matter Power Spectrum

Benchmark literature. Transfer functions and matter power spectra are evaluated using standard linear theory and structure-formation results (Ma and Bertschinger, 1995; Dodelson, 2003; Springel et al., 2005).

The late-time linear matter power spectrum may be written as

$$P_m(k, z) = P_{\mathrm{prim}}(k)T_m^2(k, z), \tag{178}$$

where:

- $P_{\mathrm{prim}}(k)$ is the primordial spectrum; and

- $T_m(k, z)$ is the matter transfer function.

For predominantly adiabatic scalar initial conditions,

$$\delta_m(k, z) = \mathcal{M}(k, z)\mathcal{R}(k). \tag{179}$$

The matter power spectrum is then

$$P_m(k, z) = \mathcal{M}^2(k, z)P_{\mathcal{R}}(k). \tag{180}$$

The scaffold affects:

- the equality scale;
- potential decay;
- the growth factor;
- small-scale suppression;
- the lensing potential;
- halo formation; and
- the shape of the matter transfer function.

A retained first- or second-leg scaffold memory may alter

$$T_m(k, z) = T_m^{(0)}(k, z) + \Delta T_{\mathrm{mem}}(k, z). \tag{181}$$

Such a correction must be propagated from the same initial covariance used in Paper II.

It cannot be inserted independently into the late-time matter spectrum.

40 Cosmological-Constant Benchmark

Benchmark literature. The cosmological-constant control and its conceptual burden follow the standard literature (Weinberg, 1989; Peebles and Ratra, 2003).

A cosmological constant contributes

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{\mathrm{matter}}. \tag{182}$$

It may be moved to the source side as an effective stress–energy tensor,

$$T_{\mu\nu}^{(\Lambda)} = -\rho_\Lambda c^2 g_{\mu\nu}, \tag{183}$$

where

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}. \tag{184}$$

Its pressure is

$$p_\Lambda = -\rho_\Lambda c^2, \tag{185}$$

so

$$w_\Lambda = -1. \tag{186}$$

Its homogeneous density is constant:

$$\dot{\rho}_\Lambda = 0. \tag{187}$$

The corresponding acceleration contribution is

$$\rho_\Lambda + \frac{3p_\Lambda}{c^2} = -2\rho_\Lambda. \tag{188}$$

A strict cosmological constant has no independent fluid perturbations in the usual general-relativistic formulation.

The minimum chasing benchmark is therefore

$$\boxed{w_{\text{chase}} = -1, \qquad \rho_{\text{chase}} = \text{constant}, \qquad \delta\rho_{\text{chase}} = 0} \tag{189}$$

in the exact cosmological-constant limit.

A chasing model may depart from this limit.

Those departures must produce consistent background and perturbative signatures.

41 Dynamical Dark-Energy Benchmark

Benchmark literature. The dynamical-dark-energy comparison follows canonical scalar-field and effective-fluid treatments, with modern baryon-acoustic-oscillation constraints reserved as observational tests (Ratra and Peebles, 1988; Wetterich, 1988; Copeland et al., 2006; DESI Collaboration et al., 2025b,a).

A dynamical dark-energy component has

$$w_{\rm de} = w_{\rm de}(a). \tag{190}$$

When uncoupled, its density evolves as

$$\rho_{\rm de}(a) = \rho_{\rm de,0} \exp\left[-3\int_1^a (1 + w_{\rm de}(\tilde{a}))\, \mathrm{d}\ln\tilde{a}\right]. \tag{191}$$

A common phenomenological parameterization is

$$w_{\rm de}(a) = w_0 + w_a\,(1 - a)\,. \tag{192}$$

This is a benchmark parameterization.

It is not itself a microscopic theory.

A dynamical chasing sector must also specify:

$$\left\{c^2_{s,\rm chase}, c^2_{a,\rm chase}, \sigma_{\rm chase}, \delta_{\rm chase}, \theta_{\rm chase}\right\}. \tag{193}$$

The adiabatic sound speed is

$$c^2_{a,\rm chase} = \frac{\dot{p}_{\rm chase}}{\dot{\rho}_{\rm chase}}. \tag{194}$$

The rest-frame sound speed need not equal the adiabatic sound speed:

$$c^2_{s,\rm chase} \neq c^2_{a,\rm chase} \tag{195}$$

in a nonadiabatic model.

A chasing model that matches $H(z)$ but develops unstable perturbations is not viable.

42 Scalar-Field Dark-Energy Benchmark

Benchmark literature. Canonical scalar dark energy and scaling solutions are treated using the standard quintessence literature (Ratra and Peebles, 1988; Wetterich, 1988; Copeland et al., 1998, 2006).

A canonical scalar field ϕ has action

$$S_\phi = \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \tag{196}$$

Its homogeneous density is

$$\rho_\phi = \frac{1}{2c^2} \dot{\phi}^2 + \frac{V(\phi)}{c^2}, \tag{197}$$

and its pressure is

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi). \tag{198}$$

Its equation of state is

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}. \tag{199}$$

Acceleration occurs when potential energy dominates sufficiently:

$$\dot{\phi}^2 < V(\phi). \tag{200}$$

The field equation is

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0 \tag{201}$$

in the uncoupled canonical case.

A scalar chasing realization must provide:

- a physically motivated potential;
- stable perturbations;
- an acceptable equation-of-state history;
- controlled coupling to matter and scaffolding;
- a consistent early-time density; and

- a defined role across the hidden legs.

43 Interacting Dark-Sector Benchmark

Benchmark literature. The interaction benchmark follows coupled-quintessence and relativistic interacting-dark-sector perturbation studies (Amendola, 2000; Väliviita et al., 2008).

The scaffolding and chasing sectors may exchange energy and momentum.

At the background level,

$$\dot{\rho}_{\text{scaf}} + 3H\left(1 + w_{\text{scaf}}\right)\rho_{\text{scaf}} = Q_{\text{scaf}}, \tag{202}$$

$$\dot{\rho}_{\text{chase}} + 3H\left(1 + w_{\text{chase}}\right)\rho_{\text{chase}} = Q_{\text{chase}}. \tag{203}$$

For exchange restricted to these two sectors,

$$Q_{\text{scaf}} + Q_{\text{chase}} = 0. \tag{204}$$

A phenomenological example is

$$Q_{\text{scaf}} = 3H\xi\rho_X, \tag{205}$$

where ρ_X may be one sector density or a combination of densities.

This form is illustrative.

It is not adopted as the Gravity Triune interaction law.

The perturbative transfer must also specify

$$\delta Q_i, \qquad F_i^{\mu}. \tag{206}$$

Different covariant completions of the same background function $Q_i(t)$ can produce different perturbation equations.

The interaction benchmark therefore requires

$$\boxed{\text{background transfer} + \text{energy perturbation} + \text{momentum transfer} + \text{covariant origin.}} \tag{207}$$

A background-only interaction law is incomplete.

44 Effective Gravitational Coupling and Gravitational Slip

Benchmark literature. Effective gravitational response and slip are compared with standard modified-gravity and effective-field-theory parameterizations (Clifton et al., 2012; Gubitosi et al., 2013; Bellini and Sawicki, 2014).

A useful phenomenological description of modified growth is

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho_m \Delta_m, \tag{208}$$

where Δ_m is a gauge-invariant comoving matter-density perturbation.

The gravitational-slip parameter may be defined as

$$\eta_{\rm slip}(k,a) = \frac{\Phi}{\Psi}. \tag{209}$$

The lensing modification may be written as

$$k^2\left(\Phi+\Psi\right) = -8\pi G a^2 \Sigma(k,a)\rho_m \Delta_m. \tag{210}$$

These functions satisfy

$$\Sigma = \frac{\mu\left(1+\eta_{\rm slip}\right)}{2} \tag{211}$$

under the stated parameterization.

In general relativity with negligible anisotropic stress,

$$\boxed{\mu = 1, \qquad \eta_{\rm slip} = 1, \qquad \Sigma = 1.} \tag{212}$$

A scaffolding sector with anisotropic stress may produce

$$\Phi - \Psi \neq 0. \tag{213}$$

A modified binding law may produce

$$\mu \neq 1. \tag{214}$$

A triune theory must predict these functions from its field equations.

They cannot be independently adjusted for lensing and structure growth.

45 Background Expansion Benchmark

Benchmark literature. The distance–redshift and expansion-history targets are anchored to supernova, microwave-background, and baryon-acoustic-oscillation measurements (Riess et al., 1998; Perlmutter et al., 1999; Planck Collaboration et al., 2020; Alam et al., 2021; DESI Collaboration et al., 2025a).

For a homogeneous third-leg universe,

$$H^2(a) = \frac{8\pi G}{3} \sum_i \rho_i(a) - \frac{kc^2}{a^2}. \tag{215}$$

The dimensionless expansion function is

$$E(z) = \frac{H(z)}{H_0}. \tag{216}$$

A standard background may be represented schematically as

$$E^2(z) = \Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{\Lambda 0}. \tag{217}$$

A dynamic chasing model gives

$$\begin{aligned} E^2(z) = \Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 \\ + \Omega_{\text{chase}0} \exp\left[3 \int_0^z \frac{1 + w_{\text{chase}}(\tilde{z})}{1+\tilde{z}} \mathrm{d}\tilde{z}\right]. \end{aligned} \tag{218}$$

Interactions alter the density evolution and therefore modify this expression.

The background observables include:

$$\{H(z), D_L(z), D_A(z), D_M(z), D_V(z), t(z)\}. \tag{219}$$

The luminosity and angular-diameter distances satisfy

$$D_L(z) = (1+z)^2 D_A(z) \tag{220}$$

when photon number is conserved and photons propagate along standard null geodesics.

A triune model modifying light propagation must test this relation rather than assume it.

46 Gravitational-Wave Propagation Benchmark

Benchmark literature. The tensor-propagation target includes direct gravitational-wave detection and the multimessenger speed constraint (LIGO Scientific Collaboration and Virgo Collaboration, 2016; LIGO Scientific Collaboration et al., 2017).

Tensor perturbations are defined by

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right], \tag{221}$$

with

$$\partial_i h_{ij} = 0, \qquad h_i{}^i = 0. \tag{222}$$

In general relativity, the source-free tensor equation is

$$h_\lambda'' + 2\mathcal{H}h_\lambda' + c^2k^2h_\lambda = 0 \tag{223}$$

on a simple homogeneous background.

A generalized propagation equation is

$$h_\lambda'' + (2+\alpha_M)\,\mathcal{H}h_\lambda' + \left(c_T^2k^2 + a^2m_T^2\right)h_\lambda = a^2\Pi_\lambda^{\mathrm{T}}. \tag{224}$$

Here:

- α_M modifies gravitational-wave damping;
- c_T is the tensor propagation speed;
- m_T is an effective tensor mass; and
- Π_λ^{T} is a tensor source.

The general-relativistic propagation limit is

$$\boxed{\alpha_M = 0, \qquad c_T = c, \qquad m_T = 0} \tag{225}$$

in the absence of anisotropic stress.

Modified damping can produce a gravitational-wave luminosity distance

$$D_L^{\mathrm{GW}}(z) \neq D_L^{\mathrm{EM}}(z). \tag{226}$$

A schematic relation is

$$\frac{D_L^{\mathrm{GW}}(z)}{D_L^{\mathrm{EM}}(z)} = \exp\left[\frac{1}{2}\int_0^z \frac{\alpha_M(\tilde{z})}{1+\tilde{z}}\mathrm{d}\tilde{z}\right]. \tag{227}$$

A Gravity Triune theory must determine whether the scaffold or chasing sector modifies tensor propagation.

47 Perturbative Stability Benchmark

Benchmark literature. Ghost, gradient, sound-speed, and higher-derivative stability conditions follow standard scalar and degenerate-theory analyses (Garriga and Mukhanov, 1999; Deffayet et al., 2011; Langlois and Noui, 2016).

A background solution is not viable if its perturbations contain fatal instabilities.

For independent perturbation variables $\boldsymbol{q}$, write the quadratic action as

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\left[\dot{\boldsymbol{q}}^{\mathsf{T}}\mathsf{K}\dot{\boldsymbol{q}} - \frac{1}{a^2}\partial_i\boldsymbol{q}^{\mathsf{T}}\mathsf{G}\partial_i\boldsymbol{q} - \boldsymbol{q}^{\mathsf{T}}\mathsf{M}^2\boldsymbol{q}\right]. \tag{228}$$

No-ghost stability requires

$$\lambda_A(\mathsf{K}) > 0. \tag{229}$$

Gradient stability requires

$$\lambda_A\left(\mathsf{K}^{-1}\mathsf{G}\right) \geq 0. \tag{230}$$

A temporarily negative effective mass squared may be allowed only when its growth is controlled and physically interpreted.

Strong coupling is indicated when

$$\det\mathsf{K} \longrightarrow 0. \tag{231}$$

The stability requirements apply to:

- the binding realization;
- the scaffolding sector;
- the chasing sector;
- the interaction modes;
- the reversal transitions; and

- the observable third-leg background.

A sector cannot be declared stable solely because its homogeneous density remains positive.

48 Binding Functional Targets

The binding function must recover the following targets.

48.1 Weak-field attraction

$$\nabla^2 \Phi_N = 4\pi G\rho \tag{232}$$

within the tested local regime.

48.2 Universal free fall

$$u^\nu \nabla_\nu u^\mu \approx 0 \tag{233}$$

for unforced ordinary test bodies.

48.3 Relativistic corrections

$$\gamma_{\text{triune}} \approx 1, \qquad \beta_{\text{triune}} \approx 1. \tag{234}$$

48.4 Light propagation

The metric potentials must reproduce gravitational redshift, time delay, and light deflection.

48.5 Strong-field systems

The model must provide stable compact-object solutions and acceptable binary dynamics.

48.6 Gravitational waves

The tensor sector must reproduce acceptable generation, propagation, polarization, and damping.

The complete binding target is

$$\boxed{\mathfrak{G}_{\text{bind}} \longrightarrow \{\text{Newtonian}, \text{post-Newtonian}, \text{strong-field}, \text{lensing}, \text{wave}\}\ \text{ recovery.}} \tag{235}$$

49 Scaffolding Functional Targets

The scaffolding function must recover or replace the successful phenomenology of cold dark matter.

49.1 Matter-like background

$$w_{\rm scaf} \approx 0, \tag{236}$$

and

$$\rho_{\rm scaf} \propto a^{-3} \tag{237}$$

when the cold limit is required.

49.2 Efficient clustering

$$c^2_{s,\rm scaf} \approx 0 \tag{238}$$

over cosmologically relevant clustering scales, unless a derived scale-dependent departure improves the fit.

49.3 Controlled anisotropic stress

$$\sigma_{\rm scaf} \tag{239}$$

must remain consistent with lensing and gravitational-slip constraints.

49.4 Lensing

The scaffold must source the Weyl potential required by observed lensing.

49.5 Structure growth

The theory must reproduce

$$\{D(a), f(a), f\sigma_8(z), P_m(k,z)\}\,. \tag{240}$$

49.6 Early-universe behavior

The scaffold must contribute consistently to:

$$\{z_{\mathrm{eq}}, r_s, \text{CMB potentials}, \text{CMB lensing}\}. \tag{241}$$

49.7 Cross-leg memory

The distinct Triple Bounce burden is

$$\mathcal{W}_1 \longrightarrow \mathcal{W}_{12} \longrightarrow \mathcal{W}_3 \tag{242}$$

through finite, stable, and conserving transfer laws.

The complete scaffolding target is

$$\boxed{\text{matter-like background} + \text{clustering} + \text{lensing} + \text{growth} + \text{memory.}} \tag{243}$$

50 Chasing Functional Targets

The chasing function must recover or replace the successful effective role of dark energy.

50.1 Accelerated expansion

The total background must satisfy

$$\frac{\ddot{a}}{a} > 0 \tag{244}$$

during the observed accelerating era.

50.2 Distance–redshift recovery

The same chasing parameters must reproduce:

$$\{H(z), D_L(z), D_A(z), D_M(z), \text{BAO}\}. \tag{245}$$

50.3 Stable perturbations

The chasing sector must avoid ghosts, gradient instabilities, and unacceptable clustering.

50.4 Growth consistency

Its effect on structure growth must agree with the same $H(z)$ used for the background fit.

50.5 Early-time control

The chasing density must not disrupt the required early-third-leg radiation and matter eras unless the model derives an acceptable alternative.

50.6 Cross-leg persistence

The distinctive Triple Bounce hypothesis requires a well-defined chasing state throughout:

$$\mathcal{I}_1 \longrightarrow \mathcal{I}_{\mathrm{O}} \longrightarrow \mathcal{I}_2 \longrightarrow \mathcal{I}_{\mathrm{C}} \longrightarrow \mathcal{I}_3. \tag{246}$$

Its persistence cannot be asserted solely by using one name across all five intervals.

The complete chasing target is

$$\boxed{\begin{aligned}&\text{acceleration} + \text{distance recovery} + \text{stable perturbations}\\ &+ \text{growth consistency} + \text{cross-leg continuity.}\end{aligned}} \tag{247}$$

51 Cross-Functional Consistency Targets

The three functions cannot be validated independently.

The combined field equations determine all observables.

The required joint mapping is

$$\{\mathfrak{G}_{\mathrm{bind}}, \mathfrak{G}_{\mathrm{scaf}}, \mathfrak{G}_{\mathrm{chase}}\} \longrightarrow \{g_{\mu\nu}, H, \Phi, \Psi, \delta_i, h_\lambda\}. \tag{248}$$

A valid joint solution must satisfy:

$$\nabla_\mu T^{\mu\nu}_{\mathrm{tot}} = 0, \tag{249}$$

$$\text{no duplicated source contribution,} \tag{250}$$

$$\mathcal{S}_{\text{stable}} = 1, \tag{251}$$

and

$$\boldsymbol{\Theta}_{\text{local}} = \boldsymbol{\Theta}_{\text{cosmological}} \tag{252}$$

after any derived environmental dependence is included.

A model that uses one effective gravitational coupling for local systems and an unrelated coupling for cosmology has not yet supplied a unified triune.

52 Benchmark Matrix

Table 1: Principal general-relativistic and ΛCDM benchmarks for the Gravity Triune.

Domain	**Standard benchmark**	**Gravity Triune target**	**Primary failure mode**
Local free fall	Universal geodesic response to one metric.	Recover metric universality or derive a sufficiently suppressed extra force.	Composition-dependent acceleration.
Newtonian gravity	$\nabla^2\Phi_N = 4\pi G\rho$.	Recover the same local potential in tested regimes.	Incorrect orbital or laboratory gravity.
Post-Newtonian gravity	General-relativistic post-Newtonian parameters.	Recover acceptable γ, β, and preferred-frame behavior.	Relativistic solar-system inconsistency.
Compact systems	Stable relativistic stars, black holes, and binaries.	Produce regular strong-field solutions and acceptable radiation.	Scalarization, collapse, or extra radiation beyond allowed levels.
Light deflection	Lensing from $\Phi + \Psi$.	Match dynamical and lensing mass simultaneously.	Correct motion but incorrect lensing.
Dark-matter background	Pressureless density scaling approximately as a^{-3}.	Produce matter-like scaffold behavior when required.	Incorrect equality or expansion history.

Domain	Standard benchmark	Gravity Triune target	Primary failure mode
Dark-matter perturbations	Small pressure, sound speed, and anisotropic stress.	Cluster efficiently and source observed potentials.	Suppressed growth or excessive slip.
Matter power	One primordial spectrum propagated through a common transfer function.	Predict $P_m(k, z)$ from the Paper II central covariance.	Independent late-time spectrum inserted by hand.
Dark-energy background	Cosmological constant or stable dynamical equation of state.	Produce observed acceleration and distances.	Incorrect $H(z)$ or distance relation.
Dark-energy perturbations	No perturbations for exact Λ; controlled perturbations for dynamical models.	Maintain stability and acceptable clustering.	Ghost, gradient instability, or excessive clustering.
Structure growth	Growth determined jointly by expansion and gravitational potentials.	Fit D, f, $f\sigma_8$, and lensing with one model.	Background and growth require incompatible parameters.
Gravitational slip	$\Phi \approx \Psi$ without significant anisotropic stress.	Predict $\eta_{\rm slip}$ and Σ consistently.	Arbitrary or excessive slip.
Gravitational waves	Luminal massless tensor propagation with standard damping.	Recover or predict allowed c_T, m_T, and α_M.	Incorrect speed, damping, or polarization.
Cross-leg behavior	No standard benchmark because ΛCDM begins from third-leg initial data.	Derive persistent scaffold and chasing states through both reversals.	Narrative persistence without equations.
Distinctive residue	No Triple Bounce residue.	Predict at least one linked observable not reducible to relabeling.	Complete observational degeneracy with ΛCDM.

53 Benchmark Claim-Specific Failure Conditions

A Gravity Triune realization fails the benchmark stage if any of the following is unavoidable.

1. The binding sector violates universal free fall without a sufficiently suppressed or screened extra force.
2. The Newtonian limit fails to recover the observed inverse-distance potential in its tested regime.
3. The post-Newtonian parameters cannot approach their general-relativistic values.
4. The model has no stable compact-object solutions.
5. The theory produces unacceptable additional gravitational radiation.
6. The gravitational-wave speed, damping, mass, or polarization is inconsistent with observation.
7. The scaffold reproduces galactic motion but not gravitational lensing.
8. The scaffold has a sound speed large enough to erase required structure.
9. The scaffold produces excessive anisotropic stress.
10. The scaffold density disrupts matter–radiation equality or the microwave-background acoustic pattern.
11. The chasing sector reproduces accelerated expansion only through a ghost or gradient instability.
12. The chasing sector fits distance data but gives unacceptable structure growth.
13. The dark-sector interaction conserves the homogeneous energy density but violates covariant momentum conservation.
14. The same interaction parameter cannot fit background and perturbation data.
15. The effective gravitational coupling required by cosmology violates local gravity.
16. The model requires independent $\mu(k, a)$, $\Sigma(k, a)$, and $H(z)$ functions unrelated to a common field theory.
17. The exact cosmological-constant limit is counted both as Λ and as an identical chasing stress–energy tensor.
18. The scaffold is introduced as a conventional cold-dark-matter component while its claimed inherited-memory role has no additional dynamics.
19. The chasing function is described as persistent across all three legs but is defined only during the third leg.
20. The theory reproduces ΛCDM exactly through a change of terminology and makes no new calculable statement.

54 Formal Summary of the Benchmark

The general-relativistic field equation is

$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.} \tag{253}$$

The local weak-field benchmark is

$$\boxed{\nabla^2 \Phi_N = 4\pi G\rho.} \tag{254}$$

The post-Newtonian benchmark is

$$\boxed{\gamma = 1, \qquad \beta = 1} \tag{255}$$

in general relativity.

The cold-scaffolding benchmark is

$$\boxed{w_{\text{scaf}} \approx 0, \qquad c^2_{s,\text{scaf}} \approx 0, \qquad \rho_{\text{scaf}} \propto a^{-3}.} \tag{256}$$

The cosmological-constant chasing benchmark is

$$\boxed{w_{\text{chase}} = -1, \qquad \rho_{\text{chase}} = \text{constant}, \qquad \delta\rho_{\text{chase}} = 0.} \tag{257}$$

The phenomenological growth and lensing functions are

$$\boxed{\mu(k,a), \qquad \eta_{\text{slip}}(k,a), \qquad \Sigma(k,a),} \tag{258}$$

with the general-relativistic limit

$$\boxed{\mu = 1, \qquad \eta_{\text{slip}} = 1, \qquad \Sigma = 1.} \tag{259}$$

The tensor propagation benchmark is

$$\boxed{\alpha_M = 0, \qquad c_T = c, \qquad m_T = 0.} \tag{260}$$

The complete Gravity Triune benchmark is

$$\boxed{\begin{array}{l}\text{local binding recovery}\\ +\text{scaffolding background, clustering, and lensing}\\ +\text{chasing acceleration and perturbative stability}\\ +\text{common covariant conservation}\\ +\text{one shared parameter system}\\ +\text{at least one distinctive prediction.}\end{array}} \tag{261}$$

55 Benchmark Status

The benchmark stage establishes the following decision sequence.

1. Determine whether binding is entirely metric or contains an additional degree of freedom.
2. Derive the weak-field metric.
3. Calculate the Newtonian and post-Newtonian limits.
4. Construct compact-object and gravitational-wave solutions.
5. Specify the scaffold background equation of state.
6. Calculate scaffold sound speed and anisotropic stress.
7. Calculate dynamical and lensing potentials.
8. Propagate scaffold perturbations into structure growth.
9. Specify whether chasing is a cosmological constant, dynamical field, interaction effect, or geometric modification.
10. Calculate its background equation of state.
11. Calculate its perturbative stability and clustering behavior.
12. Derive all interaction currents covariantly.
13. Calculate μ, η_{slip}, and Σ from the field equations.
14. Calculate tensor speed, damping, mass, and source terms.
15. Fit local, astrophysical, and cosmological observables with one shared parameter hierarchy.
16. Compare the complete model fairly with general relativity and ΛCDM.
17. Determine whether the result is a genuine alternative, a coupled extension, or only a functional relabeling.

The benchmark does not prejudge which Gravity Triune realization is viable.

It defines what every realization must accomplish.

56 Benchmark Consolidation and Dependency Lock

The benchmark material above is consolidated into one ordered test hierarchy. A candidate Gravity Triune realization must pass the hierarchy in the following order:

$$\boxed{\begin{aligned}&\text{covariant closure} \longrightarrow \text{background closure} \longrightarrow \text{local-gravity recovery}\\&\qquad\qquad\qquad\quad \longrightarrow \text{linear stability and structure growth} \longrightarrow \text{joint observational comparison.}\end{aligned}} \tag{262}$$

No later success repairs an earlier failure. In particular, an acceptable late-time expansion curve cannot compensate for double-counted binding, nonconservation, a missing matter era, a ghost or gradient instability, or an unacceptable local gravitational limit.

The third-leg control problem and the historical Triple Bounce problem remain distinct:

$$\boxed{\mathcal{G}^{(3)}_{\mathrm{GT1}} \neq \mathcal{G}^{(123)}_{\mathrm{GT1}}.} \tag{263}$$

A successful third-leg realization establishes only that the functional binding–scaffolding–chasing decomposition can be represented consistently over the observable branch. Historical identification additionally requires finite outer and central transitions, an explicit *Electro* sector, a covariant scaffold-memory carrier, and at least one residue that cannot be absorbed into unrestricted third-leg initial data.

The exact control requirement for the background stage is

$$\boxed{\mathcal{G}^{(3)}_{\mathrm{GT1}} \xrightarrow[A(\phi)=\text{constant},\ \dot{\phi}=0]{\beta=0,\ V=\rho_\Lambda} \mathcal{G}^{\text{background}}_{\mathrm{GR}+\Lambda\mathrm{CDM}}.} \tag{264}$$

This is a background control statement. Exact perturbative identity also requires the additional scalar perturbation to be absent, decoupled, or initialized in a state that produces no observable departure. Paper III does not infer historical support from recovery of the control limit.

57 Candidate Realizations of the Gravity Triune

The functional definitions of binding, scaffolding, and chasing do not select a unique microscopic theory.

Several covariant realizations may produce similar homogeneous behavior while differing substantially in:

- degrees of freedom;
- local gravitational effects;
- clustering;
- anisotropic stress;
- propagation speeds;
- transition behavior;
- stability;
- memory retention; and
- observational signatures.

The candidate-theory problem is therefore

$$\boxed{\begin{array}{l}\text{Which minimal covariant field architecture can realize all three}\\ \text{functions without duplicating gravitational effects or}\\ \text{introducing fatal instability?}\end{array}} \tag{265}$$

The realization classes considered in this analysis are:

I. minimal hybrid models;

II. three-distinct-sector models;

III. unified dark-sector scalar models;

IV. covariant effective-fluid models;

V. elastic and solid scaffolding models;

VI. vector-field scaffolding models;

VII. modified-geometric binding models;

VIII. conformal interaction models;

IX. disformal interaction models; and

X. higher-derivative or generalized effective theories.

The purpose is not to retain all possibilities indefinitely.

The purpose is to rank them and select one closed system for the first background calculation.

58 Realization Selection Criteria

A candidate realization is evaluated according to nine criteria.

1. **Covariant closure.**

   The equations must follow from a covariant action or an equivalent closed variational system.

2. **Minimal degrees of freedom.**

   The realization should introduce no more propagating modes than are required by its physical claims.

3. **Binding recovery.**

   The tested local, post-Newtonian, and strong-field limits must remain recoverable.

4. **Scaffold clustering.**

   The scaffolding sector must possess a cold or effectively cold regime.

5. **Chasing acceleration.**

   The chasing sector must produce stable negative-pressure or acceleration-driving behavior.

6. **Controlled coupling.**

   Energy and momentum exchange must remain covariant, conserving, and perturbatively well defined.

7. **Transition extensibility.**

   The variables must remain meaningful through the outer and central reversals.

8. **Memory capacity.**

   The realization must state whether and how a scaffold can retain structural information.

9. **Numerical tractability.**

   The first model should admit practical background, perturbation, and parameter-space calculations.

Define a provisional realization score,

$$\mathfrak{R}_A = \sum_{n=1}^{9} w_n s_{nA}, \tag{266}$$

where:

- A labels a realization;
- s_{nA} is its score under criterion n; and
- w_n is the declared importance weight.

This score is a research-management device.

It is not a substitute for physical calculation.

59 Action Conventions for the Candidate Models

Formal basis. The candidate actions are organized using standard scalar–tensor, effective-field-theory, and physical-metric conventions (Bekenstein, 1993; Horndeski, 1974; Gubitosi et al., 2013).

For the action-level constructions in this analysis, natural units are used:

$$c = \hbar = 1. \tag{267}$$

The reduced Planck mass is defined by

$$M_{\mathrm{Pl}}^{-2} = 8\pi G. \tag{268}$$

The standard Einstein–Hilbert term is

$$S_{\mathrm{EH}} = \int \mathrm{d}^4x \sqrt{-g}\, \frac{M_{\mathrm{Pl}}^2}{2} R. \tag{269}$$

The generic candidate action is

$$S = S_{\mathrm{EH}} + S_{\mathrm{vis}} + S_{\mathrm{rad}} + S_{\mathrm{scaf}} + S_{\mathrm{chase}} + S_{\mathrm{bind}}^{\mathrm{extra}} + S_{\mathrm{int}}. \tag{270}$$

The conservative metric-binding realization sets

$$S_{\mathrm{bind}}^{\mathrm{extra}} = 0. \tag{271}$$

Any additional binding action must demonstrate that it does not duplicate the metric response already generated by S_{EH}.

60 GT1 Minimal Hybrid Reference Realization

The selected GT1 third-leg reference realization assigns:

$$\boxed{\begin{aligned} \textit{binding} &\longrightarrow \text{general-relativistic metric response,} \\ \textit{scaffolding} &\longrightarrow \text{cold non-luminous matter or fluid,} \\ \textit{chasing} &\longrightarrow \text{one dynamical scalar field.} \end{aligned}} \tag{272}$$

Its action is

$$\begin{aligned} S^{(3)}_{\mathrm{GT1}} = &\int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \\ &+ S_{\mathrm{vis}}\left[g_{\mu\nu}, \psi_{\mathrm{vis}}\right] + S_{\mathrm{rad}}\left[g_{\mu\nu}, \psi_{\mathrm{rad}}\right] \\ &+ S_{\mathrm{scaf}}\left[\widetilde{g}^{(\mathrm{scaf})}_{\mu\nu}, \psi_{\mathrm{scaf}}\right]. \end{aligned} \tag{273}$$

The chasing field is

$$\phi = \phi_{\mathrm{chase}}. \tag{274}$$

The scaffold fields are collectively denoted

$$\psi_{\mathrm{scaf}}. \tag{275}$$

The scaffold metric may initially be chosen as

$$\widetilde{g}^{(\mathrm{scaf})}_{\mu\nu} = A^2(\phi)g_{\mu\nu}. \tag{276}$$

Visible matter and radiation remain minimally coupled to $g_{\mu\nu}$.

This restricts the scalar-mediated fifth-force-like interaction to the dark sector in the minimum model. The action in equation (273) is the single reference action propagated into the background, local-gravity, perturbation, and inference sections.

61 Why GT1 Is the First Reference Baseline

The selected GT1 realization has several advantages.

1. Binding is not double counted.
2. The local gravitational limit begins from general relativity.
3. A pressureless scaffold can reproduce the cold-dark-matter benchmark.
4. A canonical scalar can produce dynamical chasing behavior.
5. A conformal dark-sector coupling provides covariant exchange.
6. Visible matter may remain protected from a direct fifth force.
7. The background system is low dimensional.
8. The perturbation equations are known in general form.
9. The model can be reduced continuously to ΛCDM.

The corresponding standard limit is

$$A(\phi) \longrightarrow 1, \qquad \dot{\phi} \longrightarrow 0, \qquad V(\phi) \longrightarrow \rho_\Lambda. \tag{277}$$

In that limit,

$$\mathcal{M}_{\mathrm{GT1}} \longrightarrow \Lambda\mathrm{CDM}. \tag{278}$$

This property makes the model suitable for controlled comparison.

Its principal limitation is that a conventional cold scaffold does not automatically possess the cross-leg memory proposed by the Triple Bounce framework.

Memory must be added through a derived state variable, interaction law, or later scaffold generalization.

62 Field Equations of GT1

Variation with respect to the metric gives

$$M_{\mathrm{Pl}}^2 G_{\mu\nu} = T_{\mu\nu}^{\mathrm{vis}} + T_{\mu\nu}^{\mathrm{rad}} + T_{\mu\nu}^{\mathrm{scaf}} + T_{\mu\nu}^{\phi}. \tag{279}$$

The scalar stress–energy tensor is

$$T^{\phi}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right]. \tag{280}$$

Define the dimensionless coupling

$$\beta(\phi) = M_{\rm Pl}\frac{\mathrm{d}\ln A(\phi)}{\mathrm{d}\phi}. \tag{281}$$

With metric signature $(-+++)$, variation of the common action fixes the exchange current:

$$\boxed{Q^\nu_{\rm scaf} = \frac{\beta(\phi)}{M_{\rm Pl}}T_{\rm scaf}\nabla^\nu\phi, \qquad Q^\nu_\phi = -Q^\nu_{\rm scaf}.} \tag{282}$$

Here

$$T_{\rm scaf} = T_{{\rm scaf}\,\mu}{}^{\mu}. \tag{283}$$

The covariant sector equations are therefore

$$\nabla_\mu T^{\mu\nu}_{\rm scaf} = Q^\nu_{\rm scaf}, \tag{284}$$

$$\nabla_\mu T^{\mu\nu}_{\phi} = -Q^\nu_{\rm scaf}. \tag{285}$$

For pressureless scaffolding, $T_{\rm scaf} = -\rho_{\rm scaf}$ and $\nabla^0\phi = -\dot{\phi}$. Consequently,

$$Q^0_{\rm scaf} = \frac{\beta}{M_{\rm Pl}}\rho_{\rm scaf}\dot{\phi}, \tag{286}$$

which fixes the background sign used throughout Paper III. Replacing β by $-\beta$ is a permissible reparameterization only if every background, perturbation, mass-evolution, and fixed-point equation is transformed with it.

Total conservation remains exact:

$$\nabla_\mu\left(T^{\mu\nu}_{\rm vis} + T^{\mu\nu}_{\rm rad} + T^{\mu\nu}_{\rm scaf} + T^{\mu\nu}_{\phi}\right) = 0. \tag{287}$$

63 GT1 Background Equations

For a homogeneous scalar,

$$\phi = \phi(t), \tag{288}$$

the scalar density and pressure are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{289}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{290}$$

The Friedmann equation is

$$3M_{\rm Pl}^2 H^2 = \rho_{\rm vis} + \rho_{\rm rad} + \rho_{\rm scaf} + \rho_\phi - 3M_{\rm Pl}^2 \frac{k}{a^2}. \tag{291}$$

For a cold scaffold,

$$p_{\rm scaf} \approx 0, \qquad T_{\rm scaf} \approx -\rho_{\rm scaf}. \tag{292}$$

The action-derived background equations are

$$\boxed{\dot{\rho}_{\rm scaf} + 3H\rho_{\rm scaf} = \frac{\beta(\phi)}{M_{\rm Pl}}\rho_{\rm scaf}\dot{\phi},} \tag{293}$$

and

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\frac{\beta(\phi)}{M_{\rm Pl}}\rho_{\rm scaf}.} \tag{294}$$

These equations remain regular at $\dot{\phi} = 0$; no division by $\dot{\phi}$ is used. The scalar energy equation is the equal-and-opposite transfer required by total conservation.

The system reduces to uncoupled quintessence plus cold dark matter when

$$Q_{\rm scaf} = 0. \tag{295}$$

64 Three-Distinct-Sector Realization

The most literal interpretation introduces three independent field sectors,

$$\{\psi_{\text{bind}}, \chi_{\text{scaf}}, \phi_{\text{chase}}\}\,. \tag{296}$$

A generic action is

$$\begin{aligned} S_{3\text{F}} = S_{\text{EH}} + S_{\text{vis}} + S_{\text{rad}} + S_{\text{bind}}\left[g, \psi_{\text{bind}}\right] \\ + S_{\text{scaf}}\left[g, \chi_{\text{scaf}}\right] + S_{\text{chase}}\left[g, \phi_{\text{chase}}\right] \\ + S_{\text{int}}\left[g, \psi_{\text{bind}}, \chi_{\text{scaf}}, \phi_{\text{chase}}\right]. \end{aligned} \tag{297}$$

The stress–energy tensor is

$$T_{\mu\nu}^{\text{tot}} = T_{\mu\nu}^{\text{vis}} + T_{\mu\nu}^{\text{rad}} + T_{\mu\nu}^{\text{bind}} + T_{\mu\nu}^{\text{scaf}} + T_{\mu\nu}^{\text{chase}}. \tag{298}$$

This realization offers maximal functional independence.

It also has four major burdens.

1. The binding field may duplicate ordinary metric gravity.
2. The theory introduces several new propagating modes.
3. The interaction space becomes large.
4. Parameter identification becomes difficult.

The three-field realization is therefore not preferred for the first numerical study.

It remains useful as an upper-complexity comparison.

65 Independent Binding-Field Realizations

An additional binding field may be scalar, vector, tensorial, or nonlocal.

A simple scalar–tensor example is

$$S_{\text{bind}} = \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{M_{\text{Pl}}^2}{2}F(\varphi)R - \frac{1}{2}Z(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - U(\varphi)\right]. \tag{299}$$

The effective Planck mass is

$$M_*^2(\varphi) = M_{\mathrm{Pl}}^2 F(\varphi). \tag{300}$$

The effective gravitational coupling may therefore vary with the background state.

A binding modification must recover

$$F(\varphi) \longrightarrow 1, \tag{301}$$

and

$$\dot{F} \longrightarrow 0 \tag{302}$$

in the local general-relativistic regime unless an allowed screened departure remains.

The same scalar may also contribute to chasing through

$$U(\varphi). \tag{303}$$

In that case, binding and chasing are functions of one field rather than two additive substances.

This can improve parsimony but complicates functional interpretation.

66 Screening Requirements for Modified Binding

Benchmark literature. The need for environmental suppression of additional long-range forces is illustrated by chameleon screening and local-gravity constraints (Khoury and Weltman, 2004; Will, 2014).

An additional long-range binding field generally produces a fifth force.

A schematic scalar-mediated force is

$$\boldsymbol{F}_\varphi \propto -\beta(\varphi)\nabla\varphi. \tag{304}$$

Local recovery may require screening through:

- environment-dependent effective mass;
- nonlinear derivative interactions;
- density-dependent coupling;
- small scalar charge of compact bodies;
- a symmetry-restored local phase; or

- another derived suppression mechanism.

Define the ratio of additional to Newtonian acceleration,

$$\epsilon_5 = \frac{|\boldsymbol{a}_\varphi|}{|\boldsymbol{a}_N|}. \tag{305}$$

A successful screened regime requires

$$\epsilon_5 \ll 1 \tag{306}$$

where local tests demand general-relativistic behavior.

The screening mechanism must emerge from the same action used in cosmology.

It may not be inserted as a separate local rule.

67 Unified Dark-Sector Scalar

A unified dark-sector model assigns scaffolding and chasing to different regimes of one scalar field

$$\Theta. \tag{307}$$

Define

$$X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \Theta \partial_\nu \Theta. \tag{308}$$

The action is

$$S_{\mathrm{U}} = \int \mathrm{d}^4 x \sqrt{-g}\, P\left(X, \Theta\right). \tag{309}$$

The stress–energy tensor is

$$T^{\mathrm{U}}_{\mu\nu} = P_{,X} \partial_\mu \Theta \partial_\nu \Theta + P g_{\mu\nu}. \tag{310}$$

For a homogeneous timelike configuration,

$$\rho_{\mathrm{U}} = 2XP_{,X} - P, \tag{311}$$

$$p_{\mathrm{U}} = P. \tag{312}$$

The equation of state is

$$w_{\mathrm{U}} = \frac{P}{2XP_{,X} - P}. \tag{313}$$

The scalar propagation speed is

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}. \tag{314}$$

A unified model must possess both:

$$w_{\mathrm{U}} \approx 0, \qquad c_s^2 \approx 0 \tag{315}$$

during the scaffolding regime, and

$$w_{\mathrm{U}} < -\frac{1}{3} \tag{316}$$

during the chasing regime.

Producing both background behaviors is insufficient if the transition between them damages the matter power spectrum.

68 Stability of the Unified Scalar

For the $P(X, \Theta)$ model, a basic no-ghost condition is

$$P_{,X} + 2XP_{,XX} > 0. \tag{317}$$

A basic gradient-stability condition is

$$c_s^2 \geq 0. \tag{318}$$

Positive energy density requires

$$2XP_{,X} - P > 0 \tag{319}$$

in the relevant state.

A cold scaffold requires extremely small effective pressure support.

A chasing regime may instead require potential- or pressure-dominated behavior.

The unified-field trajectory must therefore avoid:

- a kinetic singularity;

- a sound-speed spike;
- a ghost crossing;
- excessive nonadiabatic pressure;
- unacceptable scale-dependent clustering; and
- a strongly coupled transition.

This realization is economical in degrees of freedom but demanding in perturbation design.

69 Covariant Effective-Fluid Realization

A less microscopic starting point treats scaffolding and chasing as interacting covariant fluids.

Each sector has stress–energy

$$T_i^{\mu\nu} = (\rho_i + p_i)\, u_i^\mu u_i^\nu + p_i g^{\mu\nu} + \pi_i^{\mu\nu}, \tag{320}$$

where

$$u_{i\mu}\pi_i^{\mu\nu} = 0, \qquad \pi_{i\mu}{}^{\mu} = 0. \tag{321}$$

The sector equations are

$$\nabla_\mu T_i^{\mu\nu} = Q_i^\nu, \qquad \sum_i Q_i^\nu = 0. \tag{322}$$

Closure requires constitutive relations for:

$$\left\{p_i(\rho_i, s_i), c_{s,i}^2, \pi_i^{\mu\nu}, Q_i^\nu\right\}. \tag{323}$$

A phenomenological fluid model is useful for the first background study.

It becomes incomplete if the pressure, sound speed, anisotropic stress, and exchange current are specified independently without an underlying state-space relation.

The effective-fluid realization should therefore be treated as an intermediate model rather than the final theory.

70 Particle Scaffold with a Field-Dependent Mass

Benchmark literature. A field-dependent scaffold mass and conformal dark-sector coupling follow the coupled-quintessence construction used as the GT1 reference point (Amendola, 2000; Bekenstein, 1993).

A concrete scaffold realization uses nonrelativistic particles whose mass depends on the chasing field:

$$m_{\mathrm{scaf}} = m_{\mathrm{scaf}}(\phi). \tag{324}$$

The point-particle action is

$$S_{\mathrm{scaf}} = -\sum_{A} \int m_{\mathrm{scaf}}(\phi)\, \mathrm{d}s_A. \tag{325}$$

Define

$$\beta_{\mathrm{scaf}}(\phi) = \frac{\mathrm{d} \ln m_{\mathrm{scaf}}}{\mathrm{d}\phi}. \tag{326}$$

The field-dependent mass produces dark-sector exchange.

At the background level, a cold particle density obeys schematically

$$\dot{\rho}_{\mathrm{scaf}} + 3H\rho_{\mathrm{scaf}} = \beta_{\mathrm{scaf}}\rho_{\mathrm{scaf}}\dot{\phi} \tag{327}$$

up to the adopted sign convention.

The scalar receives the opposite transfer.

This realization has three useful properties:

1. The scaffold remains particle-like.
2. The coupling has a clear microscopic interpretation.
3. The standard cold limit is recovered when $\beta_{\mathrm{scaf}} \to 0$.

Its cross-leg memory capacity remains limited unless the particle state includes additional labels, correlations, or a pre-existing spatial configuration.

71 Elastic and Solid Scaffolding

Benchmark literature. Elastic and solid dark-sector possibilities have precedents in relativistic solid and elastic-dark-energy models (Bucher and Spergel, 1999; Battye and Moss, 2006).

A scaffold intended to retain structural memory may be modeled as an effective relativistic solid or elastic medium.

Introduce three scalar fields

$$\varphi^I, \qquad I = 1, 2, 3, \tag{328}$$

representing comoving material coordinates.

Define

$$B^{IJ} = g^{\mu\nu} \partial_\mu \varphi^I \partial_\nu \varphi^J. \tag{329}$$

A generic solid action is

$$S_{\text{solid}} = \int \mathrm{d}^4 x \sqrt{-g}\, F\left(B^{IJ}, \phi\right). \tag{330}$$

Statistical isotropy of the background may be preserved when the internal action is invariant under spatial rotations in field space and

$$\varphi^I = \alpha x^I. \tag{331}$$

The solid can possess:

- a bulk modulus;
- a shear modulus;
- longitudinal modes;
- transverse modes;
- anisotropic stress; and
- persistent material-coordinate memory.

This makes the solid realization especially relevant to the inherited scaffold hypothesis.

Its principal danger is excessive anisotropic stress or altered tensor propagation.

72 Memory Variables in an Elastic Scaffold

An elastic scaffold can encode structural memory through deformation variables.

Define a reference internal metric

$$\overline{B}^{IJ}. \tag{332}$$

The strain tensor is schematically

$$\varepsilon^{IJ} = \frac{1}{2}\left(B^{IJ} - \overline{B}^{IJ}\right). \tag{333}$$

A scalar memory invariant may be

$$\mathcal{W} = \varepsilon^{IJ}\varepsilon_{IJ}. \tag{334}$$

Its evolution may satisfy

$$u^{\mu}\nabla_{\mu}\mathcal{W} = -\Gamma_{\mathrm{mem}}\mathcal{W} + \mathcal{S}_{\mathrm{mem}}, \tag{335}$$

where:

- Γ_{mem} is the memory-relaxation rate; and
- $\mathcal{S}_{\mathrm{mem}}$ is a source generated by the E history or transition dynamics.

Persistent memory requires

$$\Gamma_{\mathrm{mem}}\Delta t_{12} \lesssim 1 \tag{336}$$

over the first two legs, at least for the relevant large-scale modes.

Small-scale memory may relax more rapidly:

$$\Gamma_{\mathrm{mem}} = \Gamma_{\mathrm{mem}}(k, t). \tag{337}$$

This provides a covariant route toward selective large-scale retention.

73 Vector-Field Scaffolding

Benchmark literature. Vector-sector realizations require the degree counting and stability discipline developed in dynamical-vector gravity and generalized Proca theories (Jacobson and Mattingly, 2001; Heisenberg, 2014).

A vector scaffold may carry directional and transport information.

A simple massive-vector action is

$$S_A = \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_A^2 A_\mu A^\mu - V_A \left(A_\mu A^\mu \right) \right], \tag{338}$$

where

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu. \tag{339}$$

A single nonzero homogeneous spatial vector generally selects a preferred direction.

Background isotropy may instead be preserved through:

1. a purely temporal background component;
2. a triad of equal-norm orthogonal spatial vectors;
3. a statistically isotropic vector ensemble; or
4. a vector whose homogeneous expectation value vanishes.

A temporal background has

$$A_\mu = \left(A_0(t), 0, 0, 0 \right). \tag{340}$$

A spatial triad may be written as

$$A_i^{(a)} = A(t) \delta_i^a. \tag{341}$$

Vector realizations naturally carry transverse modes.

Those modes may help encode pathway orientation or vorticity, but they also produce strong isotropy and stability constraints.

74 Vector Stability and Degree Counting

A massive Proca field generally carries three propagating polarizations:

$$N_{\mathrm{dof}}^{\mathrm{Proca}} = 3. \tag{342}$$

A massless gauge vector carries two physical transverse polarizations:

$$N_{\mathrm{dof}}^{\mathrm{gauge}} = 2. \tag{343}$$

Additional derivative self-interactions may alter the constraint structure.

The theory must verify that the temporal component does not become an uncontrolled propagating ghost.

The quadratic action must satisfy

$$Q_V > 0, \tag{344}$$

and

$$c_V^2 \geq 0. \tag{345}$$

The longitudinal mode must also satisfy

$$Q_L > 0, \qquad c_L^2 \geq 0. \tag{346}$$

A vector scaffold is not preferred as the first model because it adds degrees of freedom before the scalar and fluid possibilities have been excluded.

75 Tensorial and Geometric Scaffold Possibilities

A scaffold may also be represented by a symmetric tensor field

$$S_{\mu\nu} \tag{347}$$

or by an effective geometric response.

A generic tensor action is highly constrained because an arbitrary rank-two field may introduce:

- additional tensor polarizations;
- a ghostlike degree of freedom;

- an incorrect constraint count;
- modified gravitational-wave speed;
- strong coupling; or
- a second physical metric.

A bimetric realization would contain two metrics,

$$g_{\mu\nu}, \qquad f_{\mu\nu}, \tag{348}$$

with an interaction potential

$$V\left(g^{-1}f\right). \tag{349}$$

Such models can support massive and massless spin-two combinations.

They are not suitable as a first Gravity Triune realization because the constraint and observational burdens are substantially larger than in the minimal hybrid model.

They remain possible later-stage candidates if simpler scaffolds cannot support the required memory structure.

76 Chasing as a Canonical Scalar

Benchmark literature. The canonical chasing realization follows the standard quintessence action and its cosmological dynamics (Ratra and Peebles, 1988; Wetterich, 1988; Copeland et al., 2006).

The simplest dynamical chasing realization is a canonical scalar

$$S_{\text{chase}} = \int \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]. \tag{350}$$

Its equation of state is

$$w_\phi = \frac{\dot\phi^2/2 - V}{\dot\phi^2/2 + V}. \tag{351}$$

Acceleration occurs when

$$V(\phi) > \dot\phi^2. \tag{352}$$

The canonical scalar has unit rest-frame propagation speed,

$$c_{s,\phi}^2 = 1 \tag{353}$$

in natural units.

This suppresses significant scalar clustering on subhorizon scales.

The model is therefore well suited to a smooth chasing component but not to a cold scaffold.

The potential must support:

- acceptable early-time behavior;
- late-time acceleration;
- stability;
- controlled interaction with scaffolding;
- cross-leg continuation; and
- a defined response at both reversals.

77 Noncanonical Chasing Fields

Benchmark literature. Noncanonical scalar alternatives are assessed against k-essence perturbation and stability results (Garriga and Mukhanov, 1999; Armendáriz-Picón et al., 2001).

A noncanonical chasing field may use

$$S_{\text{chase}} = \int \mathrm{d}^4x \sqrt{-g}\, P_{\text{chase}}\left(X_\phi, \phi\right), \tag{354}$$

where

$$X_\phi = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi. \tag{355}$$

Its density is

$$\rho_{\text{chase}} = 2X_\phi P_{\text{chase},X} - P_{\text{chase}}, \tag{356}$$

and its propagation speed is

$$c_{s,\text{chase}}^2 = \frac{P_{\text{chase},X}}{P_{\text{chase},X} + 2X_\phi P_{\text{chase},XX}}. \tag{357}$$

This broadens the available chasing behavior.

It also introduces greater risk of:

- a ghost;
- a gradient instability;
- superluminal effective characteristics;
- strong coupling;
- caustic formation; and
- poor parameter identifiability.

A noncanonical chasing field should be introduced only if the canonical model fails a defined requirement.

78 Conformal Couplings

Benchmark literature. The conformal matter metric and resulting exchange structure follow established scalar–matter coupling constructions (Bekenstein, 1993; Amendola, 2000).

A conformal interaction allows one sector to couple to

$$\widetilde{g}^{(i)}_{\mu\nu} = C_i(\phi) g_{\mu\nu}. \tag{358}$$

For positive metric signature preservation,

$$C_i(\phi) > 0. \tag{359}$$

The sector action is

$$S_i = S_i \left[C_i(\phi) g_{\mu\nu}, \psi_i \right]. \tag{360}$$

The interaction current is generated automatically by variation.

Define

$$\alpha_i(\phi) = \frac{1}{2} \frac{\mathrm{d} \ln C_i}{\mathrm{d}\phi}. \tag{361}$$

The current is schematically proportional to

$$Q_i^{\nu} \propto \alpha_i(\phi) T_i \nabla^{\nu} \phi. \tag{362}$$

Radiation with vanishing stress–energy trace may couple differently from nonrelativistic matter.

A dark-only conformal coupling uses

$$C_{\text{vis}} = C_{\text{rad}} = 1, \qquad C_{\text{scaf}} \neq 1. \tag{363}$$

This is the preferred first interaction architecture because it protects ordinary matter from direct composition-dependent coupling.

79 Disformal Couplings

Benchmark literature. The disformal extension is treated as a physical-metric generalization in the sense introduced by Bekenstein (1993).

A more general sector metric is

$$\widetilde{g}^{(i)}_{\mu\nu} = C_i(\phi, X_\phi)g_{\mu\nu} + D_i(\phi, X_\phi)\partial_\mu\phi\partial_\nu\phi. \tag{364}$$

The conformal model is recovered when

$$D_i = 0. \tag{365}$$

For the transformation to remain invertible, one requires

$$C_i \neq 0, \tag{366}$$

and, under the stated definition of X_ϕ,

$$C_i - 2D_iX_\phi \neq 0. \tag{367}$$

A disformal coupling can modify:

- sector propagation cones;
- momentum transfer;
- effective pressure;
- sound speeds;
- transition behavior; and
- the relation between matter and gravitational observables.

Its additional freedom also makes stability and causality harder to control.

Disformal interactions are therefore deferred until the conformal model has been tested.

80 Direct Interaction Actions

The sectors may interact through direct scalar operators.

A generic interaction action is

$$S_{\text{int}} = \int \mathrm{d}^4x \sqrt{-g}\, \mathcal{L}_{\text{int}}\left(\phi, \chi, \nabla\phi, \nabla\chi, J^{\mu}_{\text{scaf}}, \mathcal{W}\right). \tag{368}$$

Examples include

$$\mathcal{L}^{(1)}_{\text{int}} = -\lambda\phi^2\chi^2, \tag{369}$$

$$\mathcal{L}^{(2)}_{\text{int}} = -\zeta(\phi) J^{\mu}_{\text{scaf}} \nabla_{\mu}\phi, \tag{370}$$

and

$$\mathcal{L}^{(3)}_{\text{int}} = -\lambda_{\mathcal{W}} F_{\mathcal{W}}(\phi)\mathcal{W}. \tag{371}$$

The current

$$J^{\mu}_{\text{scaf}} = n_{\text{scaf}} u^{\mu}_{\text{scaf}} \tag{372}$$

may represent scaffold number transport.

Direct derivative couplings require special care because they can alter the canonical momenta and constraint structure.

Every candidate interaction must be tested for:

$$\{\text{covariance}, \text{conservation}, \text{kinetic positivity}, \text{gradient stability}, \text{causality}, \text{strong coupling}\}. \tag{373}$$

81 Interaction Currents from a Common Action

When the action is diffeomorphism invariant, total conservation follows from the complete equations of motion.

For two interacting sectors,

$$\nabla_\mu T^{\mu\nu}_{\text{scaf}} = Q^\nu, \tag{374}$$

$$\nabla_\mu T^{\mu\nu}_{\text{chase}} = -Q^\nu. \tag{375}$$

Relative to a chosen four-velocity,

$$Q^\mu = Qu^\mu + F^\mu, \qquad u_\mu F^\mu = 0. \tag{376}$$

At the homogeneous level,

$$F^\mu = 0 \tag{377}$$

by isotropy in the common cosmological frame.

At the perturbative level,

$$\delta F^\mu \neq 0 \tag{378}$$

in general.

Two models with the same background function

$$Q(t) \tag{379}$$

can have different perturbative momentum transfer.

Therefore,

$$\boxed{Q(t) \text{ alone does not define an interacting cosmological theory.}} \tag{380}$$

The covariant action or full current must be specified.

82 Frame Transformations

A scalar–tensor or conformally coupled theory may be written in different field frames.

Let the Jordan-frame metric be

$$g^{\mathrm{J}}_{\mu\nu}. \tag{381}$$

Define the Einstein-frame metric by

$$g^{\mathrm{E}}_{\mu\nu} = F(\varphi) g^{\mathrm{J}}_{\mu\nu}. \tag{382}$$

In the Jordan frame, matter may be minimally coupled while the gravitational coefficient varies.

In the Einstein frame, the gravitational term may be canonical while matter becomes explicitly coupled to the scalar.

The two descriptions are related by a field transformation when the transformation is regular.

They should not be counted as independent physical theories.

A consistent frame transformation must also transform:

- time intervals;
- scale factors;
- particle masses;
- stress–energy tensors;
- coupling functions;
- measurement units;
- perturbations; and
- observable definitions.

Physical predictions should remain frame independent when all quantities are transformed consistently.

83 The Physical Metric

A theory with multiple conformally or disformally related metrics must identify the metric measured by ordinary clocks, rods, and photons.

Define the visible-sector metric as

$$g^{\mathrm{phys}}_{\mu\nu} = g^{(\mathrm{vis})}_{\mu\nu}. \tag{383}$$

The physical metric determines:

- visible proper time;
- visible null cones;
- redshift;

- electromagnetic luminosity distance;
- laboratory free fall;
- atomic clocks; and
- ordinary gravitational lensing.

A dark scaffold may respond to another effective metric

$$\widetilde{g}_{\mu\nu}^{(\mathrm{scaf})}. \tag{384}$$

This produces relative forces in the dark sector.

The theory must not switch between metrics when interpreting observations without applying the required transformations.

84 Degrees-of-Freedom Counting

A realization must identify its propagating physical degrees of freedom.

General relativity contains two tensor polarizations:

$$N_{\mathrm{dof}}^{\mathrm{GR}} = 2. \tag{385}$$

A canonical scalar adds

$$N_{\mathrm{dof}}^{\mathrm{scalar}} = 1. \tag{386}$$

A massive vector generally adds

$$N_{\mathrm{dof}}^{\mathrm{massive\ vector}} = 3. \tag{387}$$

A fluid carries density and velocity perturbations whose independent mode count depends on its action and constraint formulation.

A relativistic solid possesses longitudinal and transverse phonon-like modes.

A generic higher-derivative field may introduce additional degrees of freedom unless degeneracy conditions remove them.

The physical count is not obtained merely by counting field components.

It requires:

1. the canonical variables;

2. primary constraints;
3. secondary constraints;
4. gauge symmetries;
5. first-class constraints;
6. second-class constraints; and
7. the rank of the kinetic matrix.

An unexplained extra degree of freedom is a potential instability rather than additional explanatory power.

85 Higher-Derivative and Degenerate Theories

Benchmark literature. Higher-derivative candidates are restricted by the Horndeski, generalized-Galileon, and degenerate-theory stability literature (Horndeski, 1974; Deffayet et al., 2011; Langlois and Noui, 2016).

Higher-derivative interactions may broaden the available gravitational and chasing behavior.

A generic higher-derivative action is

$$S_{\mathrm{HD}} = \int \mathrm{d}^4x\, \sqrt{-g}\, \mathcal{L}\left(\phi, \nabla\phi, \nabla\nabla\phi, g_{\mu\nu}, R_{\mu\nu\rho\sigma}\right). \tag{388}$$

Generic nondegenerate higher-time-derivative theories may contain an Ostrogradsky-like instability.

A viable theory must possess a constraint or degeneracy structure that removes the unwanted mode.

Let

$$\mathsf{H}_{AB} = \frac{\partial^2 \mathcal{L}}{\partial \ddot{q}_A \partial \ddot{q}_B} \tag{389}$$

be the highest-derivative Hessian.

A degenerate theory has

$$\det \mathsf{H} = 0 \tag{390}$$

in the relevant constrained formulation.

Degeneracy alone does not prove stability.

The remaining kinetic and gradient matrices must still be checked.

Higher-derivative realizations should be introduced only after the minimal second-order candidates have been evaluated.

86 Non-Double-Counting Audit

Every realization must pass a formal non-double-counting audit.

86.1 Step 1: identify the fundamental variables

List

$$\left\{g_{\mu\nu}, \psi_{\rm vis}, \psi_{\rm rad}, \psi_{\rm scaf}, \phi_{\rm chase}, \psi^{\rm extra}_{\rm bind}\right\}. \tag{391}$$

86.2 Step 2: specify one total action

Every source and interaction must appear in

$$S_{\rm tot}. \tag{392}$$

86.3 Step 3: vary the action once

Derive the field equations and stress–energy tensors from that action.

86.4 Step 4: define the physical metric

State which metric governs ordinary measurements.

86.5 Step 5: identify effective reinterpretations

If a geometric term is moved to the source side, mark it as an effective stress–energy contribution rather than a new independent substance.

86.6 Step 6: remove duplicate cosmological-constant terms

Verify that the same vacuum contribution is not included in both

$$\Lambda g_{\mu\nu} \tag{393}$$

and

$$T_{\mu\nu}^{\text{chase}}. \tag{394}$$

86.7 Step 7: verify source independence

Determine whether two named sectors actually represent distinct degrees of freedom or only different descriptions of one response.

The audit result is

$$\mathbb{I}_{\text{NDC}} = \begin{cases} 1, & \text{no duplicated contribution is present,} \\ 0, & \text{at least one physical effect is counted more than once.} \end{cases} \tag{395}$$

A candidate with

$$\mathbb{I}_{\text{NDC}} = 0 \tag{396}$$

is rejected before numerical fitting.

87 Transition Compatibility

Matching basis. Thin-surface limits are compared with standard junction conditions, while the preferred construction remains a finite-width evolution (Israel, 1966).

A Paper III realization must eventually be continued through both *Electro* reversals.

Let the complete field state be

$$\boldsymbol{\Upsilon} = \left(g_{\mu\nu}, \psi_{\text{scaf}}, \phi_{\text{chase}}, \psi_{\text{bind}}^{\text{extra}}, \mathrm{E}, \mathcal{W}\right). \tag{397}$$

At each transition,

$$\boldsymbol{\Upsilon}_j^+ = \mathcal{T}_j\left[\boldsymbol{\Upsilon}_j^-\right]. \tag{398}$$

A candidate realization is transition compatible when:

1. its fields remain finite;
2. its kinetic matrix remains nonsingular;
3. its stress–energy remains finite;
4. its total current remains conserved;

5. its physical metric remains invertible;
6. its perturbations remain well posed; and
7. its scaffold variables can define a retention law.

The first background study need not solve the complete reversal microphysics.

It should nevertheless avoid variables that become undefined when the *Electro* direction changes.

88 Candidate Realization Matrix

Table 2: Candidate covariant Gravity Triune realizations.

Realization	**Principal advantage**	**Principal burden**	**Recommended role**
Metric binding + cold fluid scaffold + canonical scalar chasing	Minimal degrees of freedom and direct ΛCDM limit.	No automatic structural-memory mechanism.	First background and perturbation baseline.
Metric binding + conformally coupled particle scaffold + scalar chasing	Covariant dark-sector exchange with microscopic interpretation.	Fifth force and growth constraints within the dark sector.	Preferred first interacting model.
Metric binding + elastic solid scaffold + scalar chasing	Natural strain and memory variables.	Anisotropic stress, transverse modes, and greater numerical complexity.	Primary second-stage memory model.
Metric binding + vector scaffold + scalar chasing	Carries orientation, transport, and vector modes.	Background isotropy and longitudinal stability.	Secondary candidate if vector residue is required.
Unified $P(X,\Theta)$ dark sector	One field may realize both scaffolding and chasing.	Cold clustering and negative pressure must coexist without harmful sound speed.	Economical comparison model.
Scalar–tensor binding + scaffold + chasing	Binding and acceleration may share one geometric scalar.	Local screening, Planck-mass evolution, and double-counting risk.	Later modified-gravity comparison.

Realization	Principal advantage	Principal burden	Recommended role
Three independent fields	Maximum functional separation.	High parameter count and weak identifiability.	Upper-complexity benchmark, not first model.
Bimetric or tensor scaffold	Rich geometric memory and additional spin-two structure.	Constraint consistency, tensor propagation, and strong coupling.	Deferred high-complexity possibility.
Disformal dark-sector coupling	Flexible momentum exchange and transition behavior.	Causal cones, invertibility, and perturbative stability.	Deferred extension after conformal coupling.
Higher-derivative degenerate theory	Broad dynamical behavior with constrained extra modes.	Complex stability and degree-counting analysis.	Use only if second-order models fail.

89 Ranked Realizations for Numerical Study

The candidate realizations are ranked for computational development.

89.1 Rank 1: minimal uncoupled hybrid

The first control model is

$$\boxed{\text{general relativity} + \text{cold scaffold fluid} + \text{canonical chasing scalar.}} \tag{399}$$

This model establishes the background code and standard limits.

It does not yet implement the strongest triune interaction claim.

89.2 Rank 2: conformally coupled minimal hybrid

The first interacting model is

$$\boxed{\text{general relativity} + \text{conformally coupled cold scaffold} + \text{canonical chasing scalar.}} \tag{400}$$

This is the preferred first genuine Gravity Triune candidate.

89.3 Rank 3: elastic-memory scaffold

The first dedicated cross-leg memory model is

$$\boxed{\text{general relativity} + \text{elastic scaffold} + \text{canonical chasing scalar.}} \tag{401}$$

It should be attempted after the minimal interacting background is understood.

89.4 Rank 4: unified dark scalar

The unified $P(X, \Theta)$ model is retained as a parsimonious comparison.

89.5 Rank 5: modified binding

Scalar–tensor or other modified-binding models are deferred until the metric-binding architecture has been tested.

This ordering protects the theory from introducing unnecessary gravitational modifications before ordinary general relativity has been shown inadequate for the proposed triune functions.

90 First Numerical Realization

The first interacting numerical realization is selected as

$$\boxed{S^{(3)}_{\mathrm{GT1}} = S_{\mathrm{EH}} + S_{\mathrm{vis}}[g] + S_{\mathrm{rad}}[g] + S_{\mathrm{scaf}}[A^2(\phi)g] + S_{\phi}[g, \phi].} \tag{402}$$

The model contains:

$$\{g_{\mu\nu}, \psi_{\mathrm{vis}}, \psi_{\mathrm{rad}}, \psi_{\mathrm{scaf}}, \phi\}\,. \tag{403}$$

Its functional interpretation is

$$\boxed{\begin{aligned} \mathfrak{G}_{\mathrm{bind}} &= \text{metric response of general relativity,} \\ \mathfrak{G}_{\mathrm{scaf}} &= \text{cold conformally coupled scaffold,} \\ \mathfrak{G}_{\mathrm{chase}} &= \text{canonical scalar chasing field.} \end{aligned}} \tag{404}$$

The model parameters are provisionally

$$\boldsymbol{\theta}_{\mathrm{GT1}} = \left(\boldsymbol{\theta}_V, \boldsymbol{\theta}_A, \rho_{\mathrm{scaf,ini}}, \phi_{\mathrm{ini}}, \dot{\phi}_{\mathrm{ini}}\right), \tag{405}$$

where:

- $\boldsymbol{\theta}_V$ parameterizes the chasing potential; and
- $\boldsymbol{\theta}_A$ parameterizes the conformal coupling.

The ΛCDM control point is included in the parameter space.

91 Initial Potential and Coupling Families

The first numerical study should compare a small number of simple potential and coupling families.

A constant-potential control is

$$V(\phi) = V_0. \tag{406}$$

A slowly varying exponential potential is

$$V(\phi) = V_0 \exp\left(-\lambda \frac{\phi}{M_{\rm Pl}}\right). \tag{407}$$

A simple power-law potential is

$$V(\phi) = V_0 \left(\frac{\phi}{M_{\rm Pl}}\right)^n. \tag{408}$$

The uncoupled control has

$$A(\phi) = 1. \tag{409}$$

A constant logarithmic coupling may be represented by

$$A(\phi) = \exp\left(\beta \frac{\phi}{M_{\rm Pl}}\right). \tag{410}$$

Then

$$\alpha(\phi) = \frac{\beta}{M_{\rm Pl}}. \tag{411}$$

These families are not assumed to be fundamental.

They form controlled baseline tests.

The first study should avoid scanning many unrelated potential and coupling shapes.

92 First-Study Reduction Limits

The selected model must reproduce several nested limits.

92.1 General relativity with cold scaffold and constant chasing

$$\beta = 0, \qquad \dot{\phi} = 0, \qquad V(\phi) = V_0. \tag{412}$$

This is the ΛCDM-like control.

92.2 Uncoupled dynamical chasing

$$\beta = 0, \qquad \dot{\phi} \neq 0. \tag{413}$$

This gives uncoupled scalar dark energy plus a cold scaffold.

92.3 Coupled scaffold and chasing

$$\beta \neq 0. \tag{414}$$

This is the first interacting triune realization.

92.4 Memory-free baseline

$$\mathcal{W} = 0. \tag{415}$$

This isolates ordinary coupled dark-sector behavior.

92.5 Memory extension

A later extension includes

$$\mathcal{W} \neq 0. \tag{416}$$

The memory extension should be evaluated against the same background control rather than introduced simultaneously with every other modification.

93 Required Outputs of the First Realization

The first numerical model must produce:

$$\left\{a(t), H(t), \phi(t), \dot{\phi}(t), \rho_{\text{scaf}}(t), \rho_{\text{rad}}(t), \rho_{\text{vis}}(t)\right\}. \tag{417}$$

It must also produce:

$$\{w_\phi(t), w_{\text{eff}}(t), \Omega_{\text{scaf}}(t), \Omega_\phi(t)\}. \tag{418}$$

The first stability outputs are

$$\left\{\rho_i > 0, Q_s > 0, c_s^2 \geq 0, \text{finite coupling}, \text{finite background}\right\}. \tag{419}$$

The first perturbative outputs are

$$\{\delta_{\text{scaf}}, \theta_{\text{scaf}}, \delta\phi, \Phi, \Psi, D(a), f\sigma_8\}. \tag{420}$$

The first observational comparison is against:

$$\{H(z), D_M(z), D_L(z), f\sigma_8(z), \text{CMB background consistency}\}. \tag{421}$$

Cross-leg continuation is a later extension after the third-leg system has been verified.

94 Candidate-Realization Rejection Criteria

A candidate realization fails or is deprioritized if any of the following is unavoidable.

1. The action is not covariant or mathematically closed.
2. The total stress–energy tensor is not conserved.
3. Binding is counted both as ordinary metric response and as an identical extra source.
4. The visible sector couples to an additional long-range field without an acceptable local limit.
5. The physical metric is not defined.
6. A conformal or disformal metric becomes noninvertible.
7. The kinetic matrix contains a negative eigenvalue.
8. The kinetic matrix becomes singular within the required history.

9. A physical scalar, vector, or tensor mode has an uncontrolled gradient instability.
10. The scaffold cannot reach a cold clustering regime.
11. The chasing field cannot produce acceleration without destabilizing the background.
12. The model reproduces acceleration but gives unacceptable structure growth.
13. A unified dark field develops excessive sound speed during the scaffold regime.
14. A vector scaffold requires an unacceptably anisotropic background.
15. An elastic scaffold generates excessive anisotropic stress.
16. A modified-binding field cannot recover local gravity.
17. A higher-derivative realization contains an unremoved extra mode.
18. The background exchange function has no perturbative or covariant completion.
19. The model introduces more free functions than independently constrained observables.
20. The model has no regular reduction to a known control limit.
21. The fields become undefined at an E reversal.
22. The claimed memory variable has no evolution law.
23. The only successful realization is exactly ΛCDM with renamed components and no additional calculation.

95 Formal Summary of Candidate Realizations

The general candidate action is

$$\boxed{S = S_{\rm EH} + S_{\rm vis} + S_{\rm rad} + S_{\rm scaf} + S_{\rm chase} + S_{\rm bind}^{\rm extra} + S_{\rm int}.} \tag{422}$$

The conservative metric-binding choice is

$$\boxed{S_{\rm bind}^{\rm extra} = 0.} \tag{423}$$

The selected first interacting realization is

$$\boxed{S_{\rm GT1} = S_{\rm EH} + S_{\rm vis}[g] + S_{\rm rad}[g] + S_{\rm scaf}[A^2(\phi)g] + S_\phi[g,\phi].} \tag{424}$$

Its functional interpretation is

$$\boxed{\begin{aligned} \textit{binding} &= \text{general-relativistic metric response}, \\ \textit{scaffolding} &= \text{cold conformally coupled dark sector}, \\ \textit{chasing} &= \text{canonical scalar field}. \end{aligned}} \tag{425}$$

The dark-sector conservation equations are

$$\boxed{\nabla_\mu T^{\mu\nu}_{\text{scaf}} = Q^\nu, \qquad \nabla_\mu T^{\mu\nu}_{\text{chase}} = -Q^\nu.} \tag{426}$$

The unified-scalar alternative is

$$\boxed{S_{\text{U}} = \int \mathrm{d}^4x \sqrt{-g}\, P(X, \Theta).} \tag{427}$$

The memory-oriented scaffold alternative is

$$\boxed{S_{\text{solid}} = \int \mathrm{d}^4x \sqrt{-g}\, F(B^{IJ}, \phi).} \tag{428}$$

The non-double-counting condition is

$$\boxed{\mathbb{I}_{\text{NDC}} = 1.} \tag{429}$$

The candidate-selection hierarchy is

$$\boxed{\begin{aligned} &\text{minimal uncoupled hybrid} \\ &\longrightarrow \text{minimal conformally coupled hybrid} \\ &\longrightarrow \text{elastic-memory scaffold} \\ &\longrightarrow \text{unified or modified-gravity alternatives.} \end{aligned}} \tag{430}$$

The decisive realization question is

$$\boxed{\begin{aligned} &\text{Can the minimal metric-binding, cold-scaffold, scalar-chasing} \\ &\text{model produce a stable and observationally acceptable common} \\ &\text{background before additional memory, vector, tensor, or} \\ &\text{modified-gravity degrees of freedom are introduced?} \end{aligned}} \tag{431}$$

96 Candidate-Realization Status

The realization problem is now reduced to a controlled development sequence.

1. Implement the minimal uncoupled hybrid model.
2. Verify its ΛCDM reduction.
3. Implement one conformally coupled dark-sector current.
4. Construct the homogeneous autonomous system.
5. Find radiation-, matter-, and chasing-dominated fixed points.
6. Test background stability.
7. Calculate the third-leg expansion history.
8. Calculate linear scaffold and scalar perturbations.
9. Test growth and lensing consistency.
10. Determine whether the conformal interaction improves or degrades the benchmark fit.
11. Introduce one memory variable only after the minimal interaction is controlled.
12. Compare particle, elastic, and unified scaffold realizations.
13. Introduce modified binding only if metric binding proves insufficient.
14. Reject higher-complexity models that do not provide a required new function.
15. Carry the surviving variables into the outer- and central-reversal analysis.

The preferred first realization is therefore selected without claiming that it is the final ontology of the Gravity Triune.

97 The Gravity Triune Background Dynamical System

Dynamical-systems basis. The autonomous-variable and fixed-point analysis follows standard scalar-field and coupled-quintessence dynamical-systems methods (Copeland et al., 1998; Amendola, 2000).

The candidate-realization analysis selected the first interacting Gravity Triune realization,

$$S_{\mathrm{GT1}} = S_{\mathrm{EH}} + S_{\mathrm{vis}}[g] + S_{\mathrm{rad}}[g] + S_{\mathrm{scaf}}[A^2(\phi)g] + S_{\phi}[g,\phi]. \tag{432}$$

Its functional interpretation is

$$\boxed{\begin{aligned} \textit{binding} &= \text{the metric response of general relativity,} \\ \textit{scaffolding} &= \text{a cold conformally coupled non-luminous sector,} \\ \textit{chasing} &= \text{a canonical scalar field.} \end{aligned}} \tag{433}$$

This part converts that action into a background dynamical system.

The immediate objectives are to determine whether the model admits:

- a radiation-dominated epoch;
- a sufficiently matter-like scaffold epoch;
- a late accelerated chasing epoch;
- finite energy exchange;
- a continuous reduction to ΛCDM;
- stable fixed points or attractors;
- a nonzero basin of acceptable initial states; and
- state variables that remain finite at both *Electro* reversals.

The first dynamical study is restricted primarily to the third-leg background.

The hidden first and second legs require an additional E stress–energy sector and are introduced later in this part as an extension of the same state-space architecture.

98 Background Sign Convention and Energy Exchange

Natural units,

$$c = \hbar = 1, \tag{434}$$

continue to be used.

The scaffold metric is

$$\tilde{g}^{(\mathrm{scaf})}_{\mu\nu} = A^2(\phi) g_{\mu\nu}. \tag{435}$$

Define the dimensionless logarithmic coupling

$$\beta(\phi) = M_{\mathrm{Pl}} \frac{\mathrm{d} \ln A}{\mathrm{d}\phi}. \tag{436}$$

The covariant convention inherited from the GT1 action is

$$\boxed{\nabla_\mu T^{\mu\nu}_{\rm scaf} = \frac{\beta}{M_{\rm Pl}} T_{\rm scaf} \nabla^\nu \phi, \qquad \nabla_\mu T^{\mu\nu}_{\phi} = -\frac{\beta}{M_{\rm Pl}} T_{\rm scaf} \nabla^\nu \phi.} \tag{437}$$

For signature $(-+++)$, pressureless scaffolding obeys $T_{\rm scaf} = -\rho_{\rm scaf}$ and a homogeneous field obeys $\nabla^0 \phi = -\dot{\phi}$. The background exchange convention is therefore

$$\dot{\rho}_{\rm scaf} + 3H\rho_{\rm scaf} = \frac{\beta(\phi)}{M_{\rm Pl}} \rho_{\rm scaf} \dot{\phi} \tag{438}$$

for a pressureless scaffold.

The scalar energy equation is

$$\dot{\rho}_\phi + 3H \left(\rho_\phi + p_\phi\right) = -\frac{\beta(\phi)}{M_{\rm Pl}} \rho_{\rm scaf} \dot{\phi}. \tag{439}$$

The scalar equation of motion is therefore

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\frac{\beta(\phi)}{M_{\rm Pl}} \rho_{\rm scaf}. \tag{440}$$

The total dark-sector exchange is zero:

$$Q_{\rm scaf} + Q_\phi = 0. \tag{441}$$

The adopted sign convention implies

$$\beta\dot{\phi} > 0 \quad \Longrightarrow \quad \text{energy is transferred from the scalar sector to the scaffold.} \tag{442}$$

An alternative sign convention can be obtained through

$$\beta \longrightarrow -\beta. \tag{443}$$

All fixed-point and stability relations must be transformed consistently if that alternative convention is used.

99 Background Closure and Constraint Propagation

The homogeneous GT1 system is closed by the Friedmann equation, the Raychaudhuri equation, the visible-matter and radiation continuity equations, the scaffold continuity equation, and the scalar equation of motion. The Raychaudhuri equation is

$$\boxed{\dot{H} = -\frac{1}{2M_{\mathrm{Pl}}^2}\left(\rho_{\mathrm{vis}} + \frac{4}{3}\rho_{\mathrm{rad}} + \rho_{\mathrm{scaf}} + \dot{\phi}^2\right) + \frac{k}{a^2}.} \tag{444}$$

The total homogeneous density and pressure obey

$$\dot{\rho}_{\mathrm{tot}} + 3H\left(\rho_{\mathrm{tot}} + p_{\mathrm{tot}}\right) = 0, \tag{445}$$

because the scaffold and scalar exchange terms cancel exactly.

Define the dimensionless Friedmann-constraint residual

$$\mathcal{C}_F = 1 - x^2 - y^2 - z^2 - u^2 - \Omega_{\mathrm{scaf}} - \Omega_k. \tag{446}$$

For the exact autonomous system derived below,

$$\boxed{\mathcal{C}_F' = 2\epsilon_H \mathcal{C}_F.} \tag{447}$$

Therefore the physical constraint surface $\mathcal{C}_F = 0$ is invariant. A numerical trajectory that begins on the constraint surface must remain there up to integration error. Equation (447) also shows why a small uncontrolled residual can grow during decelerating epochs; the constraint should be enforced or monitored rather than treated as a passive diagnostic.

100 Dimensionless Autonomous Variables

Define the e-fold variable

$$N = \ln a. \tag{448}$$

A prime in this analysis denotes

$$X' = \frac{\mathrm{d}X}{\mathrm{d}N}. \tag{449}$$

Introduce the scalar kinetic and potential variables

$$x = \frac{\dot{\phi}}{\sqrt{6} M_{\mathrm{Pl}} H}, \tag{450}$$

and

$$y = \frac{\sqrt{V(\phi)}}{\sqrt{3} M_{\mathrm{Pl}} H}. \tag{451}$$

Define the radiation and visible-pressureless-matter variables

$$z = \frac{\sqrt{\rho_{\mathrm{rad}}}}{\sqrt{3} M_{\mathrm{Pl}} H}, \tag{452}$$

and

$$u = \frac{\sqrt{\rho_{\mathrm{vis}}}}{\sqrt{3} M_{\mathrm{Pl}} H}. \tag{453}$$

The corresponding density parameters are

$$\Omega_{\phi} = x^2 + y^2, \tag{454}$$

$$\Omega_{\mathrm{rad}} = z^2, \tag{455}$$

$$\Omega_{\mathrm{vis}} = u^2. \tag{456}$$

The curvature parameter is

$$\Omega_k = -\frac{k}{a^2 H^2}. \tag{457}$$

The scaffold density parameter is

$$\Omega_{\mathrm{scaf}} = \frac{\rho_{\mathrm{scaf}}}{3 M_{\mathrm{Pl}}^2 H^2}. \tag{458}$$

101 Friedmann Constraint and Physical Phase Space

The Friedmann equation gives

$$\boxed{1 = x^2 + y^2 + z^2 + u^2 + \Omega_{\text{scaf}} + \Omega_k.} \tag{459}$$

Therefore,

$$\Omega_{\text{scaf}} = 1 - x^2 - y^2 - z^2 - u^2 - \Omega_k. \tag{460}$$

For the spatially flat submanifold,

$$\Omega_k = 0, \tag{461}$$

the positive-density phase space is

$$\mathcal{P}_{\text{flat}} = \left\{(x, y, z, u) : y \geq 0, z \geq 0, u \geq 0, x^2 + y^2 + z^2 + u^2 \leq 1\right\}. \tag{462}$$

The scaffold occupies the remaining fraction.

The scalar equation of state is

$$w_\phi = \frac{x^2 - y^2}{x^2 + y^2} \tag{463}$$

when

$$x^2 + y^2 \neq 0. \tag{464}$$

The material effective equation of state in the flat model is

$$w_{\text{eff}} = x^2 - y^2 + \frac{z^2}{3}. \tag{465}$$

102 The Hubble Slow-Evolution Parameter

Define

$$\epsilon_H = -\frac{\dot{H}}{H^2}. \tag{466}$$

For the GT1 background,

$$\boxed{\epsilon_H = 3x^2 + \frac{3}{2}\Omega_{\mathrm{scaf}} + \frac{3}{2}u^2 + 2z^2 + \Omega_k.} \tag{467}$$

On the flat submanifold,

$$\epsilon_H = \frac{3}{2}\left(1 + w_{\mathrm{eff}}\right). \tag{468}$$

The deceleration parameter is

$$q_{\mathrm{dec}} = -\frac{\ddot{a}}{aH^2} = \epsilon_H - 1. \tag{469}$$

Accelerated expansion requires

$$\boxed{\epsilon_H < 1.} \tag{470}$$

On the flat submanifold, this is equivalent to

$$w_{\mathrm{eff}} < -\frac{1}{3}. \tag{471}$$

The geometric effective equation of state may be defined by

$$w_{\mathrm{geo}} = -1 + \frac{2\epsilon_H}{3}. \tag{472}$$

This definition remains convenient when curvature is nonzero.

103 Potential and Coupling Slopes

Define the dimensionless potential slope

$$\lambda(\phi) = -M_{\rm Pl}\frac{V_{,\phi}}{V}. \tag{473}$$

Define the potential-shape function

$$\Gamma_V(\phi) = \frac{VV_{,\phi\phi}}{V_{,\phi}^2}. \tag{474}$$

The potential slope evolves as

$$\lambda' = -\sqrt{6}x\lambda^2\left(\Gamma_V - 1\right). \tag{475}$$

For an exponential potential,

$$V(\phi) = V_0 \exp\left(-\lambda\frac{\phi}{M_{\rm Pl}}\right), \tag{476}$$

one has

$$\Gamma_V = 1, \tag{477}$$

and therefore

$$\lambda' = 0. \tag{478}$$

Define the coupling-slope function

$$\kappa_\beta(\phi) = M_{\rm Pl}\frac{{\rm d}\beta}{{\rm d}\phi}. \tag{479}$$

Then

$$\beta' = \sqrt{6}x\kappa_\beta. \tag{480}$$

The autonomous fixed-point analysis below initially assumes

$$\lambda = \text{constant}, \qquad \beta = \text{constant}. \tag{481}$$

Variable slopes enlarge the phase space by adding λ and β as dynamical variables.

104 Autonomous Background Equations

Using the definitions above, the autonomous system is

$$x' = -3x + \sqrt{\frac{3}{2}}\lambda y^2 - \sqrt{\frac{3}{2}}\beta\Omega_{\rm scaf} + x\epsilon_H, \tag{482}$$

$$y' = -\sqrt{\frac{3}{2}}\lambda xy + y\epsilon_H, \tag{483}$$

$$z' = -2z + z\epsilon_H, \tag{484}$$

$$u' = -\frac{3}{2}u + u\epsilon_H, \tag{485}$$

$$\Omega_k' = 2\left(\epsilon_H - 1\right)\Omega_k. \tag{486}$$

The scaffold density evolves as

$$\boxed{\Omega_{\rm scaf}' = \Omega_{\rm scaf}\left[-3 + \sqrt{6}\beta x + 2\epsilon_H\right].} \tag{487}$$

The scalar itself is reconstructed through

$$\phi' = \sqrt{6}M_{\rm Pl}x. \tag{488}$$

The Hubble rate satisfies

$$\frac{H'}{H} = -\epsilon_H. \tag{489}$$

Equations (482)–(489) form a closed background system once

$$V(\phi), \qquad A(\phi) \tag{490}$$

and the initial state are specified.

105 Exact General-Relativity plus LCDM Background Control

The exact homogeneous control is obtained by choosing

$$\boxed{A(\phi) = A_0 = \text{constant}, \qquad \beta = 0, \qquad \dot{\phi} = 0, \qquad V(\phi) = V_0 = \rho_\Lambda.} \tag{491}$$

A constant positive factor A_0 can be absorbed into the scaffold mass normalization; the convention $A_0 = 1$ is therefore adopted in the control model. The scaffold and visible pressureless matter then satisfy

$$\rho_{\text{scaf}} \propto a^{-3}, \qquad \rho_{\text{vis}} \propto a^{-3}, \tag{492}$$

while radiation and the scalar vacuum contribution satisfy

$$\rho_{\text{rad}} \propto a^{-4}, \qquad \rho_\phi = V_0 = \rho_\Lambda. \tag{493}$$

The background equations reduce to

$$\boxed{H^2 = \frac{1}{3M_{\text{Pl}}^2} (\rho_{\text{rad}} + \rho_{\text{vis}} + \rho_{\text{scaf}} + \rho_\Lambda) - \frac{k}{a^2},} \tag{494}$$

which is the general-relativistic ΛCDM background after identifying the total pressureless density with $\rho_{\text{vis}} + \rho_{\text{scaf}}$.

The control is an evolving orbit rather than one fixed point. In the dimensionless variables, $x = 0$ remains invariant, while $y^2 = \Omega_\Lambda$ changes as H changes. The numerical control must reproduce the radiation–matter–vacuum sequence and the standard curvature evolution before nonzero coupling or a rolling potential is tested.

Limitation 105.1 (Background versus full control)**.** *Recovery of equation* (494) *establishes the background control only. Exact perturbative recovery additionally requires that $\delta\phi$ be absent, dynamically decoupled, or observationally irrelevant under declared initial conditions.*

106 Effective Sector Equations of State

The scaffold continuity equation may be written as

$$\dot{\rho}_{\text{scaf}} + 3H \left(1 + w_{\text{scaf}}^{\text{eff}}\right) \rho_{\text{scaf}} = 0. \tag{495}$$

Therefore,

$$\boxed{w_{\rm scaf}^{\rm eff} = -\sqrt{\frac{2}{3}}\beta x.} \tag{496}$$

The scalar effective equation of state is defined through

$$\dot{\rho}_\phi + 3H\left(1 + w_\phi^{\rm eff}\right)\rho_\phi = 0, \tag{497}$$

which gives

$$w_\phi^{\rm eff} = w_\phi + \frac{\sqrt{6}\beta x \Omega_{\rm scaf}}{3\Omega_\phi} \tag{498}$$

when

$$\Omega_\phi \neq 0. \tag{499}$$

At an exact scaffold–scalar scaling solution,

$$w_{\rm scaf}^{\rm eff} = w_\phi^{\rm eff} = w_{\rm eff}. \tag{500}$$

This equality is a useful numerical diagnostic.

107 Invariant Submanifolds and Phase-Space Boundaries

Several boundaries of the physical phase space are invariant.

The radiation-free surface

$$z = 0 \tag{501}$$

is invariant because

$$z' \propto z. \tag{502}$$

The visible-matter-free surface

$$u = 0 \tag{503}$$

is invariant.

The potential-free surface

$$y = 0 \tag{504}$$

is invariant for regular finite λ.

The scaffold-free surface

$$\Omega_{\text{scaf}} = 0 \tag{505}$$

is invariant because

$$\Omega'_{\text{scaf}} \propto \Omega_{\text{scaf}}. \tag{506}$$

The flat surface

$$\Omega_k = 0 \tag{507}$$

is invariant under the standard fixed-curvature dynamics.

Positive initial densities therefore remain nonnegative unless the solution reaches a singular boundary or the effective theory itself fails.

The invariant surfaces partition the phase space into dynamically distinct regions.

They also prevent a numerical trajectory from generating a missing sector from exactly zero unless an explicit source term is introduced.

108 Fixed-Point Conditions

A fixed point

$$\boldsymbol{X}_* = (x_*, y_*, z_*, u_*, \Omega_{k,*}) \tag{508}$$

satisfies

$$\boldsymbol{F}(\boldsymbol{X}_*) = \boldsymbol{0}. \tag{509}$$

The principal flat fixed points for constant λ and β are:

I. radiation domination;

II. scalar kinetic domination;

III. visible pressureless-matter domination;

IV. a coupled scaffold kinetic or ϕ-matter-dominated epoch;

V. scalar chasing domination;

VI. a scaffold–chasing scaling state; and

VII. a scalar–radiation scaling state.

The fixed points are not all required to be future attractors.

A successful cosmic history generally requires a sequence of saddles and attractors,

$$\text{radiation saddle} \longrightarrow \text{matter-like saddle} \longrightarrow \text{accelerated attractor}. \tag{510}$$

109 Fixed-Point Table

For compactness, define

$$\mathcal{A} = \lambda + \beta. \tag{511}$$

The principal flat fixed points are summarized below.

Table 3: Principal fixed points of the constant-slope GT1 background.

Point	**Coordinates**	**Effective state**	**Existence condition**
R	$x = 0,\ y = 0,\ z = 1,\ u = 0,$ $\Omega_{\rm scaf} = 0$	$w_{\rm eff} = 1/3$	Always
$K_\pm$	$x = \pm 1,\ y = z = u = 0,$ $\Omega_{\rm scaf} = 0$	$w_{\rm eff} = 1$	Always
B	$u = 1,\ x = y = z = 0,$ $\Omega_{\rm scaf} = 0$	$w_{\rm eff} = 0$	Always
M_ϕ	$x = -\sqrt{2/3}\,\beta,\ y = z = u = 0,$ $\Omega_{\rm scaf} = 1 - 2\beta^2/3$	$w_{\rm eff} = 2\beta^2/3$	$\beta^2 \leq 3/2$
D_ϕ	$x = \lambda/\sqrt{6},\ y = \sqrt{1 - \lambda^2/6},$ $z = u = 0,\ \Omega_{\rm scaf} = 0$	$w_{\rm eff} = \lambda^2/3 - 1$	$\lambda^2 \leq 6$
$C_{{\rm scaf}\phi}$	$x = \sqrt{3/2}/\mathcal{A},$ $y^2 = [3/2 + \beta\mathcal{A}]/\mathcal{A}^2,$ $\Omega_{\rm scaf} = [\lambda\mathcal{A} - 3]/\mathcal{A}^2$	$w_{\rm eff} = -\beta/\mathcal{A}$	$\mathcal{A} \neq 0,$ $3/2 + \beta\mathcal{A} \geq 0,$ $\lambda\mathcal{A} \geq 3$

Point	Coordinates	Effective state	Existence condition
R_ϕ	$x = 2\sqrt{2/3}/\lambda$, $y = 2/(\sqrt{3}\,\lambda)$, $z^2 = 1 - 4/\lambda^2$, $\Omega_{\text{scaf}} = u = 0$	$w_{\text{eff}} = 1/3$	$\lambda \neq 0$, $\lambda^2 \geq 4$

The table does not by itself determine whether a point is dynamically reached.

That requires linear stability and basin analysis.

The coordinates in table 3 were substituted back into the autonomous equations using the same plus-sign current convention fixed by the GT1 action. The listed eigenvalues in the sections below are the eigenvalues of the constrained flat-system Jacobian $\partial F_A/\partial X_B$, with $\Omega_{\text{scaf}} = 1 - x^2 - y^2 - z^2 - u^2$. This convention prevents the scaffold constraint direction from being counted twice.

Remark 109.1 (Branch convention)**.** *Most late-time inequalities below are stated on the positive-slope branch $\lambda > 0$. The algebraic existence condition of the scalar–radiation point is $\lambda^2 \geq 4$ with $\lambda \neq 0$; changing the sign of λ changes the sign of x but not its density fractions.*

110 Radiation-Dominated Point

At the radiation point,

$$R: \qquad (x, y, z, u) = (0, 0, 1, 0). \tag{512}$$

The scaffold fraction vanishes:

$$\Omega_{\text{scaf}} = 0. \tag{513}$$

The expansion parameters are

$$\epsilon_H = 2, \tag{514}$$

and

$$q_{\text{dec}} = 1. \tag{515}$$

The linear eigenvalues in the (x, y, z, u) system are

$$\boxed{m_R = \left\{-1, 2, 1, \frac{1}{2}\right\}.} \tag{516}$$

The point is therefore not a future attractor.

It has growing scalar-potential, scaffold-density, and pressureless-matter directions.

This is appropriate for an early radiation epoch that later exits toward matter and chasing domination.

The scaffold fraction grows initially as

$$\Omega_{\text{scaf}} \propto e^N \tag{517}$$

near the exact radiation point.

111 Kinetic-Dominated Points

The scalar kinetic points are

$$K_\pm : \qquad x = \pm 1, \qquad y = z = u = \Omega_{\text{scaf}} = 0. \tag{518}$$

They have

$$w_{\text{eff}} = 1, \tag{519}$$

and

$$\epsilon_H = 3. \tag{520}$$

For K_+, the eigenvalues are

$$m_{K_+} = \left\{ 3 + \sqrt{6}\,\beta,\, 3 - \frac{\sqrt{6}}{2}\lambda,\, 1,\, \frac{3}{2} \right\}. \tag{521}$$

For K_-, the eigenvalues are

$$m_{K_-} = \left\{ 3 - \sqrt{6}\,\beta,\, 3 + \frac{\sqrt{6}}{2}\lambda,\, 1,\, \frac{3}{2} \right\}. \tag{522}$$

Both points possess positive radiation or matter eigenvalues in the forward-time system.

They are therefore not acceptable late-time attractors.

They may occur as transient or past-asymptotic kinetic states.

A prolonged kinetic epoch would redshift too rapidly and would not reproduce the required radiation–matter–acceleration sequence.

112 Visible-Matter Point

The uncoupled visible pressureless-matter point is

$$B: \qquad u = 1, \qquad x = y = z = \Omega_{\text{scaf}} = 0. \tag{523}$$

It has

$$w_{\text{eff}} = 0, \tag{524}$$

and

$$\epsilon_H = \frac{3}{2}. \tag{525}$$

The linear eigenvalues are

$$\boxed{m_B = \left\{-\frac{3}{2}, \frac{3}{2}, -\frac{1}{2}, 0\right\}.} \tag{526}$$

The zero eigenvalue reflects the degeneracy between pressureless visible matter and an infinitesimal uncoupled pressureless scaffold direction.

When

$$\beta \neq 0, \tag{527}$$

the scaffold perturbs the scalar through the coupling term.

The visible-matter point is consequently a nonhyperbolic saddle rather than the generic dark-sector matter epoch.

113 Coupled Scaffold Matter Epoch

Comparison basis. The coupled matter-era branch is the GT1 analogue of the ϕ-matter-dominated epoch in coupled quintessence (Amendola, 2000).

The coupled scaffold matter point is

$$M_\phi: \qquad x = -\sqrt{\frac{2}{3}}\beta, \qquad y = z = u = 0. \tag{528}$$

Its scaffold fraction is

$$\Omega_{\mathrm{scaf}} = 1 - \frac{2\beta^2}{3}. \tag{529}$$

The scalar fraction is kinetic:

$$\Omega_\phi = \frac{2\beta^2}{3}. \tag{530}$$

The effective equation of state is

$$\boxed{w_{\mathrm{eff}} = \frac{2\beta^2}{3}.} \tag{531}$$

The expansion parameter is

$$\epsilon_H = \frac{3}{2} + \beta^2. \tag{532}$$

The eigenvalues are

$$\boxed{m_{M_\phi} = \left\{\beta^2 - \frac{3}{2},\ \beta^2 + \beta\lambda + \frac{3}{2},\ \beta^2 - \frac{1}{2},\ \beta^2\right\}.} \tag{533}$$

For ordinary positive λ and modest β, the scalar-potential direction is unstable.

This allows the matter-like state to exit toward a chasing-dominated state.

A near-standard matter epoch requires

$$\frac{2\beta^2}{3} \ll 1. \tag{534}$$

Introduce a maximum allowed matter-era deviation

$$\epsilon_{\mathrm{mat}}. \tag{535}$$

Then the requirement

$$\Omega_\phi^{(M_\phi)} < \epsilon_{\mathrm{mat}} \tag{536}$$

implies

$$\boxed{|\beta| < \sqrt{\frac{3\epsilon_{\mathrm{mat}}}{2}}.} \tag{537}$$

The observational value of ϵ_{mat} must be selected through the full third-leg data analysis rather than inserted at this stage.

114 Scalar-Dominated Chasing Point

The scalar-dominated point is

$$D_\phi : \qquad x = \frac{\lambda}{\sqrt{6}}, \qquad y = \sqrt{1 - \frac{\lambda^2}{6}}. \tag{538}$$

The scaffold, radiation, and visible-matter fractions vanish:

$$\Omega_{\mathrm{scaf}} = z = u = 0. \tag{539}$$

The scalar equation of state is

$$w_\phi = w_{\mathrm{eff}} = -1 + \frac{\lambda^2}{3}. \tag{540}$$

The expansion parameter is

$$\epsilon_H = \frac{\lambda^2}{2}. \tag{541}$$

Acceleration requires

$$\boxed{\lambda^2 < 2.} \tag{542}$$

The four eigenvalues are

$$\boxed{m_{D_\phi} = \left\{ \frac{\lambda^2 - 6}{2},\ \lambda^2 + \beta\lambda - 3,\ \frac{\lambda^2 - 4}{2},\ \frac{\lambda^2 - 3}{2} \right\}.} \tag{543}$$

The eigenvalues correspond respectively to:

- the internal scalar direction;
- the scaffold direction;
- the radiation direction; and
- the visible pressureless-matter direction.

A stable accelerated scalar attractor therefore requires

$$\boxed{\lambda^2 < 2, \qquad \lambda^2 + \beta\lambda < 3.} \tag{544}$$

The second condition prevents the coupled scaffold from destabilizing the scalar-dominated state.

For small $|\beta|$, this point supplies the simplest acceptable late chasing attractor.

115 Coupled Scaffold–Chasing Scaling Point

Comparison basis. The coupled scaling solution is compared with established exponential-potential and coupled-quintessence fixed points (Copeland et al., 1998; Amendola, 2000).

The coupled scaling point is

$$x_C = \frac{\sqrt{3/2}}{\lambda + \beta}. \tag{545}$$

Its potential fraction is

$$y_C^2 = \frac{\frac{3}{2} + \beta(\lambda + \beta)}{(\lambda + \beta)^2}. \tag{546}$$

Its scaffold fraction is

$$\Omega_{\mathrm{scaf},C} = \frac{\lambda(\lambda + \beta) - 3}{(\lambda + \beta)^2}. \tag{547}$$

The scalar and scaffold sectors redshift together.

The effective equation of state is

$$\boxed{w_{\mathrm{eff},C} = -\frac{\beta}{\lambda + \beta}.} \tag{548}$$

The expansion parameter is

$$\epsilon_{H,C} = \frac{3\lambda}{2(\lambda + \beta)}. \tag{549}$$

For

$$\lambda + \beta > 0, \tag{550}$$

acceleration requires

$$\boxed{2\beta > \lambda.} \tag{551}$$

Existence requires

$$\lambda(\lambda + \beta) \geq 3, \tag{552}$$

and

$$\frac{3}{2} + \beta(\lambda + \beta) \geq 0. \tag{553}$$

The scaling point is mathematically attractive because it maintains nonzero scaffold and chasing fractions during acceleration.

Its compatibility with a preceding matter-like epoch is considerably more restrictive.

116 Stability of the Coupled Scaling Point

The two internal scaffold–scalar eigenvalues satisfy

$$m^2 - \tau_C m + \Delta_C = 0, \tag{554}$$

where

$$\tau_C = -\frac{3(\lambda + 2\beta)}{2(\lambda + \beta)}, \tag{555}$$

and

$$\Delta_C = \frac{3\left[2\beta^2 + 2\beta\lambda + 3\right]\left[\lambda(\lambda + \beta) - 3\right]}{2(\lambda + \beta)^2}. \tag{556}$$

The internal eigenvalues are

$$m_\pm = \frac{\tau_C \pm \sqrt{\tau_C^2 - 4\Delta_C}}{2}. \tag{557}$$

Internal stability requires

$$\tau_C < 0, \tag{558}$$

and

$$\Delta_C > 0. \tag{559}$$

The radiation eigenvalue is

$$m_{\mathrm{rad},C} = -\frac{\lambda + 4\beta}{2(\lambda + \beta)}. \tag{560}$$

The visible-matter eigenvalue is

$$m_{\mathrm{vis},C} = -\frac{3\beta}{2(\lambda + \beta)}. \tag{561}$$

For

$$\lambda > 0, \qquad \beta > 0, \tag{562}$$

and the existence conditions above, the radiation and visible-matter directions decay.

The coupled scaling point can therefore become a future attractor.

The question is whether the coupling required for accelerated scaling prevents recovery of the required near-standard pressureless-matter epoch within the observable third-leg benchmark.

117 Matter-Era versus Accelerated-Scaling Tension

For positive λ and β, an accelerated coupled scaling point requires

$$\lambda < 2\beta. \tag{563}$$

Existence of a nonzero scaffold fraction requires

$$\lambda(\lambda + \beta) > 3. \tag{564}$$

From

$$\lambda < 2\beta \tag{565}$$

it follows that

$$\lambda + \beta < 3\beta. \tag{566}$$

Therefore,

$$\lambda(\lambda+\beta) < 6\beta^2. \tag{567}$$

Combining this with the existence requirement gives

$$\boxed{\beta^2 > \frac{1}{2}.} \tag{568}$$

The coupled matter epoch then has

$$w_{\text{eff}}^{(M_\phi)} = \frac{2\beta^2}{3} > \frac{1}{3}. \tag{569}$$

Proposition 117.1 (Constant-coupling scaling obstruction)**.** *Within the spatially flat GT1 model with an exponential potential, constant positive λ, constant positive β, and a coupled scaffold–scalar scaling endpoint, simultaneous existence and acceleration of that endpoint imply $\beta^2 > 1/2$. The associated coupled scaffold matter point then obeys $w_{\text{eff}}^{(M_\phi)} > 1/3$ and cannot serve as a near-standard matter epoch.*

Equivalently,

$$\boxed{\begin{array}{l}\text{accelerated constant-coupling scaffold–chasing scaling}\\ \Longrightarrow \beta^2 > \dfrac{1}{2} \Longrightarrow w_{\text{eff}}^{(M_\phi)} > \dfrac{1}{3}.\end{array}} \tag{570}$$

This is a conditional obstruction for the stated branch of the simplest constant-parameter realization. It is not a no-go theorem for GT1 as a whole.

It does not exclude:

- a scalar-dominated accelerated attractor;
- a field-dependent coupling;
- a non-exponential potential;
- a coupling that activates only at late times;
- a more general scaffold equation of state;
- a multi-field chasing sector; or
- a different sign or regime structure.

The result strongly favors the scalar-dominated late attractor in the first GT1 numerical study.

118 Scalar–Radiation Scaling Point

For

$$\lambda^2 \geq 4, \tag{571}$$

a scalar–radiation scaling point exists:

$$x_{R_\phi} = \frac{2}{\lambda}\sqrt{\frac{2}{3}}, \tag{572}$$

$$y_{R_\phi} = \frac{2}{\sqrt{3}\,\lambda}, \tag{573}$$

$$z_{R_\phi}^2 = 1 - \frac{4}{\lambda^2}. \tag{574}$$

The scalar fraction is

$$\Omega_\phi = \frac{4}{\lambda^2}. \tag{575}$$

The effective equation of state remains

$$w_{\text{eff}} = \frac{1}{3}. \tag{576}$$

An infinitesimal scaffold perturbation evolves with exponent

$$m_{\text{scaf}} = 1 + \frac{4\beta}{\lambda}. \tag{577}$$

For positive λ and β, the scaffold grows in forward time.

The point may therefore serve as an early scaling saddle but not as the late-time attractor.

A large scalar fraction during radiation domination would alter the early-third-leg expansion history.

Its allowed value must be tested against the full thermal and microwave-background calculations.

119 Linear Stability Matrix

Let

$$\boldsymbol{X} = (x, y, z, u)^{\mathsf{T}} \tag{578}$$

on the flat constant-slope submanifold.

The autonomous system is

$$\boldsymbol{X}' = \boldsymbol{F}\left(\boldsymbol{X}; \lambda, \beta\right). \tag{579}$$

Perturb a fixed point:

$$\boldsymbol{X} = \boldsymbol{X}_* + \delta \boldsymbol{X}. \tag{580}$$

The linearized system is

$$\delta \boldsymbol{X}' = \mathsf{J}_* \delta \boldsymbol{X}, \tag{581}$$

where

$$\mathsf{J}_* = \left. \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{X}} \right|_{\boldsymbol{X}_*}. \tag{582}$$

A hyperbolic fixed point is a future attractor when

$$\operatorname{Re}\left(m_A\right) < 0 \tag{583}$$

for every eigenvalue.

It is a repeller when

$$\operatorname{Re}\left(m_A\right) > 0 \tag{584}$$

for every eigenvalue.

Mixed signs define a saddle.

A zero-real-part eigenvalue requires a center-manifold, nonlinear, or higher-order analysis.

Linear stability is local.

It does not determine the size of the basin of attraction.

120 Basin Structure

Let

$$\Sigma_{\mathrm{ini}} \subset \mathcal{P} \tag{585}$$

be a declared initial-condition surface.

For attractor A, define its basin

$$\mathcal{B}_A = \left\{ \boldsymbol{X}_{\mathrm{ini}} \in \Sigma_{\mathrm{ini}} : \lim_{N\to\infty} \boldsymbol{X}(N) = \boldsymbol{X}_A \right\}. \tag{586}$$

Given a specified measure μ, the basin fraction is

$$f_A = \frac{\mu(\mathcal{B}_A)}{\mu(\Sigma_{\mathrm{ini}})}. \tag{587}$$

The measure must be declared.

A coordinate-volume fraction is not automatically a physically preferred probability.

For the third-leg background, define the successful-history basin

$$\mathcal{B}_{\mathrm{RMA}} = \left\{ \boldsymbol{X}_{\mathrm{ini}} : R \to M_\phi \to D_\phi \text{ with acceptable durations} \right\}. \tag{588}$$

A robust first model requires

$$\mu\left(\mathcal{B}_{\mathrm{RMA}}\right) > 0. \tag{589}$$

A single finely selected trajectory is not enough to establish background naturalness.

Criterion 120.1 (Viable-background basin gate)**.** *A parameter region passes the homogeneous GT1 gate only when a nonzero-measure set of declared initial states simultaneously satisfies:*

(a) the Friedmann constraint within a predeclared numerical tolerance;

(b) positive physical densities and $H > 0$ throughout the third-leg control;

(c) radiation and matter-like intervals of sufficient duration;

(d) a stable accelerated late interval;

(e) a controlled curvature history;

(f) finite exchange, potential, and conformal-factor functions; and

(g) an acceptable background likelihood relative to the ΛCDM control.

Possession of an acceptable fixed point without an orbit satisfying these conditions does not pass the gate.

121 Heteroclinic Cosmic Histories

A realistic background is not represented by one fixed point.

It is an orbit connecting several quasi-stationary regimes.

A desired third-leg trajectory is

$$\boxed{R \longrightarrow M_\phi \longrightarrow D_\phi.} \tag{590}$$

The first transition occurs when

$$\rho_{\rm rad} \sim \rho_{\rm scaf} + \rho_{\rm vis}. \tag{591}$$

The second occurs when

$$\rho_\phi \sim \rho_{\rm scaf} + \rho_{\rm vis}. \tag{592}$$

A viable orbit must spend enough e-folds near the radiation and matter saddles before reaching the accelerated attractor.

Define the matter-like interval

$$\mathcal{I}_{\rm mat} = \left\{N : |w_{\rm eff}(N)| < w_{\rm mat}^{\rm max}\right\}. \tag{593}$$

Its duration is

$$\Delta N_{\rm mat} = \int_{\mathcal{I}_{\rm mat}} \mathrm{d}N. \tag{594}$$

Similarly, define the accelerated interval

$$\mathcal{I}_{\rm acc} = \left\{N : \epsilon_H(N) < 1\right\}. \tag{595}$$

The model must reproduce the observed third-leg history through the entire orbit rather than merely possess the correct asymptotic endpoint.

122 Initial Parameter Regions for Numerical Integration

The first numerical scan should be divided into controlled regions.

122.1 Region I: exact LCDM-like control

Use

$$\beta = 0, \qquad \lambda = 0, \qquad V = V_0. \tag{596}$$

Then

$$x = 0, \qquad y^2 = \Omega_\Lambda. \tag{597}$$

This control must reproduce the complete general-relativistic radiation–matter–vacuum background orbit, not merely its late de Sitter endpoint. It validates the integration, constraint propagation, distance reconstruction, and parameter-normalization pipeline before any GT1 departure is introduced.

122.2 Region II: uncoupled scalar chasing

Use

$$\beta = 0, \qquad 0 < \lambda^2 < 2. \tag{598}$$

The future attractor is accelerated and scalar dominated.

122.3 Region III: weakly coupled scalar chasing

Use

$$|\beta| \ll 1, \qquad \lambda^2 < 2, \qquad \lambda^2 + \beta\lambda < 3. \tag{599}$$

This is the preferred first genuine triune region.

It preserves a near-standard matter epoch while permitting a stable accelerated scalar attractor.

122.4 Region IV: coupled scaling comparison

Use parameters satisfying

$$\lambda(\lambda + \beta) > 3 \tag{600}$$

and

$$2\beta > \lambda. \tag{601}$$

This region is retained as a mathematical comparison.

The matter-era tension derived above makes it unlikely to provide the first acceptable full history under constant coupling.

122.5 Region V: variable late-activation coupling

Introduce a coupling function satisfying

$$\beta(\phi) \approx 0 \tag{602}$$

during radiation and matter domination, while

$$\beta(\phi) \neq 0 \tag{603}$$

during the late chasing epoch.

This region is tested only after the constant-coupling system is verified.

123 Initial Conditions for the Third-Leg Control Study

Choose an initial e-fold

$$N_{\mathrm{ini}} \tag{604}$$

inside the early radiation-dominated third leg.

A control initial state satisfies

$$z_{\mathrm{ini}}^2 \approx 1, \tag{605}$$

with

$$x_{\text{ini}}^2, \quad y_{\text{ini}}^2, \quad u_{\text{ini}}^2, \quad \Omega_{\text{scaf,ini}} \ll 1. \tag{606}$$

The Friedmann constraint fixes one of these quantities.

The scalar initial velocity should be scanned over both signs:

$$x_{\text{ini}} \gtrless 0. \tag{607}$$

The initial scalar potential fraction should not be chosen directly from the desired late dark-energy fraction without integrating the trajectory.

The initial state must also satisfy

$$H_{\text{ini}} > 0, \tag{608}$$

and

$$\rho_i \geq 0. \tag{609}$$

A later central-transition calculation must derive, rather than freely select, these third-leg initial conditions.

124 Curvature Extension

The flat subsystem is useful for fixed-point analysis but does not remove the Paper II curvature burden.

For nonzero curvature,

$$\Omega_k' = 2\,(\epsilon_H - 1)\,\Omega_k. \tag{610}$$

The curvature-free surface is invariant.

A small nonzero curvature perturbation near a fixed point evolves as

$$\delta\Omega_k \propto e^{m_k N}, \tag{611}$$

where

$$m_k = 2\,(\epsilon_{H,*} - 1)\,. \tag{612}$$

At radiation domination,

$$m_k^{(R)} = 2. \tag{613}$$

At a matter-like epoch,

$$m_k^{(M)} \approx 1. \tag{614}$$

At the scalar-dominated point,

$$m_k^{(D_\phi)} = \lambda^2 - 2. \tag{615}$$

Therefore, the accelerated scalar attractor with

$$\lambda^2 < 2 \tag{616}$$

suppresses $|\Omega_k|$ during its accelerated interval.

The ordinary radiation and matter eras amplify the curvature fraction.

The complete present value still depends on the full transfer from the geometry-emergence boundary through every hidden and visible interval.

125 Background Sensitivity

Let the numerical parameter vector be

$$\boldsymbol{\theta}_{\mathrm{GT1}} = (\lambda, \beta, V_0, x_{\mathrm{ini}}, y_{\mathrm{ini}}, \Omega_{\mathrm{scaf,ini}}, \Omega_{k,\mathrm{ini}}) \,. \tag{617}$$

For an output $\mathcal{O}$, define the logarithmic sensitivity

$$\mathcal{S}_{\mathcal{O},a} = \left| \frac{\partial \ln \mathcal{O}}{\partial \ln \theta_a} \right| . \tag{618}$$

A background sensitivity matrix is

$$\mathsf{S}_{ia} = \frac{\partial \mathcal{O}_i}{\partial \theta_a}. \tag{619}$$

Priority outputs include:

$$\{H(z), z_{\mathrm{eq}}, N_{\mathrm{mat}}, N_{\mathrm{acc}}, w_{\mathrm{eff},0}, \Omega_{\mathrm{scaf},0}, \Omega_{\phi,0}\} \,. \tag{620}$$

A parameter point is not robust when small fractional changes in one initial condition produce qualitatively different cosmic histories.

The basin and sensitivity analyses should therefore be reported together.

126 Three-Leg Background Extension

The GT1 system above describes the ordinary third-leg sectors.

The hidden first and second legs require an E sector with density and pressure

$$\rho_E, \qquad p_E. \tag{621}$$

Introduce

$$\Omega_E = \frac{\rho_E}{3M_{\mathrm{Pl}}^2 H^2}. \tag{622}$$

The extended Friedmann constraint becomes

$$1 = x^2 + y^2 + \Omega_{\mathrm{scaf}} + \Omega_E + \Omega_{\mathrm{other}} + \Omega_k. \tag{623}$$

The E continuity equation is

$$\dot{\rho}_E + 3H\left(\rho_E + p_E\right) = Q_E. \tag{624}$$

The complete exchange condition is

$$Q_E + Q_{\mathrm{scaf}} + Q_\phi + Q_{\mathrm{other}} = 0. \tag{625}$$

The sectoral trajectory is represented independently by

$$R_E(t), \tag{626}$$

with directional sequence

$$\dot{R}_E: \quad + \longrightarrow - \longrightarrow +. \tag{627}$$

The sign of $\dot{R}_E$ does not by itself determine

$$H. \tag{628}$$

A dynamical connection exists only when the stress–energy or interaction laws depend on

$$\left\{R_E, \dot{R}_E, \rho_E, p_E\right\}. \tag{629}$$

This preserves the distinction between global expansion and sectoral directional reversal.

127 Leg-Dependent Functional Weights

The effective amplitudes of binding, scaffolding, and chasing may differ among the three legs.

Define

$$f^{(j)}_{\mathrm{bind}}, \qquad f^{(j)}_{\mathrm{scaf}}, \qquad f^{(j)}_{\mathrm{chase}}, \tag{630}$$

for leg

$$j \in \{1, 2, 3\}. \tag{631}$$

These weights are not additional arbitrary density parameters.

They must be derived projections of the underlying state:

$$f^{(j)}_A = f_A\left[\phi, \dot{\phi}, \rho_{\mathrm{scaf}}, \rho_E, R_E, \dot{R}_E, \mathcal{W}\right]. \tag{632}$$

A regime change may occur through:

$$\left\{V_{\mathrm{eff}}, A_{\mathrm{eff}}, Q_i, w_i, c^2_{s,i}, \Gamma_{\mathrm{mem}}\right\}. \tag{633}$$

The same fundamental action must generate those changes.

Assigning a separate unrelated potential and coupling to each leg would weaken the claim of one unified Gravity Triune.

128 Reversal-Ready Background States

Define the background state immediately before or after transition j as

$$\mathcal{B}^{\pm}_j = \left\{a^{\pm}_j, H^{\pm}_j, \phi^{\pm}_j, \dot{\phi}^{\pm}_j, \rho^{\pm}_{\mathrm{scaf},j}, \rho^{\pm}_{E,j}, R^{\pm}_{E,j}, \dot{R}^{\pm}_{E,j}, \Omega^{\pm}_{k,j}, \mathcal{W}^{\pm}_j\right\}. \tag{634}$$

A reversal-ready state must satisfy

$$0 < a_j^{\pm} < \infty, \tag{635}$$

$$|H_j^{\pm}| < \infty, \tag{636}$$

$$|\rho_{i,j}^{\pm}| < \infty, \tag{637}$$

and

$$|\phi_j^{\pm}|, \quad |\dot{\phi}_j^{\pm}|, \quad |\mathcal{W}_j^{\pm}| < \infty. \tag{638}$$

For the globally expanding foundational model,

$$H_j^{\pm} > 0. \tag{639}$$

The sectoral reversal condition is

$$\dot{R}_{E,j}^{-}\dot{R}_{E,j}^{+} < 0. \tag{640}$$

The transition must preserve total stress–energy conservation even though individual sector densities may change rapidly.

129 Finite-Width Reversal Extension

A smooth directional reversal may be represented by

$$\dot{R}_E^{(j)}(t) = \frac{v_j^- + v_j^+}{2} + \frac{v_j^+ - v_j^-}{2} \tanh\left(\frac{t - t_j}{\tau_j}\right). \tag{641}$$

The reversal width is

$$\tau_j > 0. \tag{642}$$

A triune interaction may depend on the reversal state through

$$\beta_{\text{eff}} = \beta_{\text{eff}}\left(\phi, R_E, \dot{R}_E\right), \tag{643}$$

or

$$V_{\text{eff}} = V_{\text{eff}}\left(\phi, R_E, \dot{R}_E\right). \tag{644}$$

Such dependence must arise from a covariant transition field or order parameter.

Directly inserting the sign function

$$\operatorname{sgn}\left(\dot{R}_E\right) \tag{645}$$

into the fundamental equations would generally produce a nondifferentiable effective theory.

A finite-width covariant transition is preferred.

130 Numerical Integration Program

The first background implementation should proceed through the following steps.

130.1 Step 1: validate the constraint

Verify throughout the integration that

$$\mathcal{C}_F = 1 - x^2 - y^2 - z^2 - u^2 - \Omega_{\text{scaf}} - \Omega_k \tag{646}$$

satisfies

$$|\mathcal{C}_F| < \varepsilon_{\text{num}}. \tag{647}$$

130.2 Step 2: reproduce the control model

Set

$$\lambda = \beta = 0 \tag{648}$$

and reproduce the standard radiation–matter–constant-chasing background.

130.3 Step 3: reproduce analytical fixed points

Initialize trajectories near each point in table 3 and verify the analytical stability directions.

130.4 Step 4: scan weak coupling

Scan the region

$$|\beta| \ll 1, \qquad \lambda^2 < 2. \tag{649}$$

130.5 Step 5: measure epoch durations

Record:

$$\{\Delta N_{\mathrm{rad}}, \Delta N_{\mathrm{mat}}, \Delta N_{\mathrm{acc}}\}. \tag{650}$$

130.6 Step 6: calculate basin fractions

Sample a declared initial-condition surface and estimate

$$f_{\mathrm{RMA}}. \tag{651}$$

130.7 Step 7: introduce curvature

Repeat the integrations for small nonzero

$$\Omega_{k,\mathrm{ini}}. \tag{652}$$

130.8 Step 8: compare constant and variable coupling

Test whether late activation of $\beta(\phi)$ avoids the constant-coupling tension.

130.9 Step 9: prepare the perturbation background

Export

$$\{H(N), \phi(N), \phi'(N), \Omega_i(N), w_{\mathrm{eff}}(N), \beta(N), \lambda(N)\} \tag{653}$$

for the later scalar, vector, and tensor perturbation system.

130.10 Step 10: construct reversal-ready extensions

Add

$$\left\{R_E, \dot{R}_E, \rho_E, p_E, Q_E\right\} \tag{654}$$

only after the third-leg control background is verified.

131 Background Diagnostics

Each numerical trajectory should report:

$$\Omega_{\rm rad}(N), \quad \Omega_{\rm vis}(N), \quad \Omega_{\rm scaf}(N), \quad \Omega_\phi(N), \quad \Omega_k(N), \tag{655}$$

$$w_\phi(N), \quad w_{\rm scaf}^{\rm eff}(N), \quad w_\phi^{\rm eff}(N), \quad w_{\rm eff}(N), \tag{656}$$

and

$$H(N), \quad \epsilon_H(N), \quad q_{\rm dec}(N). \tag{657}$$

The dimensionless transfer rate is

$$\mathcal{Q}_{\rm scaf} = \frac{Q_{\rm scaf}}{H\rho_{\rm scaf}} = \sqrt{6}\,\beta x. \tag{658}$$

The numerical conservation diagnostics should also include

$$\Delta_F(N) = \left|\mathcal{C}_F(N)\right|, \tag{659}$$

and the normalized dark-sector exchange residual

$$\Delta_Q(N) = \frac{|Q_{\rm scaf} + Q_\phi|}{H\left(\rho_{\rm scaf} + \rho_\phi\right) + \varepsilon_{\rm reg}}, \tag{660}$$

where $\varepsilon_{\rm reg} > 0$ is a declared numerical regularizer, not a physical parameter. The tolerances on Δ_F and Δ_Q must be fixed before the parameter scan.

The scalar-force balance is

$$\mathcal{F}_\phi = V_{,\phi} + \frac{\beta}{M_{\rm Pl}}\rho_{\rm scaf}. \tag{661}$$

A quasi-static effective minimum satisfies

$$\mathcal{F}_\phi \approx 0. \tag{662}$$

The effective scalar mass at that minimum is schematically

$$m_{\text{eff}}^2 = V_{,\phi\phi} + \rho_{\text{scaf}} \frac{\mathrm{d}^2 \ln A}{\mathrm{d}\phi^2} + \rho_{\text{scaf}} \left(\frac{\mathrm{d} \ln A}{\mathrm{d}\phi} \right)^2, \tag{663}$$

subject to the precise density and frame convention.

A negative or singular effective mass requires further stability analysis.

132 Background-Recovery Rejection Criteria

The GT1 background realization fails or requires revision if any of the following conditions is unavoidable.

1. The Friedmann constraint cannot be preserved numerically.
2. A positive initial sector density becomes negative without a derived physical interpretation.
3. The scalar or scaffold density diverges at finite N.
4. The physical Hubble rate passes through an unintended singularity.
5. The control limit does not reproduce the standard radiation–matter– constant-chasing sequence.
6. No radiation-dominated regime exists.
7. No sufficiently long matter-like regime exists.
8. The coupled matter epoch has an unacceptable effective equation of state.
9. The scalar dominates too early.
10. The chasing field cannot produce acceleration.
11. The accelerated state is unstable.
12. The coupled scaffold destabilizes the scalar-dominated attractor.
13. An accelerated coupled-scaling point necessarily prevents recovery of the required near-standard pressureless-matter epoch in this restricted constant-coupling branch.
14. A variable coupling avoids the matter-era problem only through an arbitrary manually timed switch.
15. The background energy-transfer rate becomes singular.

16. The potential or conformal metric becomes ill defined.
17. The conformal factor satisfies

$$A(\phi) \le 0. \tag{664}$$

18. The successful history occupies only a measure-zero initial set under the declared sampling prescription.
19. Small parameter changes destroy the radiation–matter–acceleration sequence.
20. The curvature fraction grows beyond the allowed present range.
21. The hidden-leg extension violates total stress–energy conservation.
22. The sectoral reversal forces the global scale factor to become singular.
23. The scaffold or chasing variables become undefined when $\dot{R}_E$ changes sign.
24. Different legs require unrelated actions rather than different states of one theory.
25. The only successful point is the exact ΛCDM control, with no distinct triune dynamics or testable consequence.

133 Formal Summary of the Background Dynamical System

The selected scaffold–chasing exchange convention is

$$\boxed{\dot{\rho}_{\rm scaf} + 3H\rho_{\rm scaf} = \frac{\beta}{M_{\rm Pl}}\rho_{\rm scaf}\dot{\phi},} \tag{665}$$

and

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\frac{\beta}{M_{\rm Pl}}\rho_{\rm scaf}.} \tag{666}$$

The dimensionless variables are

$$\boxed{x = \frac{\dot{\phi}}{\sqrt{6}M_{\rm Pl}H}, \qquad y = \frac{\sqrt{V}}{\sqrt{3}M_{\rm Pl}H}, \qquad z = \frac{\sqrt{\rho_{\rm rad}}}{\sqrt{3}M_{\rm Pl}H}, \qquad u = \frac{\sqrt{\rho_{\rm vis}}}{\sqrt{3}M_{\rm Pl}H}.} \tag{667}$$

The Friedmann constraint is

$$\boxed{1 = x^2 + y^2 + z^2 + u^2 + \Omega_{\rm scaf} + \Omega_k.} \tag{668}$$

The autonomous scalar equations are

$$x' = -3x + \sqrt{\frac{3}{2}}\lambda y^2 - \sqrt{\frac{3}{2}}\beta\Omega_{\rm scaf} + x\epsilon_H, \tag{669}$$

$$y' = -\sqrt{\frac{3}{2}}\lambda xy + y\epsilon_H. \tag{670}$$

The Hubble evolution is

$$\boxed{\epsilon_H = 3x^2 + \frac{3}{2}\Omega_{\rm scaf} + \frac{3}{2}u^2 + 2z^2 + \Omega_k.} \tag{671}$$

The preferred late attractor region of the constant-parameter baseline is

$$\boxed{|\beta| \ll 1, \qquad \lambda^2 < 2, \qquad \lambda^2 + \beta\lambda < 3.} \tag{672}$$

The desired third-leg sequence is

$$\boxed{R \longrightarrow M_\phi \longrightarrow D_\phi.} \tag{673}$$

The constant-coupling accelerated-scaling tension is

$$\boxed{\text{accelerated coupled scaling} \quad \Longrightarrow \quad \beta^2 > \frac{1}{2} \quad \Longrightarrow \quad w_{\rm eff}^{(M_\phi)} > \frac{1}{3}} \tag{674}$$

for positive λ and β.

The curvature equation is

$$\boxed{\Omega_k' = 2\left(\epsilon_H - 1\right)\Omega_k.} \tag{675}$$

The reversal-ready state is

$$\boxed{\mathcal{B}_j^{\pm} = \left\{a, H, \phi, \dot{\phi}, \rho_{\rm scaf}, \rho_E, R_E, \dot{R}_E, \Omega_k, \mathcal{W}\right\}_j^{\pm}.} \tag{676}$$

The decisive background question is

$$\boxed{\begin{array}{l}\text{Does a finite, non-finely selected region of the GT1 phase space}\\ \text{produce a radiation epoch, a sufficiently standard scaffold}\\ \text{matter epoch, and a stable accelerated chasing epoch while}\\ \text{remaining compatible with curvature transfer and extension}\\ \text{through both sectoral reversals?}\end{array}} \tag{677}$$

134 Background-Dynamics Status

The Gravity Triune background problem is now reduced to a definite calculation sequence.

1. Implement the autonomous GT1 system.
2. Verify the exact ΛCDM-like control limit.
3. Reproduce all analytical fixed points.
4. Verify their eigenvalues numerically.
5. Map the radiation–matter–acceleration basin.
6. Measure the duration and quality of the matter-like epoch.
7. Test the stable scalar-dominated accelerated region.
8. Confirm the constant-coupling accelerated-scaling tension.
9. Test whether a derived field-dependent coupling resolves that tension.
10. Propagate small curvature through the complete third-leg orbit.
11. Export the common background for perturbation calculations.
12. Introduce the E density, pressure, and trajectory variables.
13. Construct finite-width outer- and central-reversal backgrounds.
14. Verify total conservation through each transition.
15. Determine whether the scaffold and chasing variables remain finite and physically interpretable through all three legs.

The homogeneous GT1 audit fixes the homogeneous GT1 equations, the constraint propagation identity, the exact general-relativity plus ΛCDM background control, the constant-slope fixed points, their local stability conditions, the viable-basin gate, and the conditional constant-coupling scaling obstruction. These are analytical results inside the selected reference realization; they are not yet observational validation.

The next burden is to audit binding in the local and strong-field regimes against the same action and sign convention before the scaffold and chasing sectors are granted broader gravitational roles.

135 The Binding-Sector Problem

Benchmark literature. The local and strong-field binding audit is organized around experimental general-relativity tests and tensor–scalar compact-body results (Will, 2014; Damour and Esposito-Farèse, 1992, 1996).

The selected GT1 realization assigns the binding function to the general-relativistic metric response:

$$\boxed{\mathfrak{G}_{\mathrm{bind}} = \text{metric binding generated by the Einstein–Hilbert action.}} \tag{678}$$

No independent binding field is introduced.

The fundamental gravitational action remains

$$S_{\mathrm{EH}} = \int \mathrm{d}^4x\,\sqrt{-g}\,\frac{M_{\mathrm{Pl}}^2}{2}R. \tag{679}$$

Visible matter and radiation couple minimally to the physical metric,

$$S_{\mathrm{vis}} = S_{\mathrm{vis}}\left[g_{\mu\nu}, \psi_{\mathrm{vis}}\right], \tag{680}$$

and

$$S_{\mathrm{rad}} = S_{\mathrm{rad}}\left[g_{\mu\nu}, \psi_{\mathrm{rad}}\right]. \tag{681}$$

The scaffold instead couples to

$$\tilde{g}_{\mu\nu}^{(\mathrm{scaf})} = A^2(\phi)g_{\mu\nu}. \tag{682}$$

The local-recovery problem is therefore not primarily whether ordinary matter feels a new direct scalar force.

In the minimal GT1 construction, it does not.

The problem is whether the interacting scaffold and chasing sectors alter the physical metric, local environment, compact-object solutions, or gravitational-wave dynamics beyond acceptable levels.

The binding-sector question is

$$\boxed{\begin{array}{l}\text{Can visible matter retain the local and strong-field predictions}\\ \text{of general relativity while the scaffold experiences a}\\ \text{field-dependent interaction with the chasing scalar?}\end{array}} \tag{683}$$

Limitation 135.1 (Third-leg benchmark language)**.** *References in this part to visible matter, a radiation epoch, a pressureless-matter epoch, stars, compact objects, and binaries identify downstream observables that the GT1 third-leg realization must recover. They do not establish that all matter first originated in the third leg, and they do not establish that ordinary matter definitely existed in either hidden leg. The origin time of matter, antimatter, and particle-like excitations remains unresolved in this series.*

The local and strong-field audit distinguishes four levels that must not be collapsed:

$$\boxed{\begin{aligned}\text{action-level absence of a direct visible coupling} &\neq \text{solved local environment},\\ \text{general-relativistic control limit} &\neq \text{validated post-Newtonian fit},\\ \text{linear fifth-force kernel} &\neq \text{nonlinear halo or compact solution},\\ \text{standard tensor propagation law} &\neq \text{complete binary-wave prediction}.\end{aligned}} \tag{684}$$

136 Metric Binding in the GT1 Model

The complete GT1 metric equation is

$$M_{\mathrm{Pl}}^2 G_{\mu\nu} = T_{\mu\nu}^{\mathrm{vis}} + T_{\mu\nu}^{\mathrm{rad}} + T_{\mu\nu}^{\mathrm{scaf}} + T_{\mu\nu}^{\phi}. \tag{685}$$

The scalar stress–energy tensor is

$$T_{\mu\nu}^{\phi} = \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}\nabla_\alpha\phi\nabla^\alpha\phi + V(\phi)\right]. \tag{686}$$

The metric responds to all sectors.

Therefore, the statement that binding is general relativistic does not mean that the local metric is sourced only by visible matter.

It means that the relation between the metric and the total stress–energy retains the Einstein form.

The local gravitational field may still contain contributions from

$$\left\{T_{\mu\nu}^{\mathrm{vis}}, T_{\mu\nu}^{\mathrm{scaf}}, T_{\mu\nu}^{\phi}\right\}. \tag{687}$$

The conservative binding construction satisfies the non-double-counting condition

$$T_{\mu\nu}^{\text{bind}} \equiv 0. \tag{688}$$

Accordingly,

$$\boxed{\textit{binding} \text{ is not an additional substance in GT1.}} \tag{689}$$

It is the attractive and relativistic response of the physical metric to the complete source system.

137 Visible-Sector Geodesic Motion

Because visible matter couples minimally to $g_{\mu\nu}$, its stress–energy tensor satisfies

$$\nabla_\mu T_{\text{vis}}^{\mu\nu} = 0 \tag{690}$$

when there is no additional explicit visible-sector interaction.

A structureless visible test body follows

$$u^\mu \nabla_\mu u^\nu = 0. \tag{691}$$

Equivalently,

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau} \frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0. \tag{692}$$

No direct scalar acceleration appears:

$$f^\mu_{\phi,\text{vis}} = 0 \tag{693}$$

at the classical action level.

This removes the direct tree-level scalar-force channel for an ideal visible test body in the minimum action. It does not by itself prove the complete Einstein equivalence principle in realistic environments: radiative mixing, finite-size effects, scaffold redistribution, scalar stress–energy, and any later microscopic completion must be checked separately.

The action-level result depends on the visible coupling condition

$$A_{\text{vis}}(\phi) = 1. \tag{694}$$

If a later realization introduces

$$A_{\text{vis}}(\phi) \neq 1, \tag{695}$$

then universal free fall and post-Newtonian recovery must be recalculated.

138 Conditions for the General-Relativistic Local Limit

A local region approaches the standard general-relativistic limit when:

1. visible matter remains minimally coupled;
2. the gravitational coefficient M_{Pl}^2 is constant;
3. no independent binding field is present;
4. the local scalar gradient is negligible;
5. the scalar energy density is locally subdominant or behaves as an acceptable smooth vacuum contribution;
6. the local scaffold distribution is included as an ordinary source of the Einstein equation; and
7. the background evolves slowly over the local dynamical timescale.

These conditions may be summarized as

$$\boxed{\begin{array}{l} A_{\text{vis}} = 1, \qquad \dot{M}_{\text{Pl}} = 0, \qquad T_{\mu\nu}^{\text{bind}} = 0, \\ |\nabla_i \phi|_{\text{local}} \ll \dfrac{M_{\text{Pl}} |\nabla_i \Phi_N|}{|\beta|}, \\ \dfrac{\rho_\phi^{\text{local}}}{\rho_{\text{source}}} \ll 1 \quad \text{or} \quad T_{\mu\nu}^{\phi} \propto -g_{\mu\nu} \text{ locally.} \end{array}} \tag{696}$$

Under these conditions, the visible-sector metric approaches the general-relativistic metric sourced by the visible and scaffold mass–energy.

The local limit is not established merely by setting

$$\beta \neq 0 \tag{697}$$

or

$$\beta = 0. \tag{698}$$

The coupling may remain nonzero because it acts directly on the scaffold rather than on visible matter.

Its indirect metric consequences must still be calculated.

139 Weak-Field Expansion

Consider a weak-field, quasistatic metric,

$$\mathrm{d}s^2 = -\left(1 + 2\Psi\right)\mathrm{d}t^2 + \left(1 - 2\Phi\right)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{699}$$

where

$$|\Phi|, \quad |\Psi| \ll 1. \tag{700}$$

The nonrelativistic visible acceleration is

$$\frac{\mathrm{d}^2\boldsymbol{x}_{\mathrm{vis}}}{\mathrm{d}t^2} = -\boldsymbol{\nabla}\Psi. \tag{701}$$

The linearized gravitational constraint is schematically

$$\nabla^2\Psi = 4\pi G\left(\delta\rho_{\mathrm{vis}} + \delta\rho_{\mathrm{scaf}} + \delta\rho_\phi^{\mathrm{eff}}\right), \tag{702}$$

where the precise scalar contribution depends on the quasistatic limit and gauge convention. For a slowly varying canonical scalar with negligible local inhomogeneity,

$$\delta\rho_\phi^{\mathrm{eff}} \approx 0. \tag{703}$$

Then

$$\nabla^2\Psi \approx 4\pi G\left(\delta\rho_{\mathrm{vis}} + \delta\rho_{\mathrm{scaf}}\right). \tag{704}$$

This is the standard Poisson equation with the scaffold included as non-luminous gravitating matter.

140 Post-Newtonian Recovery

The local metric may be expanded as

$$g_{00} = -1 + 2U - 2\beta_{\mathrm{PPN}} U^2 + \cdots , \tag{705}$$

$$g_{ij} = (1 + 2\gamma_{\mathrm{PPN}} U)\, \delta_{ij} + \cdots . \tag{706}$$

For an isolated visible source in the control regime with negligible scaffold density and gradients, a locally homogeneous scalar, and a locally negligible or constant vacuum-like scalar stress, the GT1 field equations reduce to the corresponding general-relativistic local problem. The control prediction is therefore

$$\boxed{\gamma^{\mathrm{ctrl}}_{\mathrm{PPN}} = 1, \qquad \beta^{\mathrm{ctrl}}_{\mathrm{PPN}} = 1.} \tag{707}$$

This is an analytically defined limit, not yet a derived result for a realistic scaffold-rich environment. The control follows because:

- the gravitational action is Einstein–Hilbert;
- the visible source does not directly source the scalar through a field-dependent visible mass;
- the visible test body does not feel a direct scalar force; and
- the scalar is minimally coupled to curvature.

Departures may arise from:

$$\left\{ \delta\rho_{\mathrm{scaf}}, \nabla\phi, \dot{\phi}, \delta T^{\phi}_{\mu\nu}, \text{induced visible coupling} \right\}. \tag{708}$$

Represent the corrections schematically as

$$\gamma_{\mathrm{PPN}} - 1 = \Delta\gamma_{\mathrm{scaf}} + \Delta\gamma_{\phi} + \Delta\gamma_{\mathrm{ind}}, \tag{709}$$

$$\beta_{\mathrm{PPN}} - 1 = \Delta\beta_{\mathrm{scaf}} + \Delta\beta_{\phi} + \Delta\beta_{\mathrm{ind}}. \tag{710}$$

Every correction must be derived from the same local solution.

141 Visible-Sector Equivalence-Principle Protection

For a pointlike visible body A, write

$$S_A = -\int m_A \mathrm{d}s. \tag{711}$$

Its scalar coupling is

$$\alpha_A^{\mathrm{vis}} = M_{\mathrm{Pl}} \frac{\partial \ln m_A}{\partial \phi}. \tag{712}$$

The minimal visible-sector action gives the bare, direct charge

$$\boxed{\alpha_{A,\mathrm{bare}}^{\mathrm{vis}} = 0.} \tag{713}$$

This statement concerns the explicit matter coupling. An effective environmental response inferred from the total mass of a composite system must still be solved and is not set to zero by notation.

For two ideal visible compositions at the same action level A and B,

$$\alpha_{A,\mathrm{bare}}^{\mathrm{vis}} - \alpha_{B,\mathrm{bare}}^{\mathrm{vis}} = 0. \tag{714}$$

The direct scalar contribution to their differential acceleration therefore vanishes:

$$\eta_{AB}^{(\phi,\mathrm{direct})} = 0. \tag{715}$$

This is a useful structural feature of dark-only coupling, but it is only an action-level protection until the microscopic completion and environmental solution are shown to preserve it.

The protection must nevertheless be stable under:

- radiative corrections;
- portal interactions;
- mixing with Standard Model operators;
- transition-induced couplings; and
- high-energy completion.

A complete microscopic theory must explain why

$$|\alpha_A^{\mathrm{vis}}| \ll |\beta| \tag{716}$$

remains technically controlled rather than being set to zero only at one effective scale.

142 The Dark-Sector Fifth Force

Benchmark literature. The additional scaffold force is treated as a dark-sector scalar-mediated interaction, with visible-sector limits and screening kept distinct (Amendola, 2000; Khoury and Weltman, 2004).

The scaffold does not follow geodesics of the visible metric $g_{\mu\nu}$ alone.

Its effective metric is

$$\widetilde{g}_{\mu\nu}^{(\mathrm{scaf})} = A^2(\phi) g_{\mu\nu}. \tag{717}$$

In the pressureless, nonrelativistic point-particle limit, the scaffold acceleration takes the schematic form

$$\frac{\mathrm{d}^2 \boldsymbol{x}_{\mathrm{scaf}}}{\mathrm{d}t^2} = -\boldsymbol{\nabla}\Psi - \frac{\beta(\phi)}{M_{\mathrm{Pl}}} \boldsymbol{\nabla} \delta\phi. \tag{718}$$

The second term is the direct dark-sector scalar force.

For a canonical scalar, constant local coupling, and a linear quasistatic expansion about a specified background, the scalar perturbation equation is schematically

$$\left(\nabla^2 - m_{\mathrm{eff}}^2\right) \delta\phi \approx \frac{\beta}{M_{\mathrm{Pl}}} \delta\rho_{\mathrm{scaf}}. \tag{719}$$

The sign follows the interaction-current convention fixed by the selected GT1 action. Boundary conditions, background curvature, time derivatives, nonlinearities, and screening can invalidate this local reduction.

The effective scalar mass is

$$m_{\mathrm{eff}}^2 = V_{,\phi\phi} + \Delta m_{\mathrm{int}}^2, \tag{720}$$

where $\Delta m_{\mathrm{int}}^2$ contains coupling-dependent environmental contributions.

The scalar force acts directly between scaffold concentrations.

It does not act directly between two visible bodies in GT1.

143 Effective Gravitational Constants

Because different sectors couple differently, a single measured "effective gravitational constant" need not describe every response. The quantities below are response kernels defined only within the linear, quasistatic, weak-field regime; they are not new fundamental constants.

Define:

$$\{G_{\rm visvis}, G_{\rm visscaf}, G_{\rm scafvis}, G_{\rm scafscaf}\}\,. \tag{721}$$

For visible–visible interactions,

$$\boxed{G^{\rm ctrl}_{\rm visvis} = G} \tag{722}$$

for the isolated visible-source control defined above.

A visible source does not directly source $\delta\phi$, so an isolated scaffold test body orbiting a purely visible source feels

$$G^{\rm ctrl}_{\rm scafvis} \approx G \tag{723}$$

when the visible source does not induce a scalar profile and no environmental scalar gradient is present.

Visible matter responds to the metric sourced by scaffold energy, so

$$G^{\rm metric}_{\rm visscaf} = G \tag{724}$$

at the bare metric-coupling level. The metric generated by a scaffold source also contains any scalar stress–energy and scaffold redistribution, so an inferred visible response need not equal this bare coefficient in a solved environment.

Two scaffold concentrations experience both tensor and scalar attraction.

For constant coupling, an ideal pressureless scaffold, negligible nonlinear screening, and the same quasistatic normalization used above, a useful linear Fourier-space kernel is

$$\boxed{G^{\rm lin,QS}_{\rm scafscaf}(k,a) = G\left[1 + 2\beta^2 \frac{k^2}{k^2 + a^2 m^2_{\rm eff}}\right]} \tag{725}$$

for the constant-coupling, unscreened, linear quasistatic limit.

$$\boxed{\text{this kernel is not a universal nonlinear gravitational constant.}} \tag{726}$$

In the long-range scalar limit,

$$k^2 \gg a^2 m_{\rm eff}^2, \tag{727}$$

this becomes

$$G_{\rm scafscaf} \approx G\left(1 + 2\beta^2\right). \tag{728}$$

In the short-range limit,

$$k^2 \ll a^2 m_{\rm eff}^2, \tag{729}$$

one recovers

$$G_{\rm scafscaf} \approx G. \tag{730}$$

The coefficient $2\beta^2$ depends on the declared normalization of the coupling and scalar field. Extended sources, time dependence, nonlinear screening, or a noncanonical scaffold can change the response. The convention above matches the GT1 definition of β.

144 Yukawa Form of the Scaffold Force

For a static pointlike scaffold source of mass $M_{\rm scaf}$, constant coupling, a linear canonical scalar, and no nonlinear screening, the Green-function solution generates a Yukawa correction.

The potential felt by another weakly bound scaffold test body is then approximately

$$\Psi_{\rm scafscaf}(r) = -\frac{GM_{\rm scaf}}{r}\left[1 + 2\beta^2 e^{-m_{\rm eff} r}\right]. \tag{731}$$

The scalar interaction range is

$$\lambda_\phi = m_{\rm eff}^{-1}. \tag{732}$$

The radial acceleration is

$$a_{\rm scafscaf}(r) = -\frac{GM_{\rm scaf}}{r^2}\left[1 + 2\beta^2\left(1 + m_{\rm eff} r\right)e^{-m_{\rm eff} r}\right]. \tag{733}$$

Within its domain of validity, this interaction can alter:

- scaffold collapse;
- halo concentration;
- merger dynamics;
- substructure abundance;
- matter-power growth;
- cluster potentials; and
- the relationship between lensing and scaffold motion.

The force cannot be treated as a harmless hidden interaction merely because visible matter has no direct scalar charge.

Its redistribution of scaffold stress–energy and its contribution to the metric remain potentially observable and must be calculated.

145 Dynamical Mass versus Lensing Mass

Visible photons follow null geodesics of $g_{\mu\nu}$.

They respond to the Weyl potential

$$\Phi_W = \frac{\Phi + \Psi}{2}. \tag{734}$$

The dark scalar force does not directly deflect photons.

A scaffold body, however, responds to

$$\Psi_{\rm scaf}^{\rm eff} = \Psi + \frac{\beta}{M_{\rm Pl}}\delta\phi. \tag{735}$$

Consequently, the mass inferred from scaffold dynamics can differ from the mass inferred from lensing.

Define

$$\mathcal{R}_{\rm dyn/lens} = \frac{M_{\rm dyn}^{\rm (scaf)}}{M_{\rm lens}}. \tag{736}$$

In an unscreened idealized region,

$$\mathcal{R}_{\rm dyn/lens} \sim 1 + 2\beta^2 \tag{737}$$

for scaffold tracers, subject to the source composition and mass profile.

Visible stellar tracers do not receive the direct scalar acceleration.

Therefore, a system containing both visible and scaffold tracers may exhibit a species-dependent dynamical response even though all visible compositions continue to obey universal free fall.

This is a primary observational target of the dark-only interaction.

146 Environmental Scalar Fields

An isolated visible body does not directly source $\delta\phi$.

A visible body embedded in a scaffold environment may nevertheless reside in a nonzero scalar profile generated by the scaffold.

Let the environmental scalar be

$$\phi = \bar{\phi}(t) + \phi_{\rm env}(\boldsymbol{x}) + \delta\phi_{\rm body}. \tag{738}$$

For a minimally coupled visible body,

$$\delta\phi_{\rm body} \approx 0 \tag{739}$$

at leading order.

However, the environmental scalar contributes to the metric through

$$T^{\phi}_{\mu\nu}\left[\phi_{\rm env}\right]. \tag{740}$$

A spatial gradient also changes the scaffold distribution, which then changes the metric experienced by visible matter.

Thus, local visible gravity may acquire indirect corrections through the sequence

$$\boxed{\phi_{\rm env} \longrightarrow \text{scaffold redistribution} \longrightarrow T^{\rm scaf}_{\mu\nu} \longrightarrow g_{\mu\nu} \longrightarrow \text{visible motion.}} \tag{741}$$

This indirect route must be included when comparing the model with galactic, cluster, and local-environment observations.

147 Screening and Range Suppression

The minimal GT1 model does not require screening to remove a direct visible fifth force because visible matter is not directly coupled.

Screening or range suppression may still be required to control the dark-sector force.

The scalar force is suppressed when

$$m_{\mathrm{eff}} r \gg 1. \tag{742}$$

A field-dependent coupling may instead satisfy

$$|\beta(\phi_{\mathrm{local}})| \ll |\beta(\phi_{\mathrm{cos}})|. \tag{743}$$

A nonlinear scaffold response may produce another suppression factor,

$$\mathcal{S}_{\mathrm{scaf}}\left(\rho_{\mathrm{scaf}}, \phi, |\nabla\phi|\right), \tag{744}$$

so that

$$\boldsymbol{a}_{\phi,\mathrm{scaf}} = -\mathcal{S}_{\mathrm{scaf}} \frac{\beta}{M_{\mathrm{Pl}}} \boldsymbol{\nabla}\phi. \tag{745}$$

The unscreened limit is

$$\mathcal{S}_{\mathrm{scaf}} \to 1. \tag{746}$$

The screened limit is

$$\mathcal{S}_{\mathrm{scaf}} \to 0. \tag{747}$$

A valid suppression mechanism must follow from the covariant action.

It may not be activated manually for each astrophysical system.

148 Field-Dependent Scaffold Mass

The conformal coupling may be interpreted through a field-dependent scaffold particle mass,

$$m_{\rm scaf}(\phi) = m_{\rm scaf,0} A(\phi). \tag{748}$$

For

$$A(\phi) = \exp\left(\beta \frac{\phi}{M_{\rm Pl}}\right), \tag{749}$$

one obtains

$$M_{\rm Pl} \frac{\mathrm{d} \ln m_{\rm scaf}}{\mathrm{d}\phi} = \beta. \tag{750}$$

The scaffold mass evolves as

$$\frac{\dot{m}_{\rm scaf}}{m_{\rm scaf}} = \frac{\beta}{M_{\rm Pl}} \dot{\phi}. \tag{751}$$

Using the autonomous variable x,

$$\frac{\mathrm{d} \ln m_{\rm scaf}}{\mathrm{d}N} = \sqrt{6}\, \beta x. \tag{752}$$

The same combination appeared in the background transfer rate:

$$\mathcal{Q}_{\rm scaf} = \sqrt{6}\, \beta x. \tag{753}$$

Thus, background energy exchange, scaffold mass drift, and the dark-sector fifth force are not independent effects.

They are linked by the same coupling.

A viable model must fit all three with one $\beta(\phi)$.

149 Local Measurement of Newton's Constant

In the isolated visible-source control, a Cavendish-type experiment using visible source and test masses measures

$$G_{\rm Cav}^{\rm visvis} = G \tag{754}$$

at leading classical order. This equality must be corrected if the apparatus sits in a dynamically relevant scaffold or scalar environment.

The reduced Planck mass is constant:

$$\dot{M}_{\rm Pl} = 0. \tag{755}$$

Therefore,

$$\dot{G}_{\rm tensor} = 0 \tag{756}$$

for the coefficient of the Einstein–Hilbert tensor interaction in GT1. This does not force every environment-dependent gravitational inference to be time independent.

An observer may infer an effective gravitational strength from cosmological scaffold growth,

$$G_{\rm growth} \neq G_{\rm Cav}^{\rm visvis}, \tag{757}$$

because scaffold particles experience the scalar force.

This distinction must be stated precisely:

$$\boxed{\begin{aligned}&\text{scale-dependent effective dark gravitational strength}\\ &\neq \text{time-varying fundamental Newton constant.}\end{aligned}} \tag{758}$$

Confusing the two would misidentify a dark-sector interaction as a modification of the Einstein–Hilbert coefficient.

150 Canonical Scalar Anisotropic Stress

A minimally coupled canonical scalar has no independent linear scalar anisotropic stress about the standard homogeneous background. This statement does not apply automatically to nonlinear local scalar gradients.

A pressureless scaffold also has negligible intrinsic anisotropic stress only in the ideal cold-fluid limit.

Therefore, excluding radiation and neutrino contributions,

$$\Phi - \Psi \approx 0 \tag{759}$$

in the minimal GT1 scalar sector.

Thus,

$$\boxed{\eta_{\rm slip} = \frac{\Phi}{\Psi} \approx 1 \quad \text{in the stated linear ideal-fluid limit.}} \tag{760}$$

The model can nevertheless produce a difference between growth and lensing because scaffold motion responds to

$$\Psi + \frac{\beta}{M_{\rm Pl}}\delta\phi, \tag{761}$$

while photons respond to

$$\Phi + \Psi. \tag{762}$$

This is distinct from a conventional gravitational-slip modification.

The key distinction is

$$\boxed{\text{dark fifth-force enhancement} \neq \text{metric gravitational slip.}} \tag{763}$$

151 Compact Objects in the GT1 Model

Benchmark literature. Compact-object scalar charges and nonlinear strong-field effects are compared with tensor–scalar gravity (Damour and Esposito-Farèse, 1993, 1996).

A compact visible object is described by

$$T^{\rm vis}_{\mu\nu} \tag{764}$$

together with any surrounding scaffold and scalar environment.

The full stationary or quasistationary system is

$$\left\{g_{\mu\nu}, \phi, T^{\rm vis}_{\mu\nu}, T^{\rm scaf}_{\mu\nu}\right\}. \tag{765}$$

For an isolated visible object in the control regime

$$\nabla_\mu\phi \approx 0, \qquad T^{\rm scaf}_{\mu\nu} \approx 0, \tag{766}$$

the metric equations reduce to general relativity plus the local value of the scalar potential. This is a control solution, not a statement that every compact object in GT1 has a general-relativistic exterior.

If

$$V(\phi) \approx V_{\text{loc}} = \text{constant}, \tag{767}$$

the local exterior solution is approximately the corresponding general-relativistic vacuum solution with a cosmological-constant-like correction.

Whether that smooth correction is negligible depends on the potential, the asymptotic background, and the scale of the system.

The theory must nevertheless solve the full equations when:

- the scalar is time dependent;
- a scaffold halo surrounds the object;
- the scalar mass is small;
- the dark coupling is strong; or
- the compact object contains or accretes scaffold matter.

152 Scalar Charge of Extended Bodies

For an extended object A with total mass $M_A(\phi_\infty)$, define its effective scalar charge by

$$\alpha_A = M_{\text{Pl}} \frac{\partial \ln M_A}{\partial \phi_\infty}, \tag{768}$$

where ϕ_∞ is the asymptotic environmental scalar value.

For an ideal visible body with no scaffold content in the scaffold-free, scalar-homogeneous control,

$$\alpha_A^{\text{vis,ctrl}} \approx 0. \tag{769}$$

For a scaffold body,

$$\alpha_A^{\text{scaf}} \approx \beta \tag{770}$$

only in the weakly bound, point-particle, unscreened limit. Strong self-binding, scalar mass, and environmental screening can renormalize this charge.

A mixed object containing visible and scaffold mass may have

$$\alpha_A^{\mathrm{mix}} = f_{\mathrm{scaf},A}\beta + \Delta\alpha_A^{\mathrm{bind}} + \Delta\alpha_A^{\mathrm{screen}}, \tag{771}$$

where

$$f_{\mathrm{scaf},A} = \frac{M_{\mathrm{scaf},A}}{M_A}. \tag{772}$$

The binding and screening corrections depend on the internal solution.

A compact visible object surrounded by a scaffold halo may therefore have an effective environmental scalar response even when its visible material has zero fundamental scalar charge.

153 Binary Dynamics

Consider two bodies with masses

$$M_1, \qquad M_2, \tag{773}$$

and scalar charges

$$\alpha_1, \qquad \alpha_2. \tag{774}$$

Their conservative interaction may be written schematically as

$$U(r) = -\frac{GM_1M_2}{r}\left[1 + 2\alpha_1\alpha_2 e^{-m_{\mathrm{eff}}r}\right]. \tag{775}$$

For two purely visible compact objects in the scaffold-free, scalar-homogeneous control,

$$\alpha_1 \approx \alpha_2 \approx 0. \tag{776}$$

Their conservative dynamics then approach the corresponding general-relativistic prediction. A realistic environmental statement requires the coupled exterior solution.

For two scaffold-dominated objects,

$$\alpha_1\alpha_2 \sim \beta^2 \tag{777}$$

in the unscreened limit.

A mixed visible–scaffold binary has only one directly charged body.

Its conservative scalar correction depends on whether the nominally visible object contains a scaffold halo or induced environmental charge.

The binary calculation must therefore distinguish:

$$\{\text{vis–vis}, \text{vis–scaf}, \text{scaf–scaf}, \text{mixed–mixed}\}\,. \tag{778}$$

154 Scalar Radiation

A dynamical scalar can carry energy away from a binary when the system has a time-varying scalar multipole.

The leading scalar dipole power is schematically

$$P_\phi^{\mathrm{dip}} \propto G\,(\alpha_1 - \alpha_2)^2\,\mu^2 r^2 \Omega^4\,\mathcal{Y}\,(m_{\mathrm{eff}}, \Omega)\,, \tag{779}$$

where:

- μ is the reduced mass;
- r is the orbital separation;
- Ω is the orbital frequency; and
- $\mathcal{Y}$ is a mass-dependent propagation factor.

Scalar radiation is kinematically suppressed when

$$m_{\mathrm{eff}} \gtrsim \Omega \tag{780}$$

in natural units.

For two purely visible bodies in the same scaffold-free, scalar-homogeneous control,

$$\alpha_1 - \alpha_2 \approx 0, \tag{781}$$

so direct scalar dipole radiation is absent at leading order in that control. Environmental or induced charges must be included before applying this conclusion to an observed binary.

For compositionally different dark or mixed bodies,

$$\alpha_1 - \alpha_2 \neq 0 \tag{782}$$

may produce additional energy loss.

Total energy conservation requires

$$\dot{E}_{\rm orb} = -P_{\rm tensor} - P_\phi - P_{\rm other}. \tag{783}$$

A model cannot alter the binary force while omitting the corresponding radiative channel.

155 Tensor Gravitational Waves

Benchmark literature. The tensor-wave control is compared with direct gravitational-wave observations and multimessenger propagation (LIGO Scientific Collaboration and Virgo Collaboration, 2016; LIGO Scientific Collaboration et al., 2017).

The GT1 gravitational action has a constant Planck mass and no nonminimal curvature coupling.

The source-free tensor equation is therefore

$$h''_\lambda + 2\mathcal{H}h'_\lambda + k^2 h_\lambda = \frac{2a^2}{M_{\rm Pl}^2}\Pi_\lambda^{\rm TT}, \tag{784}$$

in natural units.

The propagation parameters are

$$\boxed{\alpha_M = 0, \qquad c_T^2 = 1, \qquad m_T^2 = 0.} \tag{785}$$

Thus, at the level of the minimum action linearized about the declared background, GT1 has the standard tensor propagation cone, zero tensor mass, and constant-Planck-mass friction term. This action-level result does not determine source generation or additional scalar radiation.

The scaffold and scalar may still alter gravitational waves through:

- their contribution to the background $H(a)$;
- their anisotropic stress;
- changes to the source dynamics;
- additional scalar radiation;
- environmental lensing; and
- modified binary masses or orbital evolution.

When electromagnetic and tensor waves propagate on the same physical metric and the effective Planck mass is constant, the propagation-law control gives

$$D_L^{\rm GW} = D_L^{\rm EM}. \tag{786}$$

This equality concerns propagation. Source calibration, scalar energy loss, lensing, and an incorrectly inferred background can still change the observed comparison.

This equality would cease to hold in a later modified-binding realization with a varying effective Planck mass.

156 Black-Hole Sector

In the minimal GT1 theory, the chasing scalar is canonical and minimally coupled to curvature.

A stationary isolated black-hole exterior has a general-relativistic control branch when the field equations and boundary conditions permit:

$$\nabla_\mu \phi \approx 0, \qquad T^{\mathrm{scaf}}_{\mu\nu} \approx 0 \tag{787}$$

in the exterior region.

A time-dependent cosmological scalar, nontrivial potential, scaffold accretion, or environmental scalar gradient can modify this conclusion.

The required black-hole calculation includes:

$$\left\{ g_{\mu\nu}, \phi, T^{\mathrm{scaf}}_{\mu\nu}, \text{boundary conditions at the horizon and infinity} \right\}. \tag{788}$$

A viable solution must possess:

- a regular horizon;
- finite scalar invariants;
- finite stress–energy;
- stable linear perturbations;
- acceptable quasinormal modes;
- acceptable shadow and lensing geometry; and
- acceptable accretion behavior.

The manuscript does not infer a universal no-hair theorem from the minimum action.

It treats the absence or presence of scalar structure as a property to be derived from the potential, coupling, boundary conditions, and scaffold distribution.

157 Neutron Stars and Material Compact Objects

A static spherical visible star may be written as

$$\mathrm{d}s^2 = -e^{2\nu(r)}\mathrm{d}t^2 + \left(1 - \frac{2Gm(r)}{r}\right)^{-1} \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2. \tag{789}$$

Visible matter obeys the ordinary conservation equation

$$\nabla_\mu T^{\mu\nu}_{\mathrm{vis}} = 0. \tag{790}$$

In the scaffold-free, scalar-homogeneous control, the stellar structure equations reduce to the corresponding general-relativistic system up to any locally relevant constant potential term. This control does not replace the environmental compact-star calculation.

A scaffold halo or nontrivial scalar profile introduces additional metric sources:

$$\rho_{\mathrm{tot}} = \rho_{\mathrm{vis}} + \rho_{\mathrm{scaf}} + \rho_\phi. \tag{791}$$

The visible pressure gradient continues to respond to the physical metric.

The full compact-star problem must determine:

$$\left\{M(R), R, I, \Lambda_{\mathrm{tidal}}, \phi(r), \rho_{\mathrm{scaf}}(r)\right\}, \tag{792}$$

where:

- I is the moment of inertia; and
- Λ_{tidal} is a tidal-deformability parameter.

A dark scaffold accumulated inside or around the star could modify these quantities without a direct visible scalar coupling.

This is an environmental matter-distribution effect rather than a modification of the visible equation of motion.

158 Strong-Field Stability

A regular background solution does not guarantee strong-field stability.

Perturb the compact solution:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \tag{793}$$

$$\phi = \bar{\phi} + \delta\phi, \tag{794}$$

and

$$\psi_{\text{scaf}} = \bar{\psi}_{\text{scaf}} + \delta\psi_{\text{scaf}}. \tag{795}$$

The perturbation operator may be written schematically as

$$\mathcal{D}\left(\bar{g}, \bar{\phi}, \bar{\psi}_{\text{scaf}}\right) \delta\boldsymbol{Y} = 0. \tag{796}$$

For every physical propagating mode in a fully reduced perturbation system, necessary local conditions include

$$Q_A(r) > 0, \tag{797}$$

$$c_A^2(r) \geq 0, \tag{798}$$

together with boundary conditions that exclude unacceptable growing modes,

$$\mathrm{Im}\left(\omega_A\right) \leq 0 \tag{799}$$

under the convention

$$\delta Y_A \propto e^{-\mathrm{i}\omega_A t}. \tag{800}$$

The scalar effective mass must not generate a rapid tachyonic instability in the required environment.

A controlled long-timescale instability may still be physically relevant but must be compared with the system age.

159 Binding Recovery Hierarchy

Binding recovery should be tested in a strict order.

159.1 Level 1: visible test-body motion

Verify

$$u^\mu \nabla_\mu u^\nu = 0. \tag{801}$$

159.2 Level 2: visible-source weak field

Recover

$$\nabla^2 \Psi = 4\pi G \rho_{\text{vis}} \tag{802}$$

for an isolated visible source.

159.3 Level 3: post-Newtonian recovery

Recover acceptable values of

$$\{\gamma_{\text{PPN}}, \beta_{\text{PPN}}, \text{preferred-frame parameters}\}. \tag{803}$$

159.4 Level 4: visible plus scaffold environments

Solve the physical metric in realistic mixed environments.

159.5 Level 5: compact visible bodies

Recover acceptable neutron-star, black-hole, and binary solutions.

159.6 Level 6: dark and mixed compact systems

Calculate the scalar force, charge, radiation, and merger behavior of scaffold-rich systems.

159.7 Level 7: cross-scale consistency

Use the same

$$\{V(\phi), A(\phi), \beta(\phi)\} \tag{804}$$

for local, galactic, cluster, cosmological, and cross-leg calculations.

The model should not advance to a modified-binding theory unless the metric-binding realization fails a clearly identified requirement.

160 First Local Numerical Program

The initial binding calculation should proceed through eight controlled systems.

160.1 System A: isolated visible point source

Set

$$\rho_{\text{scaf}} = 0, \qquad \nabla\phi = 0. \tag{805}$$

Verify the Newtonian and post-Newtonian limits.

160.2 System B: visible source in a homogeneous scalar background

Allow

$$\phi = \bar{\phi}(t), \tag{806}$$

and quantify corrections from cosmological time evolution.

160.3 System C: isolated scaffold source

Solve jointly for

$$\Psi(r), \qquad \Phi(r), \qquad \delta\phi(r). \tag{807}$$

Verify the Yukawa limit.

160.4 System D: mixed visible–scaffold source

Calculate visible and scaffold test-body trajectories separately.

160.5 System E: screened or massive-scalar source

Vary

$$m_{\text{eff}} r \tag{808}$$

and recover the short-range general-relativistic limit.

160.6 System F: visible compact star

Calculate the mass–radius and tidal relations.

160.7 System G: scaffold-rich compact object

Calculate scalar charge and strong-field stability.

160.8 System H: binary system

Calculate conservative dynamics, tensor radiation, and scalar radiation.

Each system should be tested first in the uncoupled limit

$$\beta = 0, \tag{809}$$

and then continued into the weakly coupled parameter region selected in the background-dynamics analysis.

161 Binding Observables

The local and strong-field prediction vector is

$$\boldsymbol{\mathcal{O}}_{\text{bind}} = (\boldsymbol{\mathcal{O}}_{\text{WF}}, \boldsymbol{\mathcal{O}}_{\text{PPN}}, \boldsymbol{\mathcal{O}}_{\text{lens}}, \boldsymbol{\mathcal{O}}_{\text{CO}}, \boldsymbol{\mathcal{O}}_{\text{bin}}, \boldsymbol{\mathcal{O}}_{\text{GW}}) \,. \tag{810}$$

The weak-field block includes

$$\boldsymbol{\mathcal{O}}_{\text{WF}} = \{\Psi(r), \Phi(r), G_{\text{Cav}}, \boldsymbol{a}_{\text{vis}}, \boldsymbol{a}_{\text{scaf}}\} \,. \tag{811}$$

The post-Newtonian block includes

$$\mathcal{O}_{\rm PPN} = \left\{\gamma_{\rm PPN}, \beta_{\rm PPN}, \dot{G}/G, \eta_{AB}\right\}. \tag{812}$$

The lensing block includes

$$\mathcal{O}_{\rm lens} = \left\{\Phi + \Psi, M_{\rm lens}, \mathcal{R}_{\rm dyn/lens}\right\}. \tag{813}$$

The compact-object block includes

$$\mathcal{O}_{\rm CO} = \{M(R), I, \Lambda_{\rm tidal}, \alpha_A, \omega_{\rm QNM}\}. \tag{814}$$

The binary block includes

$$\mathcal{O}_{\rm bin} = \left\{\dot{P}_{\rm orb}, P_\phi, P_{\rm tensor}, \text{phase evolution}\right\}. \tag{815}$$

The same parameter set used in the background-dynamics cosmological solution must be propagated into these observables.

162 Binding-Sector Rejection Criteria

The metric-binding GT1 realization fails or requires revision if any of the following conditions is unavoidable.

1. Visible matter does not follow geodesics of one physical metric.
2. A direct visible scalar coupling is required but cannot satisfy equivalence-principle limits.
3. The local Newtonian potential differs unacceptably from the general-relativistic result.
4. The post-Newtonian parameters cannot approach their general-relativistic values.
5. The scalar background causes an unacceptable local time dependence.
6. The model predicts a varying fundamental Newton constant despite the constant Einstein–Hilbert coefficient.
7. The scaffold fifth force is too strong to preserve acceptable structure or local mixed-system dynamics.
8. The scalar interaction range is incompatible with the same coupling required by cosmology.
9. A required screening mechanism has no derivation from the action.
10. The conformal factor becomes zero or negative.

11. The scaffold particle mass becomes singular.
12. The scaffold particle mass varies too rapidly during the required third-leg eras.
13. The scalar force causes unacceptable separation between visible and scaffold tracers.
14. The model fits scaffold dynamics but fails gravitational lensing.
15. The model produces excessive environmental scalar stress around visible systems.
16. Visible compact objects acquire an unacceptable scalar charge.
17. Mixed compact systems generate excessive scalar dipole radiation.
18. The theory omits scalar radiation while retaining a dynamical scalar force.
19. No regular black-hole exterior solution exists.
20. No stable neutron-star solution exists.
21. The mass–radius or tidal predictions are unacceptable.
22. A strong-field ghost or gradient instability appears.
23. The scalar develops a rapid tachyonic instability in ordinary astrophysical environments.
24. The tensor gravitational-wave speed differs from the metric null cone in the minimal GT1 action.
25. The tensor sector develops an effective mass or altered damping without a corresponding modification of the action.
26. The gravitational-wave and electromagnetic luminosity distances differ despite the constant Planck mass and standard tensor propagation law.
27. Local, galactic, binary, and cosmological data require incompatible values of β or m_{eff}.
28. The visible-sector decoupling $A_{\text{vis}} = 1$ is unstable under the proposed microscopic completion.
29. The only viable local limit requires setting the scaffold–chasing interaction identically to zero at every scale and epoch.
30. A modified-binding sector is introduced before the metric-binding baseline has been shown to fail a specific observable requirement.

163 Formal Summary of Binding and Local Recovery

The binding assignment is

$$\boxed{\mathfrak{G}_{\text{bind}} = \text{general-relativistic metric response}, \qquad T^{\text{bind}}_{\mu\nu} = 0.} \tag{816}$$

Visible matter is minimally coupled:

$$\boxed{S_{\text{vis}} = S_{\text{vis}}[g, \psi_{\text{vis}}], \qquad A_{\text{vis}}(\phi) = 1.} \tag{817}$$

Visible test bodies obey

$$\boxed{u^\mu \nabla_\mu u^\nu = 0.} \tag{818}$$

The scaffold acceleration is

$$\boxed{\boldsymbol{a}_{\text{scaf}} = -\boldsymbol{\nabla}\Psi - \frac{\beta}{M_{\text{Pl}}}\boldsymbol{\nabla}\delta\phi.} \tag{819}$$

The scalar perturbation satisfies

$$\boxed{\left(\nabla^2 - m_{\text{eff}}^2\right)\delta\phi \approx \frac{\beta}{M_{\text{Pl}}}\delta\rho_{\text{scaf}}.} \tag{820}$$

In the same regime, the scaffold self-response is

$$\boxed{G^{\text{lin,QS}}_{\text{scafscaf}}(k, a) = G\left[1 + 2\beta^2 \frac{k^2}{k^2 + a^2 m_{\text{eff}}^2}\right]} \tag{821}$$

for the constant-coupling, unscreened, linear quasistatic limit. This kernel is not a universal nonlinear gravitational constant.

The visible Cavendish coupling is

$$\boxed{G^{\text{ctrl}}_{\text{visvis}} = G.} \tag{822}$$

The local post-Newtonian control limit is

$$\boxed{\gamma^{\text{ctrl}}_{\text{PPN}} = 1, \qquad \beta^{\text{ctrl}}_{\text{PPN}} = 1} \tag{823}$$

when the local scalar and scaffold environmental corrections are negligible.

The scaffold mass drift is

$$\boxed{\frac{\mathrm{d}\ln m_{\mathrm{scaf}}}{\mathrm{d}N} = \sqrt{6}\,\beta x.} \tag{824}$$

At the linear propagation-law level of the minimum action,

$$\boxed{\alpha_M = 0, \qquad c_T^2 = 1, \qquad m_T^2 = 0.} \tag{825}$$

The metric-slip limit is

$$\boxed{\Phi \approx \Psi} \tag{826}$$

for a canonical scalar and ideal cold scaffold, apart from other anisotropic-stress sources.

The key observational distinction is

$$\boxed{\text{scaffold growth may be fifth-force enhanced while photon lensing remains purely metric.}} \tag{827}$$

The decisive binding question is

$$\boxed{\begin{array}{l}\text{Can one common coupling and scalar potential preserve}\\ \text{general-relativistic visible gravity, compact objects, and}\\ \text{tensor-wave propagation while producing a controlled}\\ \text{scaffold-only force that remains compatible with lensing,}\\ \text{structure, binary dynamics, and the cosmological background?}\end{array}} \tag{828}$$

Proposition 163.1 (Local and strong-field control result)**.** *Within the minimum GT1 action, an isolated visible-source, scaffold-free, scalar-homogeneous control recovers metric geodesic motion, the general-relativistic weak-field and post-Newtonian limits, and standard linear tensor propagation. A scaffold-only conformal coupling can simultaneously generate a dark scalar-force kernel. This proposition establishes the existence of compatible control limits; it does not establish that one cosmological parameter set passes realistic environmental, nonlinear, compact-object, binary, and structure observations.*

Criterion 163.2 (Local and strong-field viability gate)**.** *The binding sector is viable only if one common choice of $V(\phi)$, $A(\phi)$, initial data, and environmental boundary conditions passes all of the following without manually changing the theory between systems:*

1. *the isolated visible weak-field and post-Newtonian control;*
2. *mixed visible–scaffold dynamics and lensing;*

3. *scaffold mass-drift and fifth-force bounds;*
4. *regular black-hole and material-star solutions where required;*
5. *binary conservative dynamics and every kinematically available radiative channel;*
6. *strong-field ghost, gradient, and tachyonic-stability tests; and*
7. *standard tensor-wave propagation for the minimum action.*

164 Binding-Sector Status

The binding-sector problem is now reduced to the following calculation sequence.

1. Verify visible geodesic motion from the action.
2. Solve the isolated visible weak-field limit.
3. Recover the general-relativistic post-Newtonian parameters.
4. Calculate corrections from a slowly evolving cosmological scalar.
5. Solve the scalar profile of an isolated scaffold source.
6. Verify the Yukawa and mass-suppressed limits.
7. Calculate the effective scaffold self-attraction.
8. Calculate visible and scaffold trajectories in the same mixed system.
9. Compare dynamical and lensing masses.
10. Propagate the background-dynamics parameter region into local scalar masses and interaction ranges.
11. Determine whether any dark-sector screening mechanism is required.
12. Calculate scaffold mass drift from the background scalar trajectory.
13. Solve visible compact-star configurations.
14. Solve scaffold-rich and mixed compact configurations.
15. Calculate effective scalar charges.
16. Calculate conservative binary dynamics.
17. Calculate scalar and tensor radiation together.
18. Construct black-hole solutions in cosmological and scaffold environments.

19. Verify strong-field stability.

20. Test all local and compact-system outputs with the same $V(\phi)$ and $A(\phi)$ used cosmologically.

The minimum GT1 action contains a clean control limit in which visible binding recovers general relativity without forcing the scaffold–chasing coupling to vanish identically. This is a structural result, not yet a validated local or compact-system solution.

The distinctive local and astrophysical burden is the derived difference between metric motion and any scaffold-only fifth-force motion, evaluated with the same coupling, potential, environment, and source composition used cosmologically.

The next part develops the scaffolding sector itself: its clustering, sound speed, anisotropic stress, lensing contribution, structure-growth law, web formation, and capacity to retain cross-leg structural memory.

165 The Scaffolding-Sector Problem

Benchmark literature. The third-leg scaffold burden is defined by cold-dark-matter background, perturbation, halo, and cosmic-web phenomenology (Peebles, 1980; Ma and Bertschinger, 1995; Springel et al., 2005; Bertone et al., 2005).

The scaffolding function has two distinct scientific burdens.

The first is conventional:

$$\boxed{\begin{array}{l}\text{recover the successful background, clustering, and lensing}\\ \text{functions conventionally assigned to cold dark matter.}\end{array}} \tag{829}$$

The second is specific to the Triple Bounce framework:

$$\boxed{\begin{array}{l}\text{retain and transmit part of the first-leg structural state}\\ \text{through the second leg and both reversals.}\end{array}} \tag{830}$$

These burdens must not be conflated.

A cold gravitating component can generate a third-leg cosmic web from ordinary primordial perturbations without preserving information from a prior leg.

Conversely, a medium may retain structural memory while failing to cluster like cold dark matter.

The frozen Paper I architecture treats persistence of a distributed scaffold as a working hypothesis while leaving its microscopic identity unresolved. The frozen Paper II framework requires any inherited component to enter the same central covariance, transition maps, and third-leg observables used by the causal and perturbation tests. Paper III does not upgrade either requirement into a derived result.

The minimum third-leg scaffolding burden is

$$\boxed{\mathcal{B}^{(3)}_{\rm scaf} = \left\{ \begin{array}{c} \text{cold or effectively cold background behavior,} \\ \text{stable clustering and acceptable structure growth,} \\ \text{consistent metric sourcing and photon lensing} \end{array} \right\}.} \tag{831}$$

The full historical extension has the additional burden

$$\boxed{\mathcal{B}^{(123)}_{\rm scaf} = \mathcal{B}^{(3)}_{\rm scaf} + \left\{ \begin{array}{c} \text{a covariant memory state and stress–energy law,} \\ \text{finite propagation and relaxation through the hidden legs,} \\ \text{derived transfer through both reversals,} \\ \text{a statistically acceptable observable residue} \end{array} \right\}.} \tag{832}$$

Failure of the historical extension would block scaffold-memory and historical-identification claims. It would not by itself erase a viable third-leg cold-scaffold realization.

All references in this part to radiation domination, matter–radiation equality, or a pressureless-matter epoch describe downstream observable third-leg benchmarks. They neither assert that all matter first originated in the third leg nor assert that ordinary matter definitely existed during either hidden leg.

165.1 Derivation-status and reproducibility rule

The scaffolding equations are divided into four reproducible classes:

I. **Selected-action identities.** The relations among $A(\phi)$, $m_{\rm scaf}(\phi)$, $\beta(\phi)$, number conservation, and the background exchange current follow from the declared number-conserving particle branch.

II. **Linear action-dependent equations.** The density, velocity, scalar, and metric perturbation equations require the declared metric signature, Fourier convention, gauge, coupling sign, and matter content.

III. **Controlled approximations.** The effective self-attraction, reduced growth equations, Jeans scale, and related kernels are diagnostic limits of the full system, not independent laws.

IV. **Open historical templates.** The memory tensor, relaxation law, and reversal-transfer operators are bookkeeping structures that define what must be derived. They are not claimed as a selected microscopic completion.

A later derivation may replace a displayed template, but it must preserve the frozen Papers I–II architecture, total covariant conservation, the single central covariance, and the declared control limits.

The minimal GT1 scaffold begins as a pressureless particle sector.

A dedicated memory degree of freedom is introduced only after the cold-particle baseline is defined and tested.

166 Minimal Cold-Particle Scaffold

The minimal scaffold consists of particles whose mass depends on the chasing field:

$$m_{\rm scaf} = m_{\rm scaf}(\phi). \tag{833}$$

The particle action is

$$S_{\rm scaf} = -\sum_A \int m_{\rm scaf}(\phi)\mathrm{d}s_A. \tag{834}$$

Equivalently, the particles move geodesically with respect to

$$\widetilde{g}^{(\rm scaf)}_{\mu\nu} = A^2(\phi) g_{\mu\nu}, \tag{835}$$

where

$$m_{\rm scaf}(\phi) = m_{\rm scaf,0} A(\phi). \tag{836}$$

The solved field domain must satisfy

$$\boxed{0 < A(\phi) < \infty, \qquad 0 < m_{\rm scaf}(\phi) < \infty, \qquad |\beta(\phi)| < \infty.} \tag{837}$$

These are domain conditions for the selected branch, not observational claims about a particular dark-matter particle.

The dimensionless coupling is

$$\beta(\phi) = M_{\rm Pl}\frac{\mathrm{d}\ln A}{\mathrm{d}\phi} = M_{\rm Pl}\frac{\mathrm{d}\ln m_{\rm scaf}}{\mathrm{d}\phi}. \tag{838}$$

The cold-particle stress–energy tensor is

$$T^{\mu\nu}_{\rm scaf} = \rho_{\rm scaf} u^\mu_{\rm scaf} u^\nu_{\rm scaf} \tag{839}$$

in the ideal pressureless limit.

Its trace is

$$T_{\rm scaf} = -\rho_{\rm scaf} \tag{840}$$

for the metric signature adopted in Paper III.

The ideal cold limit has

$$\boxed{p_{\rm scaf} = 0, \qquad c^2_{s,\rm scaf} = 0, \qquad \pi^{\mu\nu}_{\rm scaf} = 0.} \tag{841}$$

Corrections from velocity dispersion, self-interaction, elasticity, or memory are introduced separately.

167 Number Conservation and Mass Evolution

Define the scaffold number current

$$J^\mu_{\rm scaf} = n_{\rm scaf} u^\mu_{\rm scaf}. \tag{842}$$

In the minimal particle model, scaffold particle number is conserved:

$$\boxed{\nabla_\mu J^\mu_{\rm scaf} = 0.} \tag{843}$$

The energy density is

$$\rho_{\rm scaf} = m_{\rm scaf}(\phi) n_{\rm scaf}. \tag{844}$$

For a homogeneous FLRW background,

$$\dot{n}_{\rm scaf} + 3H n_{\rm scaf} = 0, \tag{845}$$

so

$$n_{\rm scaf} \propto a^{-3}. \tag{846}$$

The particle mass evolves as

$$\frac{\dot{m}_{\rm scaf}}{m_{\rm scaf}} = \frac{\beta}{M_{\rm Pl}} \dot{\phi}. \tag{847}$$

Therefore,

$$\boxed{\dot{\rho}_{\rm scaf} + 3H\rho_{\rm scaf} = \frac{\beta}{M_{\rm Pl}}\rho_{\rm scaf}\dot{\phi}.} \tag{848}$$

The density scales as

$$\rho_{\rm scaf}(a) = \rho_{\rm scaf,ini}\left(\frac{a_{\rm ini}}{a}\right)^3 \frac{A[\phi(a)]}{A(\phi_{\rm ini})}. \tag{849}$$

Thus, the departures from cold-dark-matter background scaling are completely linked to the scalar trajectory and conformal coupling.

Define the cumulative scaffold mass drift

$$\Delta_{\rm mass}(a_1, a_2) = \ln\left[\frac{m_{\rm scaf}(a_2)}{m_{\rm scaf}(a_1)}\right]. \tag{850}$$

For constant β,

$$\Delta_{\rm mass} = \frac{\beta}{M_{\rm Pl}}\left[\phi(a_2) - \phi(a_1)\right]. \tag{851}$$

The same quantity affects the background density, particle inertia, and perturbation evolution.

168 Perfect-Fluid and Kinetic Limits

The pressureless particle description is the cold limit of a more general phase-space distribution

$$f_{\rm scaf}(x^\mu, p^\nu). \tag{852}$$

The stress–energy tensor is

$$T^{\mu\nu}_{\rm scaf} = \int \frac{\mathrm{d}^3 p}{\sqrt{-g}\, p^0} p^\mu p^\nu f_{\rm scaf}. \tag{853}$$

The cold limit corresponds to a narrow momentum distribution.

A small velocity dispersion generates an effective pressure

$$p_{\rm scaf} \sim \frac{1}{3}\rho_{\rm scaf}\left\langle v^2_{\rm scaf}\right\rangle. \tag{854}$$

The associated effective sound speed is approximately

$$c_{s,\mathrm{scaf}}^2 \sim \frac{1}{3}\left\langle v_{\mathrm{scaf}}^2\right\rangle \tag{855}$$

in the nonrelativistic regime.

A fluid description is valid only when the relevant hierarchy can be closed.

The minimal perfect-fluid scaffold is

$$T_{\mathrm{scaf}}^{\mu\nu} = \left(\rho_{\mathrm{scaf}} + p_{\mathrm{scaf}}\right) u_{\mathrm{scaf}}^{\mu} u_{\mathrm{scaf}}^{\nu} + p_{\mathrm{scaf}} g^{\mu\nu}. \tag{856}$$

An imperfect or memory-bearing scaffold requires additional variables,

$$\left\{q_{\mathrm{scaf}}^{\mu}, \pi_{\mathrm{scaf}}^{\mu\nu}, \mathcal{W}_{\mu\nu}, \ldots\right\}. \tag{857}$$

Those additions must be accompanied by constitutive and evolution laws.

169 Linear Perturbation Variables

Use conformal Newtonian gauge,

$$\mathrm{d}s^2 = a^2(\eta)\left[-\left(1+2\Psi\right)\mathrm{d}\eta^2 + \left(1-2\Phi\right)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]. \tag{858}$$

The scaffold density contrast is

$$\delta_{\mathrm{scaf}} = \frac{\delta\rho_{\mathrm{scaf}}}{\rho_{\mathrm{scaf}}}. \tag{859}$$

The number-density contrast is

$$\delta_n = \frac{\delta n_{\mathrm{scaf}}}{n_{\mathrm{scaf}}}. \tag{860}$$

The scalar velocity divergence is

$$\theta_{\mathrm{scaf}} = \mathrm{i}k_j v_{\mathrm{scaf}}^j. \tag{861}$$

Because the particle mass depends on ϕ,

$$\delta_{\mathrm{scaf}} = \delta_n + \frac{\beta}{M_{\mathrm{Pl}}}\delta\phi \tag{862}$$

for constant β.

For a field-dependent coupling, the linear mass perturbation remains

$$\frac{\delta m_{\rm scaf}}{m_{\rm scaf}} = \frac{\beta(\bar{\phi})}{M_{\rm Pl}}\delta\phi, \tag{863}$$

while derivatives of β enter its time evolution and the effective scalar mass.

The distinction between δ_n and $\delta_{\rm scaf}$ is necessary whenever scaffold mass evolves.

170 Coupled Linear Perturbation Equations

Perturbation basis. The coupled linear system is checked against gauge-consistent cosmological perturbation theory and interacting-dark-sector stability analyses (Bardeen, 1980; Ma and Bertschinger, 1995; Malik and Wands, 2009; Väliviita et al., 2008).

The displayed equations in this section use the selected GT1 coupling sign, conformal Newtonian gauge, the Fourier convention $\nabla^2 \to -k^2$, an ideal pressureless scaffold, constant β, and no independent memory stress. Variable coupling, finite pressure, anisotropic stress, or memory must be rederived from the same action rather than appended term by term.

Particle-number conservation gives

$$\boxed{\delta_n' = -\theta_{\rm scaf} + 3\Phi'.} \tag{864}$$

For constant β,

$$\boxed{\delta_{\rm scaf}' = -\theta_{\rm scaf} + 3\Phi' + \frac{\beta}{M_{\rm Pl}}\delta\phi'.} \tag{865}$$

The scaffold Euler equation is

$$\boxed{\theta_{\rm scaf}' = -\left(\mathcal{H} + \frac{\beta}{M_{\rm Pl}}\phi'\right)\theta_{\rm scaf} + k^2\left(\Psi + \frac{\beta}{M_{\rm Pl}}\delta\phi\right)} \tag{866}$$

for the adopted coupling convention and ideal cold-particle limit.

The coupling modifies both:

- the effective velocity-damping term; and
- the force potential.

A more general scaffold fluid adds pressure and anisotropic-stress terms:

$$
\begin{aligned}
\theta'_{\mathrm{scaf}} = & -\left[\mathcal{H}\left(1-3c^2_{s,\mathrm{scaf}}\right)+\frac{\beta}{M_{\mathrm{Pl}}}\phi'\right]\theta_{\mathrm{scaf}} \\
& + \frac{c^2_{s,\mathrm{scaf}}}{1+w_{\mathrm{scaf}}}k^2\delta_{\mathrm{scaf}} - k^2\sigma_{\mathrm{scaf}} \\
& + k^2\left(\Psi+\frac{\beta}{M_{\mathrm{Pl}}}\delta\phi\right)+\mathcal{S}_{\theta,\mathrm{int}}.
\end{aligned}
\tag{867}
$$

The complete form for variable β, finite pressure, and general momentum transfer must be derived directly from the chosen action.

171 Scalar Perturbation Response

The chasing-field perturbation satisfies an equation of the form

$$
\delta\phi'' + 2\mathcal{H}\delta\phi' + \left(k^2 + a^2 m^2_{\mathrm{eff}}\right)\delta\phi = \mathcal{S}_\phi\left[\delta_{\mathrm{scaf}}, \theta_{\mathrm{scaf}}, \Phi, \Psi\right]. \tag{868}
$$

For a conserved scaffold number density, the effective potential is locally

$$
V_{\mathrm{eff}}\left(\phi, n_{\mathrm{scaf}}\right) = V(\phi) + m_{\mathrm{scaf}}(\phi) n_{\mathrm{scaf}}. \tag{869}
$$

Its effective mass at fixed scaffold number density is

$$
m^2_{\mathrm{eff}} \equiv \left.\frac{\partial^2 V_{\mathrm{eff}}}{\partial\phi^2}\right|_{n_{\mathrm{scaf}}}. \tag{870}
$$

For the selected mass relation this gives

$$
\boxed{m^2_{\mathrm{eff}} = V_{,\phi\phi} + \frac{\rho_{\mathrm{scaf}}}{M^2_{\mathrm{Pl}}}\left[\beta^2 + M_{\mathrm{Pl}}\beta_{,\phi}\right].} \tag{871}
$$

In the subhorizon quasistatic limit,

$$
k^2 \gg \mathcal{H}^2, \qquad |\delta\phi''|, \quad |\mathcal{H}\delta\phi'| \ll k^2|\delta\phi|, \tag{872}
$$

one obtains

$$
\boxed{\left(k^2 + a^2 m^2_{\mathrm{eff}}\right)\delta\phi \simeq -\frac{a^2\beta}{M_{\mathrm{Pl}}}\rho_{\mathrm{scaf}}\delta_{\mathrm{scaf}}.} \tag{873}
$$

The scalar perturbation is therefore linked to the scaffold overdensity.

The quasistatic expression must not be used near horizon crossing, during rapid transitions, or when the scalar time derivatives become important.

172 Enhanced Scaffold Self-Attraction

Substituting equation (873) into the scaffold force gives the effective self-attraction

$$\boxed{\mu_{\rm scaf}(k,a) = 1 + 2\beta^2 \frac{k^2}{k^2 + a^2 m_{\rm eff}^2}.} \tag{874}$$

For compact notation define the linear scaffold self-response kernel

$$\mathcal{K}^{(\rm QS)}_{\rm scafscaf}(k,a) = G\mu_{\rm scaf}(k,a). \tag{875}$$

This is not a new universal Newton constant and is not the coupling seen by photons or by two directly uncoupled visible test bodies. It is a linear, unscreened, quasistatic response of scaffold motion within the selected branch.

The long-range limit is

$$k^2 \gg a^2 m_{\rm eff}^2, \tag{876}$$

for which

$$\mu_{\rm scaf} \longrightarrow 1 + 2\beta^2. \tag{877}$$

The short-range or heavy-scalar limit is

$$k^2 \ll a^2 m_{\rm eff}^2, \tag{878}$$

for which

$$\mu_{\rm scaf} \longrightarrow 1. \tag{879}$$

The fifth force does not directly enhance visible–visible attraction.

It changes the scaffold trajectory and therefore the later metric source seen by visible matter and photons.

The scalar range is

$$\lambda_\phi = m_{\rm eff}^{-1}. \tag{880}$$

A viable model must use the same $m_{\rm eff}$ in local, halo, cluster, and cosmological calculations.

173 Visible and Scaffold Growth Equations

Visible pressureless matter responds directly only to the metric potential in the minimum GT1 action. It may still respond indirectly to the changed scaffold and scalar stress–energy through the metric.

The reduced equations below are subhorizon quasistatic diagnostics. A numerical result must use the full coupled Einstein–Boltzmann system, and the remainder $\mathcal{S}_{\rm time}$ may not be treated as an adjustable source.

In the quasistatic subhorizon regime, the visible density contrast satisfies approximately

$$\boxed{\delta''_{\rm vis} + \mathcal{H}\delta'_{\rm vis} - 4\pi G a^2 \left(\rho_{\rm vis}\delta_{\rm vis} + \rho_{\rm scaf}\delta_{\rm scaf}\right) \simeq 0.} \tag{881}$$

The scaffold contrast satisfies

$$\delta''_{\rm scaf} + \left(\mathcal{H} + \frac{\beta}{M_{\rm Pl}}\phi'\right)\delta'_{\rm scaf} - 4\pi G a^2 \Big[\rho_{\rm vis}\delta_{\rm vis} + \mu_{\rm scaf}(k,a)\rho_{\rm scaf}\delta_{\rm scaf}\Big] = \mathcal{S}_{\rm time}, \tag{882}$$

where $\mathcal{S}_{\rm time}$ collects terms neglected in the slowly varying, quasistatic reduction.

The scaffold therefore differs from visible matter through:

$$\boxed{\text{mass-exchange drag} + \text{scalar self-attraction} + \text{possible scalar time-derivative sources.}} \tag{883}$$

Define the separate growth rates

$$f_{\rm vis} = \frac{{\rm d}\ln\delta_{\rm vis}}{{\rm d}\ln a}, \qquad f_{\rm scaf} = \frac{{\rm d}\ln\delta_{\rm scaf}}{{\rm d}\ln a}. \tag{884}$$

Their difference is

$$\Delta f_{\rm scaf\,vis} = f_{\rm scaf} - f_{\rm vis}. \tag{885}$$

A nonzero differential growth rate is a characteristic consequence of the dark-only coupling.

174 Differential Bias and Relative Modes

Define the scaffold-to-visible density bias

$$b_{\mathrm{scaf/vis}}(k,a) = \frac{\delta_{\mathrm{scaf}}(k,a)}{\delta_{\mathrm{vis}}(k,a)}. \tag{886}$$

In the uncoupled cold limit,

$$b_{\mathrm{scaf/vis}} \longrightarrow 1 \tag{887}$$

for adiabatic growing-mode initial conditions on sufficiently large linear scales.

Define the relative density mode

$$S_{\mathrm{scaf\,vis}} = \delta_{\mathrm{scaf}} - \delta_{\mathrm{vis}}. \tag{888}$$

Define the relative velocity mode

$$V_{\mathrm{scaf\,vis}} = \theta_{\mathrm{scaf}} - \theta_{\mathrm{vis}}. \tag{889}$$

The scalar force can generate

$$S_{\mathrm{scaf\,vis}} \neq 0, \qquad V_{\mathrm{scaf\,vis}} \neq 0 \tag{890}$$

even when the central output was initially adiabatic.

The generated relative modes must remain compatible with:

- galaxy and matter clustering;
- lensing;
- peculiar velocities;
- cluster dynamics;
- redshift-space distortions; and
- the microwave-background transfer.

A relative mode generated after recombination is not identical to a primordial isocurvature mode.

Its origin and redshift dependence must be tracked explicitly.

175 Sound Speed, Jeans Scale, and Free Streaming

A finite scaffold sound speed adds pressure support.

For a generalized fluid, define

$$c_{s,\mathrm{scaf}}^2 = \left.\frac{\delta p_{\mathrm{scaf}}}{\delta \rho_{\mathrm{scaf}}}\right|_{\mathrm{rest}}. \tag{891}$$

For positive, slowly varying $c_{s,\mathrm{scaf}}^2$ in a fluid regime and positive quasistatic response, a schematic effective Jeans wavenumber is

$$k_J^2(a) \sim \frac{4\pi G a^2 \rho_{\mathrm{scaf}} \mu_{\mathrm{scaf}}}{c_{s,\mathrm{scaf}}^2}. \tag{892}$$

Modes with

$$k \ll k_J \tag{893}$$

can grow gravitationally.

Modes with

$$k \gg k_J \tag{894}$$

are pressure suppressed in the fluid approximation.

A kinetic scaffold also possesses a free-streaming scale.

A schematic comoving free-streaming distance is

$$\lambda_{\mathrm{fs}}(a) = \int^a \frac{v_{\mathrm{rms}}(\tilde{a})}{\tilde{a}^2 H(\tilde{a})} \mathrm{d}\tilde{a}. \tag{895}$$

The corresponding wavenumber is

$$k_{\mathrm{fs}} \sim \frac{2\pi}{\lambda_{\mathrm{fs}}}. \tag{896}$$

Cold scaffolding requires the relevant observational modes to satisfy

$$k \ll k_J, \qquad k \ll k_{\mathrm{fs}} \tag{897}$$

through the epochs in which cold-dark-matter-like clustering is needed. The displayed k_J is an implicit diagnostic when μ_{scaf} is scale dependent. Jeans support and kinetic free streaming are distinct closures and must not be double counted.

176 Anisotropic Stress and Gravitational Slip

For scalar perturbations, define the scaffold anisotropic-stress potential

$$\sigma_{\rm scaf}. \tag{898}$$

The metric slip equation is

$$\boxed{k^2\,(\Phi - \Psi) = 12\pi G a^2 \sum_i (\rho_i + p_i)\,\sigma_i.} \tag{899}$$

An ideal cold-particle scaffold has

$$\sigma_{\rm scaf} \approx 0. \tag{900}$$

A canonical chasing scalar also has no intrinsic linear scalar anisotropic stress.

Therefore,

$$\Phi \approx \Psi \tag{901}$$

after accounting for radiation and neutrino anisotropic stress.

An elastic, vector, or memory-bearing scaffold may instead generate

$$\sigma_{\rm scaf} \neq 0. \tag{902}$$

Its anisotropic stress must then be propagated simultaneously into:

- gravitational lensing;
- microwave-background evolution;
- matter growth;
- gravitational-wave sourcing; and
- statistical anisotropy.

A memory mechanism that alters density growth but omits its anisotropic stress is generally incomplete.

177 Metric Potentials and Lensing

Benchmark literature. Photon lensing remains a metric-potential observable and is not assigned the scaffold fifth-force kernel by hand (Bartelmann and Schneider, 2001; Bellini and Sawicki, 2014).

The metric potentials are sourced by the complete stress–energy system.

A gauge-invariant Poisson relation may be represented as

$$k^2\Phi = -4\pi G a^2 \sum_i \rho_i \Delta_i, \tag{903}$$

where

$$\Delta_i \tag{904}$$

is the comoving density perturbation of sector i.

Photons respond to the Weyl potential

$$\Phi_W = \frac{\Phi + \Psi}{2}. \tag{905}$$

The scaffold scalar force does not directly appear in the photon geodesic equation.

It modifies lensing indirectly by changing:

$$\{\delta_{\mathrm{scaf}}, \rho_{\mathrm{scaf}}, \Phi, \Psi, \text{structure formation}\}. \tag{906}$$

The lensing convergence is

$$\kappa \propto \int_0^{\chi_s} \mathrm{d}\chi\, W_{\mathrm{lens}}(\chi, \chi_s)\, \nabla_\perp^2 (\Phi + \Psi). \tag{907}$$

The key GT1 consistency condition is

$$\boxed{\begin{array}{l}\text{the scaffold distribution inferred from its enhanced dynamics} \\ \text{must source the metric lensing field measured by photons.}\end{array}} \tag{908}$$

The scalar force may not be counted as additional lensing mass unless its stress–energy itself contributes to the metric. In particular, μ_{scaf} cannot be inserted directly into a photon-lensing Poisson equation. Scaffold dynamics and photon lensing must be propagated through one common stress–energy solution.

178 Total Matter Perturbation

Define the total pressureless-matter density

$$\rho_m = \rho_{\text{vis}} + \rho_{\text{scaf}}. \tag{909}$$

Define the visible and scaffold density weights

$$\omega_{\text{vis}}^{(m)} = \frac{\rho_{\text{vis}}}{\rho_m}, \qquad \omega_{\text{scaf}}^{(m)} = \frac{\rho_{\text{scaf}}}{\rho_m}. \tag{910}$$

The symbol f_i remains reserved for logarithmic growth rates in the perturbation and observation sections.

The total matter contrast is

$$\boxed{\delta_m = \omega_{\text{vis}}^{(m)}\delta_{\text{vis}} + \omega_{\text{scaf}}^{(m)}\delta_{\text{scaf}}.} \tag{911}$$

Because scaffold mass may evolve,

$$\omega_{\text{vis}}^{(m)} = \omega_{\text{vis}}^{(m)}(a), \qquad \omega_{\text{scaf}}^{(m)} = \omega_{\text{scaf}}^{(m)}(a). \tag{912}$$

The total matter growth rate is

$$f_m = \frac{\mathrm{d}\ln\delta_m}{\mathrm{d}\ln a}. \tag{913}$$

The total contrast does not generally satisfy a closed one-component growth equation because

$$\delta_{\text{vis}} \neq \delta_{\text{scaf}} \tag{914}$$

and the fractions evolve.

A one-fluid approximation is allowed only after verifying that the relative modes remain negligible.

179 Matter Transfer Matrix and Power Spectra

Benchmark literature. Transfer matrices and power spectra are compared with standard Boltzmann and linear-structure methods (Ma and Bertschinger, 1995; Seljak and Zaldarriaga, 1996; Lewis et al., 2000; Blas et al., 2011).

Let the primordial mode vector at the central output be

$$\boldsymbol{Z}_{C^+} = (\mathcal{R}, S_A, \mathcal{W}_A, \ldots)^{\mathsf{T}} . \tag{915}$$

The visible and scaffold density contrasts are

$$\delta_{\text{vis}}(k,z) = \sum_A T_{\text{vis}A}(k,z) Z_A(k), \tag{916}$$

$$\delta_{\text{scaf}}(k,z) = \sum_A T_{\text{scaf}A}(k,z) Z_A(k). \tag{917}$$

The transfer functions must be calculated using the same central covariance defined in Paper II.

The auto- and cross-power spectra are

$$P_{\text{visvis}} = \left\langle \delta_{\text{vis}} \delta^*_{\text{vis}} \right\rangle , \tag{918}$$

$$P_{\text{scafscaf}} = \left\langle \delta_{\text{scaf}} \delta^*_{\text{scaf}} \right\rangle , \tag{919}$$

$$P_{\text{visscaf}} = \left\langle \delta_{\text{vis}} \delta^*_{\text{scaf}} \right\rangle . \tag{920}$$

The total matter power is

$$\boxed{P_m = \left(\omega^{(m)}_{\text{vis}}\right)^2 P_{\text{visvis}} + \left(\omega^{(m)}_{\text{scaf}}\right)^2 P_{\text{scafscaf}} + 2\omega^{(m)}_{\text{vis}} \omega^{(m)}_{\text{scaf}} P_{\text{visscaf}}.} \tag{921}$$

The visible–scaffold correlation coefficient is

$$r_{\text{visscaf}}(k,z) = \frac{P_{\text{visscaf}}}{\sqrt{P_{\text{visvis}} P_{\text{scafscaf}}}}. \tag{922}$$

Adiabatic cold-dark-matter-like behavior gives

$$r_{\text{visscaf}} \approx 1. \tag{923}$$

An inherited scaffold component may produce a controlled departure.

180 Equality and Acoustic-Era Requirements

The equality calculation in this section is a third-leg observational benchmark. It does not determine when matter or particle-like excitations first originated in the complete Triple Bounce history.

Matter–radiation equality is defined by

$$\rho_{\rm rad}(a_{\rm eq}) = \rho_{\rm vis}(a_{\rm eq}) + \rho_{\rm scaf}(a_{\rm eq}). \tag{924}$$

Because

$$\rho_{\rm scaf} \propto a^{-3} A[\phi(a)], \tag{925}$$

the equality epoch depends on the scalar trajectory.

The fractional scaffold-mass change before recombination is

$$\Delta_{\rm rec} = \ln \left[\frac{A(\phi_*)}{A(\phi_{\rm ini})} \right]. \tag{926}$$

A nearly standard early-third-leg scaffold requires

$$|\Delta_{\rm rec}| \ll 1 \tag{927}$$

unless the full acoustic calculation demonstrates an acceptable nonstandard alternative.

The scaffold affects the microwave background through:

- the equality scale;
- the time evolution of Φ and Ψ;
- early integrated gravitational effects;
- acoustic driving;
- the sound-horizon inference;
- lensing; and
- the central-output mode decomposition.

The coupling must therefore be included in the Einstein–Boltzmann system rather than applied only to late structure growth.

181 Nonlinear Collapse

Benchmark literature. Nonlinear collapse and halo formation require comparison with standard structure-formation calculations and simulations (Peebles, 1980; Springel et al., 2005).

Linear perturbations do not determine halo formation by themselves.

Consider a spherical scaffold overdensity with physical radius

$$R_{\rm scaf}(t). \tag{928}$$

A schematic shell equation is

$$\begin{aligned}\ddot{R}_{\rm scaf} = & -\frac{GM_{\rm metric}(< R_{\rm scaf})}{R_{\rm scaf}^2} \\ & -2\beta^2\mathcal{Y}\left(m_{\rm eff}R_{\rm scaf}\right)\frac{GM_{\rm scaf}(< R_{\rm scaf})}{R_{\rm scaf}^2}+\mathcal{A}_{\rm bg},\end{aligned} \tag{929}$$

where:

- $M_{\rm metric}$ contains all metric-sourcing mass;
- $\mathcal{Y}$ is the finite-range scalar-force kernel; and
- $\mathcal{A}_{\rm bg}$ contains the homogeneous expansion contribution.

A visible shell responds only to the metric term:

$$\ddot{R}_{\rm vis} = -\frac{GM_{\rm metric}(< R_{\rm vis})}{R_{\rm vis}^2}+\mathcal{A}_{\rm bg}. \tag{930}$$

The separate forces can produce segregation between scaffold and visible matter during nonlinear collapse.

Define the linear collapse threshold

$$\delta_{\rm coll}\left(z,\beta,m_{\rm eff}\right), \tag{931}$$

and the virial overdensity

$$\Delta_{\rm vir}\left(z,\beta,m_{\rm eff}\right). \tag{932}$$

Both must be recalculated in the coupled model.

182 Halo Formation and Internal Structure

The enhanced scaffold force may modify:

$$\left\{\begin{matrix}\text{collapse time}, & \text{halo abundance}, & \text{concentration}, & \text{substructure}, \\ \text{merger history}, & \text{velocity dispersion}, & \text{visible–scaffold offset} & \end{matrix}\right\}. \tag{933}$$

The scaffold halo density may be represented as

$$\rho_{\text{scaf}} = \rho_{\text{scaf}}\left(r; M, z, \beta, m_{\text{eff}}\right). \tag{934}$$

Visible tracers orbit in the metric potential sourced by the complete halo.

Scaffold particles additionally experience the scalar potential.

Therefore, the velocity-dispersion tensors may differ:

$$\boldsymbol{\sigma}_{v,\text{scaf}} \neq \boldsymbol{\sigma}_{v,\text{vis}}. \tag{935}$$

The mass inferred through visible motion is compared with the mass inferred through:

$$\{\text{scaffold dynamics}, \text{weak lensing}, \text{strong lensing}, \text{gas dynamics}\}. \tag{936}$$

A successful halo model must not rely on treating the scalar fifth force as metric lensing mass.

The nonlinear predictions require particle or phase-space simulations once the linear theory is validated.

183 Third-Leg Cosmic-Web Formation

Benchmark literature. Any inherited-web claim must be distinguished from ordinary gravitational cosmic-web formation (Zel'dovich, 1970; Bond et al., 1996; Cautun et al., 2014; Libeskind et al., 2018).

In conventional structure formation, the cosmic web develops from the gravitational evolution of primordial density and tidal perturbations.

Define the tidal tensor

$$\mathcal{T}_{ij} = \partial_i \partial_j \Phi_W. \tag{937}$$

Web environments may be classified through the eigenvalues

$$\lambda_1^{\mathcal{T}}, \qquad \lambda_2^{\mathcal{T}}, \qquad \lambda_3^{\mathcal{T}}. \tag{938}$$

A scaffold-only force additionally responds to the scalar Hessian,

$$\mathcal{T}_{ij}^{(\phi)} = \frac{\beta}{M_{\rm Pl}} \partial_i \partial_j \delta\phi. \tag{939}$$

The effective scaffold tidal tensor is

$$\mathcal{T}_{ij}^{(\rm scaf)} = \partial_i \partial_j \Psi + \mathcal{T}_{ij}^{(\phi)}. \tag{940}$$

The visible web responds only to the metric part.

The model may therefore generate small differences between:

$$\text{the scaffold dynamical web} \tag{941}$$

and

$$\text{the visible and lensing web.} \tag{942}$$

Such differences must remain compatible with the observed spatial coincidence of luminous and gravitating structure.

184 Why a Perfect Cold Fluid Does Not Encode Historical Memory

A perfect cold fluid is locally specified by

$$\{\rho_{\rm scaf}, u^\mu_{\rm scaf}\}. \tag{943}$$

These variables determine the present density and flow.

They do not by themselves identify:

- the path by which the fluid reached its present state;
- a reference configuration;
- first-leg route ancestry;
- persistent strain;
- junction identity;

- topological memory; or
- a record of two prior directional passes.

Two histories can produce the same

$$\rho_{\rm scaf}, \qquad u^{\mu}_{\rm scaf} \tag{944}$$

on one hypersurface.

They are then indistinguishable within the perfect-fluid state description.

Therefore,

$$\boxed{\text{ordinary cold-fluid scaffolding does not automatically constitute cross-leg historical memory.}} \tag{945}$$

A genuine memory claim requires at least one additional state variable or nonlocal functional with a defined evolution law.

185 Elastic-Memory Scaffold Extension

Introduce three comoving internal-coordinate fields,

$$\varphi^I, \qquad I = 1, 2, 3. \tag{946}$$

Define

$$B^{IJ} = g^{\mu\nu}\partial_\mu\varphi^I\partial_\nu\varphi^J. \tag{947}$$

A candidate memory-capable scaffold action template may be written as

$$S^{\rm (mem)}_{\rm scaf} = \int {\rm d}^4x\,\sqrt{-g}\,F\left(B^{IJ}, \phi, \Xi_E\right), \tag{948}$$

where

$$\Xi_E \tag{949}$$

is a placeholder for a covariant order parameter associated with the E history or reversal state. Neither Ξ_E nor the function F is selected by the minimum GT1 action. This extension becomes a physical model only after its stress–energy tensor, degrees of freedom, constraint structure, and stability conditions are derived.

A homogeneous isotropic background may use

$$\varphi^I = \alpha x^I \tag{950}$$

together with internal rotational symmetry.

The elastic medium may possess a bulk modulus

$$K_{\mathrm{scaf}}, \tag{951}$$

and shear modulus

$$\mu^{(\mathrm{sh})}_{\mathrm{scaf}}. \tag{952}$$

Its longitudinal and transverse propagation speeds are schematically

$$c_L^2 = \frac{K_{\mathrm{scaf}} + \frac{4}{3}\mu^{(\mathrm{sh})}_{\mathrm{scaf}}}{\rho_{\mathrm{scaf}} + p_{\mathrm{scaf}}}, \tag{953}$$

and

$$c_T^2 = \frac{\mu^{(\mathrm{sh})}_{\mathrm{scaf}}}{\rho_{\mathrm{scaf}} + p_{\mathrm{scaf}}}. \tag{954}$$

A cold memory scaffold requires both speeds to remain sufficiently small on the structure-forming scales or to become relevant only in a controlled hidden-leg regime.

186 Covariant Memory Tensor

Let the scaffold four-velocity be

$$u^{\mu}_{\mathrm{scaf}}. \tag{955}$$

Define the spatial projector

$$h^{(\mathrm{scaf})}_{\mu\nu} = g_{\mu\nu} + u_{\mathrm{scaf},\mu} u_{\mathrm{scaf},\nu}. \tag{956}$$

Introduce a symmetric scaffold-memory tensor

$$\mathcal{W}_{\mu\nu}, \tag{957}$$

satisfying

$$u^{\mu}_{\rm scaf}\mathcal{W}_{\mu\nu} = 0. \tag{958}$$

Decompose it into trace and trace-free parts:

$$\mathcal{W}_{\mu\nu} = \frac{1}{3}h^{(\rm scaf)}_{\mu\nu}\mathcal{W}_{\rm tr} + \mathcal{W}_{\langle\mu\nu\rangle}. \tag{959}$$

Here,

$$\mathcal{W}_{\rm tr} = h^{\mu\nu}_{\rm scaf}\mathcal{W}_{\mu\nu}, \tag{960}$$

and

$$h^{\mu\nu}_{\rm scaf}\mathcal{W}_{\langle\mu\nu\rangle} = 0. \tag{961}$$

The scalar trace may encode isotropic compression history.

The trace-free part may encode directional strain, pathway orientation, or shear ancestry.

The memory tensor must not be identified automatically with the metric tidal tensor.

It is an internal scaffold state whose stress–energy effects must be derived from the action.

187 Memory Evolution and Relaxation

A local covariant memory law may be written as

$$\boxed{h_{\mu}{}^{\rho\,(\rm scaf)}h_{\nu}{}^{\sigma\,(\rm scaf)}u^{\alpha}_{\rm scaf}\nabla_{\alpha}\mathcal{W}_{\langle\rho\sigma\rangle} + \Gamma_{\rm mem}\mathcal{W}_{\langle\mu\nu\rangle} = \mathcal{S}^{(E)}_{\langle\mu\nu\rangle} + \mathcal{S}^{(\rm grav)}_{\langle\mu\nu\rangle} + \mathcal{S}^{(\rm int)}_{\langle\mu\nu\rangle}.} \tag{962}$$

The terms represent:

- classical relaxation or information loss through $\Gamma_{\rm mem}$;
- source imprint from the E state;
- ordinary gravitational deformation; and
- interaction-driven conversion.

The trace may satisfy

$$u^{\alpha}_{\mathrm{scaf}}\nabla_{\alpha}\mathcal{W}_{\mathrm{tr}}+\Gamma_{\mathrm{tr}}\mathcal{W}_{\mathrm{tr}}=\mathcal{S}_{\mathrm{tr}}. \tag{963}$$

A passive retained-memory interval has

$$\mathcal{S}_{\mu\nu}\approx 0, \tag{964}$$

and therefore

$$\mathcal{W}_{\mu\nu}(t_b)\approx\mathcal{W}_{\mu\nu}(t_a)\exp\left[-\int_{t_a}^{t_b}\Gamma_{\mathrm{mem}}(t)\mathrm{d}t\right]. \tag{965}$$

Substantial retention requires

$$\int_{t_a}^{t_b}\Gamma_{\mathrm{mem}}\mathrm{d}t\lesssim 1 \tag{966}$$

for the modes claimed to survive.

188 Reversal Transfer of Scaffold Memory

At transition j, the linearized memory state may be represented through the bookkeeping map

$$\boxed{\mathcal{W}^{+}_{\mu\nu,j}(k)=T^{(j)}_{\mathcal{W}\mathcal{W}}(k)\mathcal{W}^{-}_{\mu\nu,j}(k)+N^{(j)}_{\mu\nu}(k).} \tag{967}$$

Here $T^{(j)}_{\mathcal{W}\mathcal{W}}$ denotes a linear operator on the decomposed scalar, vector, and tensor memory state, not an independently fitted scalar amplitude. The map is not a derived reversal solution until it follows from one finite-width action and matching calculation.

The outer transition gives

$$\mathcal{W}^{+}_{\mathrm{O}}=T^{(\mathrm{O})}_{\mathcal{W}\mathcal{W}}\mathcal{W}^{-}_{\mathrm{O}}+N^{(\mathrm{O})}_{\mathcal{W}}. \tag{968}$$

The central transition gives

$$\mathcal{W}^{+}_{\mathrm{C}}=T^{(\mathrm{C})}_{\mathcal{W}\mathcal{W}}\mathcal{W}^{-}_{\mathrm{C}}+N^{(\mathrm{C})}_{\mathcal{W}}. \tag{969}$$

The complete retained first-leg memory is

$$\mathcal{W}_3(k) = T^{(\mathrm{C})}_{\mathcal{W}\mathcal{W}}(k) U^{(2)}_{\mathcal{W}}(k) T^{(\mathrm{O})}_{\mathcal{W}\mathcal{W}}(k) U^{(1)}_{\mathcal{W}}(k) \mathcal{W}_1(k) + N_{\mathcal{W},\mathrm{tot}}(k). \tag{970}$$

Here,

$$U^{(1)}_{\mathcal{W}}, \qquad U^{(2)}_{\mathcal{W}} \tag{971}$$

are the memory propagators through the first and second legs.

A valid historical-memory claim requires the ordered product to be derived from one theory.

189 Scale-Dependent Retention Kernel

After decomposing the memory state into a specified propagation eigenmode, define the corresponding cumulative retention diagnostic

$$\mathcal{E}^{(A)}_{\mathrm{mem}}(k) = \left| T^{(\mathrm{C})}_{\mathcal{W}\mathcal{W}}(k) T^{(\mathrm{O})}_{\mathcal{W}\mathcal{W}}(k) \right| \times \exp\left[-\int_{\mathcal{I}_1 \cup \mathcal{I}_2} \Gamma_{\mathrm{mem}}(k,t) \mathrm{d}t \right]. \tag{972}$$

The inherited third-leg memory power is then

$$\mathcal{P}^{(3)}_{\mathcal{W},A}(k) = \left[\mathcal{E}^{(A)}_{\mathrm{mem}}(k) \right]^2 \mathcal{P}^{(1)}_{\mathcal{W},A}(k) + \mathcal{P}_{N_{\mathcal{W}}}(k). \tag{973}$$

A large-scale retention and small-scale relaxation pattern satisfies

$$\mathcal{E}^{(A)}_{\mathrm{mem}}(k) \to \mathcal{E}^{(A)}_0 \neq 0 \quad \text{as} \quad k \to 0, \tag{974}$$

and

$$\mathcal{E}^{(A)}_{\mathrm{mem}}(k) \to 0 \quad \text{as} \quad k \to \infty. \tag{975}$$

A provisional retention scale is

$$k_{\mathrm{mem}}. \tag{976}$$

The shape around that scale must follow from

$$\left\{\Gamma_{\rm mem}, T^{(\rm O)}_{\mathcal{W}\mathcal{W}}, T^{(\rm C)}_{\mathcal{W}\mathcal{W}}, c_L, c_T\right\}. \tag{977}$$

It may not be fitted independently for each observable.

190 Statistical Homogeneity and Isotropy of the Scaffold

A memory-bearing scaffold must preserve the observed near homogeneity and isotropy of the cosmological background.

The homogeneous expectation value of the trace-free memory must satisfy

$$\boxed{\left\langle \mathcal{W}_{\langle ij\rangle} \right\rangle = 0.} \tag{978}$$

Its two-point function may be statistically isotropic:

$$\langle \mathcal{W}_{ij}(\boldsymbol{k})\mathcal{W}^{*}_{mn}(\boldsymbol{k}')\rangle = (2\pi)^3\delta_D\left(\boldsymbol{k}-\boldsymbol{k}'\right)\mathsf{P}^{\mathcal{W}}_{ijmn}\left(k\right). \tag{979}$$

A residual anisotropic component may instead produce

$$\mathcal{P}_{\mathcal{W}}(\boldsymbol{k}) = \mathcal{P}^{(0)}_{\mathcal{W}}(k)\left[1 + g^{\mathcal{W}}_{ij}(k)\hat{k}_i\hat{k}_j + \cdots\right], \tag{980}$$

where

$$g^{\mathcal{W}}_{ii} = 0. \tag{981}$$

Observational viability requires

$$\left|g^{\mathcal{W}}_{ij}(k)\right| < g_{\mathcal{W},\max}(k). \tag{982}$$

The memory may therefore survive statistically without defining one large preferred direction.

191 Memory Conversion into Density and Potential Structure

The third-leg scaffold density may contain ordinary and inherited components:

$$\delta_{\rm scaf} = \delta_{\rm scaf}^{(\rm ad)} + \delta_{\rm scaf}^{(\rm inh)} + \delta_{\rm scaf}^{(\rm new)}. \tag{983}$$

The inherited component is

$$\delta_{\rm scaf}^{(\rm inh)}(k,z) = T_{\delta\mathcal{W}}(k,z)\mathcal{W}_3(k). \tag{984}$$

The scalar metric potential becomes

$$\Phi = \Phi^{(\rm ad)} + \Phi^{(\rm inh)} + \Phi^{(\rm other)}. \tag{985}$$

The inherited metric contribution must be sourced through the Einstein equations.

It cannot be added directly to the lensing potential without a stress–energy source.

Because the ordinary, inherited, and newly sourced components may be correlated, the total scaffold power contains auto- and cross-terms:

$$\mathcal{P}_{\delta_{\rm scaf}}^{(\rm tot)} = \sum_{A,B\in\{\rm ad,inh,new\}} \mathcal{P}_{AB}. \tag{986}$$

Define the diagonal inherited-power share

$$f_{\rm inh}^{(\rm diag)}(k,z) = \frac{\mathcal{P}_{\rm inh,inh}(k,z)}{\mathcal{P}_{\rm ad,ad} + \mathcal{P}_{\rm inh,inh} + \mathcal{P}_{\rm new,new}}. \tag{987}$$

This diagnostic lies between zero and one for positive auto-powers, but it does not replace the required cross-covariance calculation. A viable memory model requires a nonzero, bounded diagonal share over any scale range on which inherited structure is claimed.

If

$$f_{\rm inh}^{(\rm diag)} = 0 \tag{988}$$

at every observable scale, the scaffold may still function as dark matter but carries no measurable historical residue.

192 Distinguishing an Inherited Scaffold from Cold Dark Matter

Conventional cold dark matter and the minimal GT1 scaffold can be observationally degenerate when

$$\beta \to 0, \qquad \mathcal{W} \to 0, \qquad c_{s,\mathrm{scaf}}^2 \to 0, \qquad \sigma_{\mathrm{scaf}} \to 0. \tag{989}$$

An inherited scaffold requires at least one additional calculable relation.

Candidate distinctions include:

1. **Differential growth:**

$$f_{\mathrm{scaf}} - f_{\mathrm{vis}} \neq 0. \tag{990}$$

2. **Dynamical–lensing mismatch for scaffold tracers:**

$$\mathcal{R}_{\mathrm{dyn/lens}} \neq 1. \tag{991}$$

3. **Scale-dependent inherited transfer:**

$$T_{\delta\mathcal{W}}(k) \neq 0. \tag{992}$$

4. **Phase or topology correlations:**

$$\{\mathcal{P}_{\phi,\mathrm{phase}}, \Delta V_\mu, \Delta\beta_p\} \neq 0. \tag{993}$$

5. **Off-diagonal covariance:**

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle \neq C_\ell \delta_{\ell\ell'} \delta_{mm'}. \tag{994}$$

6. **Cross-correlation with central-transition observables:**

$$\mathcal{P}_{\mathcal{W}X}(k) \neq 0 \tag{995}$$

for a derived observable X.

7. **A two-reversal feature relation:**

$$\Delta_{\mathrm{mem}}(k) = \mathcal{F}\left[T_{\mathrm{O}}(k), U_2(k), T_{\mathrm{C}}(k)\right]. \tag{996}$$

The first two items test a coupled particle scaffold and can occur without any hidden-leg memory. They therefore cannot identify the Triple Bounce history by themselves. The remaining items are candidate historical residues, but none is individually unique to the Triple Bounce framework.

Specific historical support requires a linked cross-observable prediction from one derived memory-transfer theory.

193 Scaffold Initial Covariance

The scaffold state below is a block of the single central-output state and covariance defined in frozen Paper II. It may not be assigned an independent primordial spectrum or separately retuned for matter, lensing, and microwave-background calculations.

The complete central-output scaffold state is represented by

$$\boldsymbol{Z}_{\mathrm{scaf},C^+} = \left(\delta_{\mathrm{scaf}}, \theta_{\mathrm{scaf}}, \mathcal{W}_{\mathrm{tr}}, \mathcal{W}_{\mathrm{L}}, \mathcal{W}_i^{\mathrm{V}}, \mathcal{W}_{ij}^{\mathrm{TT}}\right)^{\mathsf{T}}. \tag{997}$$

Its covariance is

$$\left\langle \boldsymbol{Z}_{\mathrm{scaf}}(\boldsymbol{k}) \boldsymbol{Z}_{\mathrm{scaf}}^{\dagger}(\boldsymbol{k}') \right\rangle = (2\pi)^3 \delta_D \left(\boldsymbol{k} - \boldsymbol{k}'\right) \mathsf{P}_{\mathrm{scaf}}(k). \tag{998}$$

The covariance must be:

$$\mathsf{P}_{\mathrm{scaf}} = \mathsf{P}_{\mathrm{scaf}}^{\dagger}, \tag{999}$$

and positive semidefinite:

$$\boldsymbol{v}^{\dagger} \mathsf{P}_{\mathrm{scaf}} \boldsymbol{v} \geq 0 \tag{1000}$$

for every complex vector $\boldsymbol{v}$.

The ordinary cold-dark-matter control retains only

$$\left(\delta_{\mathrm{scaf}}, \theta_{\mathrm{scaf}}\right). \tag{1001}$$

A memory-bearing model must propagate all additional covariance blocks into the observable third leg.

It may not discard the memory perturbations after using them to motivate the background scaffold.

194 Scaffolding Observable Test Set

The candidate scaffold test vector is

$$\boldsymbol{\mathcal{O}}_{\rm scaf} = (\boldsymbol{\mathcal{O}}_{\rm bg}, \boldsymbol{\mathcal{O}}_{\rm lin}, \boldsymbol{\mathcal{O}}_{\rm lens}, \boldsymbol{\mathcal{O}}_{\rm nl}, \boldsymbol{\mathcal{O}}_{\rm mem}) \,. \tag{1002}$$

The background block includes

$$\boldsymbol{\mathcal{O}}_{\rm bg} = \{\rho_{\rm scaf}(a), m_{\rm scaf}(a), z_{\rm eq}, H(a)\} \,. \tag{1003}$$

The linear block includes

$$\boldsymbol{\mathcal{O}}_{\rm lin} = \{D_{\rm vis}, D_{\rm scaf}, f_{\rm vis}, f_{\rm scaf}, P_m, P_{\rm visscaf}\} \,. \tag{1004}$$

The lensing block includes

$$\boldsymbol{\mathcal{O}}_{\rm lens} = \left\{\Phi + \Psi, C_\ell^{\phi\phi}, \kappa, \mathcal{R}_{\rm dyn/lens}\right\} . \tag{1005}$$

The nonlinear block includes

$$\boldsymbol{\mathcal{O}}_{\rm nl} = \{\delta_{\rm coll}, \Delta_{\rm vir}, n(M,z), c(M,z), \text{offset statistics}\} \,. \tag{1006}$$

The memory block includes

$$\boldsymbol{\mathcal{O}}_{\rm mem} = \left\{\mathcal{E}_{\rm mem}, f_{\rm inh}^{\rm (diag)}, g_{ij}^{\mathcal{W}}, \mathcal{P}_{\mathcal{W}X}, \text{topology}\right\} . \tag{1007}$$

Within each declared realization, one coupling, scalar potential, initial state, and parameter ledger must generate every included block. The memory-free baseline omits $\boldsymbol{\mathcal{O}}_{\rm mem}$ rather than fitting a hidden memory law that is absent from its action.

195 Scaffolding Numerical Program

The scaffolding calculation should proceed through the following stages.

Stage 1: cold uncoupled control

Set

$$\beta = 0, \qquad \mathcal{W}_{\mu\nu} = 0. \tag{1008}$$

Reproduce the standard cold-dark-matter background and linear transfer.

Stage 2: coupled background

Integrate the background-dynamics system and record

$$\{\rho_{\rm scaf}, m_{\rm scaf}, \phi, \beta, m_{\rm eff}\}\,. \tag{1009}$$

Stage 3: coupled linear perturbations

Integrate

$$\{\delta_{\rm vis}, \theta_{\rm vis}, \delta_{\rm scaf}, \theta_{\rm scaf}, \delta\phi, \Phi, \Psi\}\,. \tag{1010}$$

Stage 4: quasistatic validation

Compare the full perturbation solution with

$$\mu_{\rm scaf} = 1 + 2\beta^2 \frac{k^2}{k^2 + a^2 m_{\rm eff}^2}. \tag{1011}$$

Stage 5: growth and lensing

Calculate

$$\left\{f_{\rm vis}, f_{\rm scaf}, P_m, \Phi + \Psi, C_\ell^{\phi\phi}\right\}. \tag{1012}$$

Stage 6: nonlinear spherical collapse

Calculate

$$\delta_{\rm coll}, \qquad \Delta_{\rm vir}. \tag{1013}$$

Stage 7: particle simulation

Implement the common metric force for visible and scaffold particles and apply the direct scalar force only to scaffold particles in the minimum GT1 action.

Stage 8: memory-free likelihood

Constrain the coupled particle scaffold before introducing historical memory.

Stage 9: elastic-memory extension

Add

$$\left\{\mathcal{W}_{\rm tr}, \mathcal{W}_{\langle ij\rangle}, \Gamma_{\rm mem}, T^{(j)}_{\mathcal{W}\mathcal{W}}\right\}. \tag{1014}$$

Stage 10: reversal transfer

Propagate the memory state through explicit finite-width outer and central transitions.

Stage 11: historical null tests

Set

$$\mathcal{E}_{\rm mem} = 0 \tag{1015}$$

and determine which observables lose the claimed historical residue.

Stage 12: comparative model testing

Compare:

$$\{\text{CDM}, \text{coupled cold scaffold}, \text{elastic scaffold}, \text{inherited scaffold}\}. \tag{1016}$$

196 Scaffolding-Sector Rejection Criteria

The following criteria have graded consequences. Background, clustering, lensing, conservation, and stability failures test the minimum third-leg scaffold. Failures involving $\mathcal{W}_{\mu\nu}$, reversal transfer, or a distinctive residue test only the historical extension. A historical-memory failure does not automatically falsify a viable cold-scaffold control.

1. The scaffold cannot reproduce an approximately pressureless background during the required matter era.
2. Its mass evolution shifts matter–radiation equality beyond the allowed third-leg history.
3. The scaffold mass becomes zero, negative, divergent, or noninvertible.
4. The number current is not conserved and no particle-creation law is provided.
5. The background transfer is not accompanied by a consistent momentum transfer.
6. The linear perturbation equations violate total covariant conservation.
7. The scalar-mediated force gives an unacceptable enhancement of scaffold growth.
8. The same coupling that fits the background fails the matter power spectrum.
9. The scalar interaction range required by cosmology fails local or halo tests.
10. The scaffold sound speed erases required structure.
11. The scaffold free-streaming scale is too large.
12. The anisotropic stress produces unacceptable gravitational slip.
13. The model fits scaffold dynamics but not gravitational lensing.
14. Visible and scaffold perturbations separate beyond observationally acceptable levels.
15. The coupling produces an unacceptable visible–scaffold velocity bias.
16. The total matter perturbation cannot be reduced to the observed large-scale clustering behavior.
17. The model requires an independently fitted primordial spectrum for the scaffold.
18. The central covariance used for the scaffold differs from the one used for the microwave background.
19. The nonlinear collapse threshold or halo abundance is unacceptable.
20. The scalar force produces excessive visible–scaffold offsets.
21. The halo concentration or substructure spectrum is unacceptable.

22. The model counts the scalar fifth force as additional lensing mass.
23. The cold-fluid state is claimed to contain historical memory without an additional state variable.
24. The memory variable has no covariant action or evolution law.
25. The memory relaxation rate is selected independently for each observable.
26. The memory survives only through a manually imposed transition rule.
27. The outer and central memory transfers are assumed identical without a derived reason.
28. The memory tensor becomes singular at either reversal.
29. The elastic scaffold contains a ghost.
30. The longitudinal or transverse propagation speed is negative.
31. The elastic propagation speeds suppress structure on required scales.
32. The memory anisotropic stress exceeds observational limits.
33. The retained memory generates an unacceptable preferred direction.
34. The inherited scaffold power is not positive semidefinite.
35. The inherited component destroys acoustic phase coherence.
36. The memory fraction must be tuned separately at every wavenumber.
37. The claimed historical signal disappears when propagated through the full third-leg transfer.
38. The claimed signal is equally well reproduced by ordinary primordial initial conditions with fewer assumptions.
39. No observable distinguishes the scaffold from conventional cold dark matter.
40. Only the control limit remains viable:

$$\beta = 0, \qquad \mathcal{W}_{\mu\nu} = 0. \tag{1017}$$

This outcome preserves ordinary cold-dark-matter recovery but supplies no support for an interacting scaffold or a historical-memory claim.

197 Formal Summary of the Scaffolding Sector

Within the selected number-conserving particle branch, the minimal scaffold density is

$$\boxed{\rho_{\rm scaf} = m_{\rm scaf}(\phi) n_{\rm scaf}.} \tag{1018}$$

In that branch, particle number is conserved:

$$\boxed{\nabla_\mu \left(n_{\rm scaf} u^\mu_{\rm scaf}\right) = 0.} \tag{1019}$$

Its background density evolves as

$$\boxed{\dot{\rho}_{\rm scaf} + 3H\rho_{\rm scaf} = \frac{\beta}{M_{\rm Pl}}\rho_{\rm scaf}\dot{\phi}.} \tag{1020}$$

The density and number perturbations satisfy

$$\boxed{\delta_{\rm scaf} = \delta_n + \frac{\beta}{M_{\rm Pl}}\delta\phi.} \tag{1021}$$

For constant β, no memory stress, and the declared linear gauge conventions, the ideal cold scaffold Euler equation is

$$\boxed{\theta'_{\rm scaf} = -\left(\mathcal{H} + \frac{\beta}{M_{\rm Pl}}\phi'\right)\theta_{\rm scaf} + k^2\left(\Psi + \frac{\beta}{M_{\rm Pl}}\delta\phi\right).} \tag{1022}$$

In the linear, unscreened, subhorizon quasistatic regime, the scalar response is

$$\boxed{\left(k^2 + a^2 m_{\rm eff}^2\right)\delta\phi \simeq -\frac{a^2\beta}{M_{\rm Pl}}\rho_{\rm scaf}\delta_{\rm scaf}.} \tag{1023}$$

The effective scaffold self-attraction is

$$\boxed{\mu_{\rm scaf}(k,a) = 1 + 2\beta^2\frac{k^2}{k^2 + a^2 m_{\rm eff}^2}.} \tag{1024}$$

The total matter contrast is

$$\boxed{\delta_m = \omega^{(m)}_{\rm vis}\delta_{\rm vis} + \omega^{(m)}_{\rm scaf}\delta_{\rm scaf}.} \tag{1025}$$

The matter power spectrum is

$$\boxed{P_m = \left(\omega_{\rm vis}^{(m)}\right)^2 P_{\rm visvis} + \left(\omega_{\rm scaf}^{(m)}\right)^2 P_{\rm scafscaf} + 2\omega_{\rm vis}^{(m)}\omega_{\rm scaf}^{(m)} P_{\rm visscaf}.} \tag{1026}$$

The cold-fluid memory limitation is

$$\boxed{\{\rho_{\rm scaf}, u^{\mu}_{\rm scaf}\} \text{ does not uniquely encode cross-leg historical ancestry.}} \tag{1027}$$

A projected candidate memory-evolution template is

$$\boxed{h_{\mu}{}^{\rho\,({\rm scaf})} h_{\nu}{}^{\sigma\,({\rm scaf})} u^{\alpha}_{\rm scaf} \nabla_{\alpha} \mathcal{W}_{\langle\rho\sigma\rangle} + \Gamma_{\rm mem} \mathcal{W}_{\langle\mu\nu\rangle} = \mathcal{S}_{\langle\mu\nu\rangle}.} \tag{1028}$$

The cumulative retention kernel is

$$\boxed{\mathcal{E}^{(A)}_{\rm mem}(k) = \left| T^{(\rm C)}_{\mathcal{WW},A} T^{(\rm O)}_{\mathcal{WW},A} \right| \exp\left[- \int \Gamma_{\rm mem}(k,t) {\rm d}t \right].} \tag{1029}$$

The complete scaffolding burden is

$$\boxed{\begin{aligned} &\text{cold background} + \text{efficient clustering} + \text{metric lensing} \\ &+ \text{acceptable nonlinear structure} + \text{stable memory dynamics} \\ &+ \text{finite two-reversal retention} + \text{observational distinction.} \end{aligned}} \tag{1030}$$

The decisive scaffolding question is

$$\boxed{\begin{aligned} &\text{Can one scaffold theory reproduce the successful gravitational} \\ &\text{functions of cold dark matter while carrying a finite, stable,} \\ &\text{statistically acceptable structural state through both} \\ &\text{reversals, and can that retained state produce a linked} \\ &\text{observable that ordinary third-leg cold dark matter does not} \\ &\text{require?} \end{aligned}} \tag{1031}$$

198 Scaffolding-Sector Status

The scaffolding problem is now divided into two development levels.

The first is the coupled cold-particle scaffold, whose selected action and analytic control equations are specified but not numerically validated:

$$\boxed{m_{\mathrm{scaf}}(\phi) + \text{number conservation} + \text{scalar-mediated dark self-attraction.}} \tag{1032}$$

The second is the historical-memory extension, which remains a covariant derivation template rather than a selected physical model:

$$\boxed{\text{cold or elastic scaffold} + \mathcal{W}_{\mu\nu} + \Gamma_{\mathrm{mem}} + T^{(\mathrm{O,C})}_{\mathcal{W}\mathcal{W}}.} \tag{1033}$$

Proposition 198.1 (Scaffolding viability and identification gate)**.** *The minimum third-leg scaffold is viable only if one action and one parameter ledger recover the required background, linear growth, metric lensing, acoustic-era transfer, nonlinear structure, and stability limits. Historical identification requires, in addition, a derived memory state, finite transfer through both reversals, and at least one linked observable residue that survives comparison with ordinary third-leg initial conditions. Passing the first gate does not imply passing the second.*

The current status is therefore

$$\boxed{\begin{aligned} \mathcal{G}^{(3)}_{\mathrm{GT1,scaf}} &: \text{selected effective branch and analytic controls specified,}\\ \mathcal{G}^{(123)}_{\mathrm{GT1,scaf}} &: \text{memory action and reversal transfer not yet derived,}\\ \text{numerical and observational viability} &: \text{not established.} \end{aligned}} \tag{1034}$$

The required calculation sequence is:

1. validate the uncoupled cold-dark-matter limit;
2. calculate the field-dependent scaffold mass;
3. integrate the coupled linear perturbations;
4. verify the quasistatic scalar-force limit;
5. calculate visible and scaffold differential growth;
6. calculate the total matter transfer matrix;
7. propagate the same central covariance into matter and lensing observables;
8. test equality and acoustic-era consistency;

9. calculate nonlinear spherical collapse;
10. perform coupled visible–scaffold simulations;
11. test halo, cluster, and cosmic-web observables;
12. determine whether the coupled particle scaffold is viable without memory;
13. introduce a covariant internal memory variable;
14. derive the memory stress–energy and propagation speeds;
15. calculate relaxation through the first and second legs;
16. derive the outer and central memory-transfer maps;
17. propagate the complete memory covariance into the third leg;
18. test statistical isotropy and anisotropic stress;
19. identify one linked historical null test; and
20. compare the inherited scaffold with conventional cold dark matter under a common likelihood and complexity analysis.

The next part develops the chasing sector: its cosmological-constant limit, canonical and noncanonical dynamics, early-time control, late-time acceleration, perturbative sound speed, coupling stability, effective minima, cross-leg persistence, reversal response, and the conditions under which chasing can remain active throughout the Triple Bounce history without being interpreted as a moving material boundary.

199 The Chasing-Sector Problem

Benchmark literature. The chasing audit is anchored to cosmological-constant, quintessence, and modern expansion-history tests (Weinberg, 1989; Peebles and Ratra, 2003; Copeland et al., 2006; DESI Collaboration et al., 2025a).

The frozen Paper I architecture assigns *chasing* to the continuing outward-support role provisionally associated with dark energy. It does not identify that role with a literal material current, a moving shell, or a measured local velocity. The frozen Paper II framework then requires any proposed realization to enter one common background, curvature history, transition system, central covariance, and observational comparison. Paper III preserves both boundaries.

In the minimum third-leg GT1 realization, the candidate mathematical representation is

$$\boxed{\mathfrak{G}^{(3)}_{\text{chase}} = \text{a canonical scalar field } \phi \text{ with potential } V(\phi).} \tag{1035}$$

This is a selected reference realization. It is not a claim that dark energy has been microscopically identified or that the same canonical field description is already valid through the hidden legs.

The minimum third-leg burden is

$$\boxed{\mathcal{B}^{(3)}_{\rm chase} = \left\{ \begin{array}{c} \text{the exact constant-vacuum control,} \\ \text{an acceptable third-leg expansion and distance history,} \\ \text{stable perturbations, growth, and lensing,} \\ \text{one conserving scaffold–chasing interaction ledger} \end{array} \right\}.} \tag{1036}$$

The full historical extension has the additional burden

$$\boxed{\mathcal{B}^{(123)}_{\rm chase} = \mathcal{B}^{(3)}_{\rm chase} + \left\{ \begin{array}{c} \text{one action-derived hidden-leg state,} \\ \text{finite continuation through both internal reversals,} \\ \text{a contribution to the Paper II central covariance,} \\ \text{a linked historical residue not implied by generic dark energy} \end{array} \right\}.} \tag{1037}$$

Cross-leg persistence does not require constant density, constant pressure, constant equation of state, or permanent accelerated metric expansion. Paper II permits accelerated hidden-leg subintervals only when the same solved background gives $\epsilon_H < 1$; it does not assume that the chasing role automatically produces such intervals.

All references in this part to radiation domination, matter–radiation equality, or a pressureless-matter epoch describe downstream observable third-leg benchmarks. They do not determine when matter, antimatter, or particle-like excitations first originated in the complete history.

199.1 Derivation-status and reproducibility rule

The chasing equations are divided into four reproducible classes:

I. **Selected-action identities.** The canonical stress–energy tensor, background field equation, conformal exchange current, scaffold mass relation, and exact constant-vacuum limit follow from the declared GT1 third-leg action.

II. **Conditional analytical branches.** Fixed points, acceleration conditions, tracking relations, and constant-slope tensions apply only to the stated potential, coupling, background, and orbit assumptions.

III. **Action-dependent perturbation equations.** Scalar, scaffold, metric, and observable transfer equations require one declared gauge, Fourier convention, sign convention, and common Einstein–Boltzmann implementation. Schematic source structures do not replace that derivation.

IV. **Open historical templates.** Dependence on a reversal order parameter, cross-leg state vectors, finite-width junction laws, and historical transfer relations remain bookkeeping structures until one closed hidden-leg and transition action derives them.

A failed candidate potential or coupling rejects that realization. It does not by itself reject the Paper I functional architecture.

200 Chasing as a Functional Role

The term *chasing* identifies a proposed function rather than a literal trajectory.

It does not imply:

- a material shell moving toward an outer boundary;
- a physical edge of the universe;
- a fluid traveling through pre-existing space;
- identity with the E front;
- identity with the cosmic microwave background;
- identity with electromagnetic radiation;
- superluminal material motion; or
- a direction defined by one preferred spatial vector.

The chasing function is defined through its contribution to the background and perturbation equations.

For a component with density ρ_{chase} and pressure p_{chase}, its direct contribution to the acceleration equation is

$$\mathcal{A}_{\text{chase}} = -\frac{4\pi G}{3}\left(\rho_{\text{chase}} + 3p_{\text{chase}}\right). \tag{1038}$$

An acceleration-supporting state has

$$\rho_{\text{chase}} + 3p_{\text{chase}} < 0. \tag{1039}$$

A persistent chasing sector need not satisfy this condition at every time.

It is useful to distinguish three regimes:

$$\boxed{\begin{aligned} \text{latent chasing} &: \rho_{\text{chase}} > 0 \text{ but } \rho_{\text{chase}} + 3p_{\text{chase}} \geq 0, \\ \text{active chasing} &: \rho_{\text{chase}} + 3p_{\text{chase}} < 0, \\ \text{dominant chasing} &: \Omega_{\text{chase}} \text{ is large enough to control the total acceleration.} \end{aligned}} \tag{1040}$$

This distinction prevents the claim of cross-leg persistence from being confused with permanent accelerated expansion.

201 Exact Cosmological-Constant Limit

The exact third-leg constant-vacuum control is obtained when

$$\boxed{A(\phi) = A_0 > 0, \qquad \beta = 0, \qquad \dot{\phi} = 0, \qquad V(\phi) = V_0 = \rho_\Lambda.} \tag{1041}$$

A constant A_0 may be absorbed into the reference scaffold-mass normalization. The scalar stress–energy tensor becomes

$$T^{\phi}_{\mu\nu} = -V_0 g_{\mu\nu}. \tag{1042}$$

Its density, pressure, and intrinsic equation of state are

$$\rho_\phi = V_0, \qquad p_\phi = -V_0, \qquad w_\phi = -1. \tag{1043}$$

Its homogeneous density is constant:

$$\dot{\rho}_\phi = 0. \tag{1044}$$

At the linear-control level, the vacuum contribution has no independent rolling-field clock or propagating dark-energy perturbation inserted into the observable system:

$$\dot{\phi} = 0, \qquad \delta\phi = 0. \tag{1045}$$

A dynamical or coupled scalar may satisfy $w_\phi \approx -1$ over a restricted interval without being exactly equivalent to a cosmological constant. Exact recovery requires the background, interaction current, metric response, perturbations, and observables to reduce together to the constant-vacuum control.

Proposition 201.1 (Constant-vacuum control is not historical identification)**.** *Recovering the exact ΛCDM control validates the reduction of the selected third-leg equations. It supplies no evidence for hidden-leg continuation, either internal reversal, or the historical identity of chasing.*

202 Canonical Scalar Chasing

Benchmark literature. The canonical scalar equations and their standard dynamical regimes follow the quintessence literature (Ratra and Peebles, 1988; Wetterich, 1988; Copeland et al., 1998).

The canonical scalar is selected because it is the minimum reproducible third-leg field realization with a positive standard kinetic term, a well-defined stress–energy tensor, and a transparent constant-vacuum limit. The selection is methodological rather than ontological.

The canonical chasing action is

$$S_\phi = \int \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right]. \tag{1046}$$

For a homogeneous field,

$$\phi = \phi(t), \tag{1047}$$

the density and pressure are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{1048}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{1049}$$

The intrinsic equation of state is

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}. \tag{1050}$$

For

$$\rho_\phi > 0, \qquad V \geq 0, \tag{1051}$$

one has

$$\boxed{-1 \leq w_\phi \leq 1.} \tag{1052}$$

The scalar contributes directly to accelerated expansion when

$$\boxed{V(\phi) > \dot{\phi}^2.} \tag{1053}$$

Potential domination gives

$$w_\phi \to -1. \tag{1054}$$

Kinetic domination gives

$$w_\phi \to 1. \tag{1055}$$

203 Coupled Background Evolution

The equations in this section use the selected GT1 sign convention, a number-conserving pressureless scaffold, the conformal mass relation $m_{\text{scaf}} = m_{\text{scaf},0} A(\phi)$, and no independent reversal source. A variable coupling remains allowed only when derived from the same $A(\phi)$.

The GT1 scalar equation is

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\frac{\beta(\phi)}{M_{\text{Pl}}}\rho_{\text{scaf}}.} \tag{1056}$$

The scalar energy equation is

$$\boxed{\dot{\rho}_\phi + 3H\left(\rho_\phi + p_\phi\right) = -\frac{\beta}{M_{\text{Pl}}}\rho_{\text{scaf}}\dot{\phi}.} \tag{1057}$$

The scaffold receives the opposite transfer:

$$\dot{\rho}_{\text{scaf}} + 3H\rho_{\text{scaf}} = \frac{\beta}{M_{\text{Pl}}}\rho_{\text{scaf}}\dot{\phi}. \tag{1058}$$

The transfer rate is

$$Q_{\text{scaf}} = \frac{\beta}{M_{\text{Pl}}}\rho_{\text{scaf}}\dot{\phi}. \tag{1059}$$

For the sign convention used here,

$$\beta\dot{\phi} > 0 \quad \Longrightarrow \quad \text{scalar energy is transferred to scaffolding.} \tag{1060}$$

The background exchange, scaffold mass drift, scalar force, and perturbative momentum transfer arise from the same conformal coupling.

They are not independent phenomenological effects.

204 Intrinsic and Effective Equations of State

The intrinsic scalar equation of state is defined by its stress–energy:

$$w_\phi = \frac{p_\phi}{\rho_\phi}. \tag{1061}$$

An effective equation of state may instead be defined by rewriting the coupled continuity equation as

$$\dot{\rho}_\phi + 3H\left(1 + w_\phi^{\rm eff}\right)\rho_\phi = 0. \tag{1062}$$

This gives

$$\boxed{w_\phi^{\rm eff} = w_\phi + \frac{\beta \rho_{\rm scaf}\dot{\phi}}{3HM_{\rm Pl}\rho_\phi}.} \tag{1063}$$

The scaffold effective equation of state is

$$w_{\rm scaf}^{\rm eff} = -\frac{\beta\dot{\phi}}{3HM_{\rm Pl}}. \tag{1064}$$

The effective quantities are derived bookkeeping summaries of the coupled dilution histories.

They are not independent fluid equations of state, and they do not replace the intrinsic stress–energy relations.

In particular,

$$w_\phi^{\rm eff} < -1 \tag{1065}$$

is possible when the interaction term is sufficiently negative, even though

$$w_\phi \geq -1. \tag{1066}$$

Therefore,

$$\boxed{\text{effective phantom-like dilution} \neq \text{a fundamental negative-kinetic-energy ghost.}} \tag{1067}$$

The full perturbative stability must nevertheless be checked.

205 Potential-Slope Variables

Define

$$\lambda(\phi) = -M_{\rm Pl} \frac{V_{,\phi}}{V}, \tag{1068}$$

and

$$\Gamma_V(\phi) = \frac{V V_{,\phi\phi}}{V_{,\phi}^2}. \tag{1069}$$

Using

$$x = \frac{\dot{\phi}}{\sqrt{6} M_{\rm Pl} H}, \tag{1070}$$

the slope evolves as

$$\frac{\mathrm{d}\lambda}{\mathrm{d}N} = -\sqrt{6} x \lambda^2 \left(\Gamma_V - 1\right). \tag{1071}$$

For an exponential potential,

$$\Gamma_V = 1, \tag{1072}$$

so

$$\lambda = \text{constant}. \tag{1073}$$

For

$$x > 0, \qquad \Gamma_V > 1, \tag{1074}$$

the slope decreases along the trajectory:

$$\frac{\mathrm{d}\lambda}{\mathrm{d}N} < 0. \tag{1075}$$

A decreasing slope permits a steep early potential and a shallow late potential within one continuous field theory.

206 Baseline Potential Families

Benchmark literature. The potential families are treated as standard candidate classes rather than as Triple Bounce predictions (Ratra and Peebles, 1988; Wetterich, 1988; Copeland et al., 2006).

The first chasing study should compare a restricted set of potential families under common initial-condition, prior, stability, and observational rules. None of these families is selected by the Paper I architecture, and none is promoted to a physical identification merely because it admits acceleration.

206.1 Constant potential

$$V(\phi) = V_0. \tag{1076}$$

This supplies the exact cosmological-constant control when the field is static and uncoupled.

206.2 Exponential potential

$$V(\phi) = V_0 \exp\left(-\lambda \frac{\phi}{M_{\mathrm{Pl}}}\right). \tag{1077}$$

This gives constant λ and admits exact fixed-point analysis.

206.3 Inverse-power potential

$$V(\phi) = M^{4+n}\phi^{-n}. \tag{1078}$$

Its slope decreases as the field evolves toward larger ϕ.

206.4 Periodic or pseudo-Goldstone-like potential

$$V(\phi) = V_0\left[1 + \cos\left(\frac{\phi}{f}\right)\right]. \tag{1079}$$

A field initially near a flat extremum can remain temporarily frozen.

206.5 Plateau potential

A simple plateau family is

$$V(\phi) = V_0 \left[1 - \exp\left(-\frac{\phi}{\mu}\right)\right]^2. \tag{1080}$$

These functions are baseline mathematical families.

They are not claimed to be the final microphysical chasing potential.

207 Thawing, Freezing, Tracking, and Scaling

Benchmark literature. Thawing, freezing, tracking, and scaling are used as established scalar-dark-energy classifications (Copeland et al., 1998, 2006).

A *thawing* chasing trajectory begins with

$$\dot{\phi} \approx 0, \qquad w_\phi \approx -1, \tag{1081}$$

because Hubble damping initially dominates.

The field later begins to roll when

$$|V_{,\phi}| \tag{1082}$$

becomes dynamically significant compared with

$$3H|\dot{\phi}|. \tag{1083}$$

A *freezing* trajectory rolls at earlier times and later approaches

$$w_\phi \to -1 \tag{1084}$$

as the potential flattens, the field approaches an effective minimum, or the interaction slows the motion.

A *tracking* solution reduces sensitivity to some initial conditions by approaching a common trajectory determined by the background and potential.

A *scaling* solution maintains approximately constant density ratios:

$$\frac{\rho_\phi}{\rho_b} \approx \text{constant}. \tag{1085}$$

These classifications concern dynamical trajectories.

They do not identify a unique potential or microscopic origin.

208 Effective Potential Generated by Scaffolding

For a conserved scaffold number density,

$$\boxed{V_{\text{eff}}(\phi, n_{\text{scaf}}) = V(\phi) + m_{\text{scaf}}(\phi) n_{\text{scaf}}.} \tag{1086}$$

Its derivative is

$$V_{\text{eff},\phi} = V_{,\phi} + \frac{\beta}{M_{\text{Pl}}} \rho_{\text{scaf}}. \tag{1087}$$

The scalar equation becomes

$$\ddot{\phi} + 3H\dot{\phi} + V_{\text{eff},\phi} = 0. \tag{1088}$$

An effective minimum satisfies

$$\boxed{V_{,\phi} + \frac{\beta}{M_{\text{Pl}}} \rho_{\text{scaf}} = 0.} \tag{1089}$$

The effective scalar mass used here is the curvature of V_{eff} at fixed scaffold number density:

$$m_{\text{eff}}^2 \equiv \left. \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2} \right|_{n_{\text{scaf}}}. \tag{1090}$$

For the selected mass relation,

$$\boxed{m_{\text{eff}}^2 = V_{,\phi\phi} + \frac{\rho_{\text{scaf}}}{M_{\text{Pl}}^2} \left[\beta^2 + M_{\text{Pl}} \beta_{,\phi} \right].} \tag{1091}$$

A stable local minimum requires

$$m_{\text{eff}}^2 > 0. \tag{1092}$$

Because

$$n_{\text{scaf}} \propto a^{-3}, \tag{1093}$$

the minimum generally moves with cosmic expansion.

209 Adiabatic Tracking of the Effective Minimum

Let

$$\phi_{\min} = \phi_{\min}(t) \tag{1094}$$

solve

$$V_{\text{eff},\phi}\left(\phi_{\min}, t\right) = 0. \tag{1095}$$

Differentiating gives schematically

$$\dot{\phi}_{\min} = -\frac{\partial_t V_{\text{eff},\phi}}{m_{\text{eff}}^2}. \tag{1096}$$

Adiabatic tracking requires

$$m_{\text{eff}}^2 \gg H^2, \tag{1097}$$

together with

$$\left|\frac{\dot{m}_{\text{eff}}}{m_{\text{eff}}^2}\right| \ll 1, \tag{1098}$$

and a slowly moving minimum:

$$\left|\dot{\phi}_{\min}\right| \ll M_{\text{Pl}} H \tag{1099}$$

when a near-vacuum equation of state is required.

A heavy field can track the minimum with suppressed large-scale fluctuations.

However, strong dependence on ρ_{scaf} can make the chasing density follow the scaffold too closely.

The effective-minimum mechanism must therefore be tested simultaneously for:

$$\{\text{acceleration}, \text{mass drift}, \text{growth}, \text{sound response}, \text{early-time density}\}. \tag{1100}$$

210 Autonomous Chasing Diagnostics

Using the background-dynamics variables,

$$x = \frac{\dot{\phi}}{\sqrt{6}M_{\mathrm{Pl}}H}, \qquad y = \frac{\sqrt{V}}{\sqrt{3}M_{\mathrm{Pl}}H}, \tag{1101}$$

the scalar fraction is

$$\Omega_\phi = x^2 + y^2. \tag{1102}$$

Its intrinsic equation of state is

$$w_\phi = \frac{x^2 - y^2}{x^2 + y^2}. \tag{1103}$$

Its contribution to the flat-background effective equation of state is

$$w_{\mathrm{eff}} = x^2 - y^2 + \frac{1}{3}z^2. \tag{1104}$$

Acceleration requires

$$x^2 - y^2 + \frac{1}{3}z^2 < -\frac{1}{3}. \tag{1105}$$

At the scalar-dominated exponential-potential point,

$$x = \frac{\lambda}{\sqrt{6}}, \qquad y^2 = 1 - \frac{\lambda^2}{6}. \tag{1106}$$

The equation of state is

$$w_\phi = -1 + \frac{\lambda^2}{3}. \tag{1107}$$

Acceleration requires

$$\boxed{\lambda^2 < 2.} \tag{1108}$$

211 Early-Time Chasing Control

The chasing sector must not disrupt the required downstream third-leg radiation-dominated and pressureless-matter benchmark epochs. These are observational recovery conditions and are neutral about the origin time of matter or particle-like excitations.

Define an allowed early chasing envelope

$$\Omega_\phi(a) < \Omega_{\phi,\max}^{\mathrm{early}}(a). \tag{1109}$$

The envelope is determined by the complete thermal, acoustic, and growth analysis.

It is kept symbolic here.

For the coupled scaffold matter point derived in the background analysis,

$$\Omega_\phi^{(M_\phi)} = \frac{2\beta^2}{3}. \tag{1110}$$

Therefore,

$$\boxed{\frac{2\beta^2}{3} < \Omega_{\phi,\max}^{\mathrm{mat}}} \tag{1111}$$

is required for a sufficiently standard matter era.

For the scalar–radiation scaling point,

$$\Omega_\phi^{(R_\phi)} = \frac{4}{\lambda^2}. \tag{1112}$$

Thus,

$$\boxed{\lambda^2 > \frac{4}{\Omega_{\phi,\max}^{\mathrm{rad}}}} \tag{1113}$$

is required if the field occupies that scaling solution during the radiation era.

The scalar may instead remain frozen with a much smaller early density.

212 Constant-Slope Early–Late Tension

A single exponential potential has constant

$$\lambda. \tag{1114}$$

A small radiation-era scaling fraction requires

$$\lambda^2 \gg 1. \tag{1115}$$

A scalar-dominated accelerated attractor requires

$$\lambda^2 < 2. \tag{1116}$$

These conditions cannot both be satisfied by one constant slope.

Therefore, within the flat, constant-slope exponential branch and under the assumption that the orbit first occupies the radiation-scaling fixed point and later approaches the scalar-dominated fixed point, the same constant λ cannot satisfy both requirements:

$$\boxed{\begin{array}{l} \Omega_\phi^{(R_\phi)} \ll 1 \quad \text{and} \quad w_\phi^{(\phi)} < -\dfrac{1}{3} \\ \text{cannot both follow from the two stated fixed-point conditions} \\ \text{for one unchanged constant slope } \lambda. \end{array}} \tag{1117}$$

This is a branch-and-orbit conditional result, not a general no-go theorem.

It does not exclude:

- a field initially away from the radiation scaling point;
- a thawing solution;
- a variable-slope potential;
- a coupling-induced moving minimum;
- a multi-field chasing sector; or
- a finite transition that resets the scalar state.

The first variable-slope target is

$$\boxed{\lambda_{\text{early}}^2 \gg 1, \qquad \lambda_{\text{late}}^2 < 2.} \tag{1118}$$

213 Late-Time Acceleration Conditions

A viable late chasing epoch must satisfy

$$\epsilon_H < 1. \tag{1119}$$

For scalar domination,

$$V > \dot{\phi}^2. \tag{1120}$$

For the scalar-dominated fixed point of the flat, constant-β, exponential-potential autonomous GT1 system, the background analysis found

$$\boxed{\lambda^2 < 2, \qquad \lambda^2 + \beta\lambda < 3.} \tag{1121}$$

The first condition supplies acceleration.

The second prevents scaffold perturbations of the homogeneous phase space from destabilizing the scalar-dominated state.

A successful late-time trajectory must also reproduce:

$$\{H(z), D_L(z), D_A(z), D_M(z), D_V(z), f\sigma_8(z)\} \tag{1122}$$

with one common parameter set.

The existence of an accelerated fixed point is not sufficient.

The physical trajectory must enter it at an acceptable epoch.

214 Scalar Perturbation Variables

In the perturbation sections below, a prime denotes differentiation with respect to conformal time:

$$X' = \frac{\mathrm{d}X}{\mathrm{d}\eta}. \tag{1123}$$

Write

$$\phi(\eta, \boldsymbol{x}) = \bar{\phi}(\eta) + \delta\phi(\eta, \boldsymbol{x}). \tag{1124}$$

The conformal Newtonian-gauge metric is

$$\mathrm{d}s^2 = a^2(\eta)\left[-(1+2\Psi)\,\mathrm{d}\eta^2 + (1-2\Phi)\,\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]. \tag{1125}$$

A useful gauge-invariant scalar fluctuation is

$$Q_\phi = \delta\phi + \frac{\bar{\phi}'}{\mathcal{H}}\Phi. \tag{1126}$$

The scalar density and pressure perturbations in Newtonian gauge are

$$\delta\rho_\phi = \frac{1}{a^2}\left(\bar{\phi}'\delta\phi' - \bar{\phi}'^2\Psi\right) + V_{,\phi}\delta\phi, \tag{1127}$$

$$\delta p_\phi = \frac{1}{a^2}\left(\bar{\phi}'\delta\phi' - \bar{\phi}'^2\Psi\right) - V_{,\phi}\delta\phi. \tag{1128}$$

These quantities must be combined with the scaffold transfer perturbations in the full interacting system.

215 Coupled Scalar Perturbation Equation

The following equation records the principal source structure implied by the selected action. It is not a substitute for the complete constraint-consistent Einstein–Boltzmann derivation used for numerical predictions.

The scalar perturbation equation has the form

$$\delta\phi'' + 2\mathcal{H}\delta\phi' + \left(k^2 + a^2 m_{\rm eff}^2\right)\delta\phi = \mathcal{S}_{\rm met} + \mathcal{S}_{\rm scaf} + \mathcal{S}_\beta. \tag{1129}$$

Here:

- $\mathcal{S}_{\rm met}$ contains the metric sources Φ, Ψ, and their derivatives;
- $\mathcal{S}_{\rm scaf}$ contains the scaffold density and velocity perturbations; and
- $\mathcal{S}_\beta$ contains derivatives of the coupling function.

The leading scaffold-density source is proportional to

$$\mathcal{S}_{\rm scaf} \supset -\frac{a^2\beta}{M_{\rm Pl}}\rho_{\rm scaf}\delta_{\rm scaf}. \tag{1130}$$

The effective mass is

$$m_{\text{eff}}^2 = V_{,\phi\phi} + \frac{\rho_{\text{scaf}}}{M_{\text{Pl}}^2}\left[\beta^2 + M_{\text{Pl}}\beta_{,\phi}\right]. \tag{1131}$$

The exact metric-source coefficients depend on the adopted perturbation and exchange-current conventions.

They must be derived from the same action rather than combined from incompatible phenomenological prescriptions.

216 Scalar Sound Speed and Clustering

A canonical scalar has rest-frame sound speed

$$\boxed{c_{s,\phi}^2 = 1} \tag{1132}$$

in natural units.

Its intrinsic perturbations therefore resist clustering substantially below the sound horizon.

The scalar can still vary on:

- horizon and superhorizon scales;
- scales comparable to its Compton wavelength;
- rapid-transition scales;
- scaffold-sourced scales; and
- scales where the quasistatic approximation fails.

The scalar Compton wavenumber is

$$k_\phi(a) = a m_{\text{eff}}(a). \tag{1133}$$

For

$$k \gg k_\phi, \tag{1134}$$

the scalar-mediated scaffold force approaches its long-range limit.

For

$$k \ll k_\phi, \tag{1135}$$

the direct scaffold fifth force is suppressed.

The same effective mass therefore controls both chasing-field clustering and scaffold self-attraction.

217 Adiabatic and Nonadiabatic Pressure

The scalar adiabatic sound speed is

$$c_{a,\phi}^2 = \frac{\dot{p}_\phi}{\dot{\rho}_\phi}. \tag{1136}$$

In general,

$$c_{a,\phi}^2 \neq c_{s,\phi}^2. \tag{1137}$$

The intrinsic nonadiabatic pressure is

$$\delta p_{\mathrm{nad},\phi} = \delta p_\phi - c_{a,\phi}^2 \delta \rho_\phi. \tag{1138}$$

For the coupled scaffold–scalar system, define the relative entropy perturbation

$$S_{\mathrm{scaf}\phi} = 3\left(\zeta_{\mathrm{scaf}} - \zeta_\phi\right). \tag{1139}$$

The total curvature perturbation evolves schematically as

$$\dot{\mathcal{R}} = -\frac{H}{\rho_{\mathrm{tot}} + p_{\mathrm{tot}}} \delta p_{\mathrm{nad}} + \mathcal{S}_Q + \mathcal{O}\left(\frac{k^2}{a^2 H^2}\right), \tag{1140}$$

where

$$\mathcal{S}_Q \tag{1141}$$

contains interaction-induced energy-transfer effects.

The chasing sector can therefore alter large-scale curvature evolution even when its intrinsic rest-frame sound speed equals unity.

218 Ghost, Gradient, and Tachyonic Stability

Benchmark literature. The scalar stability audit follows canonical and noncanonical sound-speed and kinetic-stability analyses (Garriga and Mukhanov, 1999; Armendáriz-Picón et al., 2001; Langlois and Noui, 2016).

For the canonical scalar, the quadratic kinetic coefficient is positive:

$$Q_\phi > 0. \tag{1142}$$

The gradient coefficient gives

$$c_{s,\phi}^2 = 1 > 0. \tag{1143}$$

Thus, the isolated canonical scalar contains neither a kinetic ghost nor a gradient instability.

The coupled system must still satisfy

$$\lambda_A\left(\mathsf{K}_{\mathrm{scaf}\phi}\right) > 0, \tag{1144}$$

and

$$\lambda_A\left(\mathsf{K}_{\mathrm{scaf}\phi}^{-1}\mathsf{G}_{\mathrm{scaf}\phi}\right) \geq 0. \tag{1145}$$

A negative effective mass squared,

$$m_{\mathrm{eff}}^2 < 0, \tag{1146}$$

produces tachyonic growth rather than a kinetic ghost.

It is unacceptable when

$$|m_{\mathrm{eff}}| \gg H \tag{1147}$$

over a required stable epoch.

A slow tachyonic drift may describe controlled rolling, but its timescale and observational effects must be calculated.

219 Strong Coupling and Kinetic Degeneracy

The canonical scalar action remains regular when

$$V, \quad V_{,\phi}, \quad V_{,\phi\phi}, \quad A, \quad \beta \tag{1148}$$

are finite.

A singular effective description can nevertheless arise when:

- $A(\phi)$ approaches zero;
- $\beta(\phi)$ diverges;
- $m_{\rm scaf}(\phi)$ vanishes or diverges;
- the perturbative kinetic matrix becomes degenerate;
- the transition width approaches zero with a finite impulse; or
- higher-order interaction terms become dominant below the required physical scale.

Define the scalar-sector strong-coupling scale

$$\Lambda_{\rm sc}\left(\phi, \rho_{\rm scaf}, \Xi_E\right). \tag{1149}$$

The effective theory requires

$$E_{\rm phys} \ll \Lambda_{\rm sc} \tag{1150}$$

for every mode and epoch to which the calculation is applied.

A formally smooth background is not sufficient if the perturbative cutoff falls below the Hubble, transition, or observational scale.

220 Cross-Leg Chasing State

For bookkeeping, a candidate homogeneous chasing state during leg j may be written as

$$\mathcal{C}_j^{\rm book} = \{\phi_j, \Pi_{\phi,j}, \rho_{\phi,j}, p_{\phi,j}, V_j, \beta_j, Q_{\phi,j}\}, \tag{1151}$$

where, whenever the canonical homogeneous action is valid,

$$\Pi_\phi = a^3\dot{\phi}. \tag{1152}$$

The candidate full-history branch would require an ordered map

$$\mathcal{C}_1^{\rm book} \longrightarrow \mathcal{C}_{\rm O}^{\rm book} \longrightarrow \mathcal{C}_2^{\rm book} \longrightarrow \mathcal{C}_{\rm C}^{\rm book} \longrightarrow \mathcal{C}_3^{\rm book} \tag{1153}$$

derived from one closed action and finite transition laws. The vector is not a gauge-invariant observable and does not establish that the minimum third-leg canonical action remains valid in the hidden legs. It states the information that a selected historical extension must transport or replace with a more fundamental state description.

Cross-leg persistence does not require

$$\rho_{\phi,1} = \rho_{\phi,2} = \rho_{\phi,3}, \tag{1154}$$

or

$$w_{\phi,1} = w_{\phi,2} = w_{\phi,3}. \tag{1155}$$

The field may be latent, dynamically active, or dominant in different intervals. None of those regimes is assigned until the corresponding background is solved.

221 Chasing Through the First Outward Leg

In a candidate full-history extension where the canonical scalar remains a valid effective description during the first E leg,

$$\dot{R}_E^{(1)} > 0. \tag{1156}$$

The chasing field may contribute to continued global expansion through

$$H_1^2 \supset \frac{\rho_{\phi,1}}{3M_{\rm Pl}^2}. \tag{1157}$$

Its acceleration contribution is

$$\mathcal{A}_{\phi,1} = \frac{8\pi G}{3}\left(V_1 - \dot{\phi}_1^2\right). \tag{1158}$$

The first-leg field may be:

- potential dominated;
- kinetic dominated;

- tracking an effective minimum;
- subdominant but nonzero; or
- coupled to a hidden-leg scaffold state.

Paper III does not assume that the first leg is inflationary.

If

$$\epsilon_H^{(1)} < 1, \tag{1159}$$

then that interval is accelerated metric expansion regardless of the sectoral terminology.

The duration and role of any such interval must be reported explicitly.

222 Chasing Through the Second Inward Leg

During the second E leg,

$$\dot{R}_E^{(2)} < 0. \tag{1160}$$

The foundational model may simultaneously satisfy

$$H_2 > 0. \tag{1161}$$

If the same canonical action remains valid and an additional E-related interaction has been derived, the candidate homogeneous equation has the template

$$\ddot{\phi}_2 + 3H_2\dot{\phi}_2 + V_{,\phi}^{(2)} = -\frac{\beta_2}{M_{\rm Pl}}\rho_{\rm scaf,2} + \mathcal{S}_{E\phi}^{(2)}. \tag{1162}$$

The term

$$\mathcal{S}_{E\phi}^{(2)} \tag{1163}$$

represents a possible derived interaction with the E transition or order parameter.

The inward direction of the E trajectory does not imply

$$\dot{\phi}_2 < 0, \tag{1164}$$

nor does it imply

$$H_2 < 0. \tag{1165}$$

The signs of $\dot{R}_E$, $\dot{\phi}$, and H are dynamically distinct.

223 Reversal-Dependent Chasing Dynamics

A future finite-width transition action may contain a covariant order parameter denoted

$$\Xi_E. \tag{1166}$$

The field Ξ_E is not part of minimum third-leg GT1 and is not yet a selected hidden-leg degree of freedom. In a candidate extension, the potential and coupling may depend on it:

$$V = V\left(\phi, \Xi_E\right), \tag{1167}$$

and

$$A = A\left(\phi, \Xi_E\right). \tag{1168}$$

The scalar equation becomes

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}\left(\phi, \Xi_E\right) = -\frac{\beta(\phi, \Xi_E)}{M_{\mathrm{Pl}}}\rho_{\mathrm{scaf}} + \mathcal{S}_{\phi\Xi}. \tag{1169}$$

The Ξ_E sector must carry the compensating energy and momentum implied by the transition-dependent interaction.

Total conservation requires

$$\boxed{\nabla_\mu\left(T^{\mu\nu}_{\phi} + T^{\mu\nu}_{\mathrm{scaf}} + T^{\mu\nu}_{\Xi} + T^{\mu\nu}_{\mathrm{other}}\right) = 0.} \tag{1170}$$

Changing V or A through an externally prescribed time function without including the corresponding transition sector would generally violate the closed energy accounting.

224 Thin and Finite-Width Chasing Matching

For a finite-width regular transition, the field and momentum remain finite:

$$|\phi|, \quad |\dot{\phi}|, \quad |\Pi_\phi| < \infty. \tag{1171}$$

In a thin-transition limit with no localized scalar impulse,

$$[\phi]_j = 0, \tag{1172}$$

and

$$\left[a^3 \dot{\phi}\right]_j = 0. \tag{1173}$$

If a localized transition action

$$S_{\mathrm{tr},j} = \int_{\Sigma_j} \mathrm{d}^3 x \, \sqrt{h} \, \mathcal{L}_{\mathrm{tr},j} \left(\phi, \Xi_E, \ldots\right) \tag{1174}$$

is present, a homogeneous momentum jump may be represented schematically as

$$\left[a^3 \dot{\phi}\right]_j = \mathcal{J}_{\phi,j}, \qquad \mathcal{J}_{\phi,j} \propto - \left.\frac{\delta S_{\mathrm{tr},j}}{\delta \phi}\right|_{\Sigma_j}. \tag{1175}$$

The exact sign, normal orientation, and normalization must be derived from the selected covariant junction convention. The jump and its stress–energy contribution must be solved together.

An arbitrary reset of

$$\phi, \qquad \dot{\phi}, \qquad \rho_\phi \tag{1176}$$

is not an acceptable transition law.

225 Adiabatic and Nonadiabatic Reversal Response

For a canonical perturbation mode v_k, write

$$v_k'' + \omega_k^2(\eta) v_k = 0, \tag{1177}$$

where schematically

$$\omega_k^2 = k^2 + a^2 m_{\rm eff}^2 - \frac{a''}{a} + \Delta_{\rm int}. \tag{1178}$$

The adiabaticity parameter is

$$\mathcal{A}_k = \left| \frac{\omega_k'}{\omega_k^2} \right|. \tag{1179}$$

An adiabatic transition satisfies

$$\mathcal{A}_k \ll 1. \tag{1180}$$

A nonadiabatic transition has

$$\mathcal{A}_k \gtrsim 1 \tag{1181}$$

for at least part of the relevant mode range.

The latter can produce:

- scalar excitations;
- oscillatory transfer features;
- entropy perturbations;
- modified scaffold mass;
- non-Gaussianity;
- transition reheating or energy redistribution; and
- a scale-dependent historical residue.

The transition-generated excitations must be included in the central covariance of Paper II.

226 Chasing–Scaffold Interdependence

The chasing and scaffolding sectors are dynamically linked through

$$\{A(\phi), \beta(\phi), m_{\rm scaf}(\phi), V_{\rm eff}(\phi, n_{\rm scaf})\}. \tag{1182}$$

The chasing trajectory determines:

$$\{\text{scaffold mass drift}, \text{energy transfer}, \text{scalar-force strength}, \text{effective scalar range}\}\,. \tag{1183}$$

The scaffold density determines:

$$\{\text{effective minimum}, \text{effective scalar mass}, \text{chasing trajectory}, \text{scalar perturbation source}\}\,. \tag{1184}$$

Therefore,

$$\boxed{\begin{array}{l}\text{the chasing background and scaffold growth may not be fitted}\\ \text{with independent coupling histories.}\end{array}} \tag{1185}$$

The same $\beta(\phi)$ must appear in:

$$\{Q_{\text{scaf}}, m_{\text{scaf}}, m_{\text{eff}}, \delta\phi, G_{\text{scafscaf}}, \text{transition response}\}\,. \tag{1186}$$

This common dependence is both a constraint and a source of predictive power.

227 Background Observational Tests

A candidate chasing background test vector is

$$\boldsymbol{\mathcal{O}}^{(\text{bg})}_{\text{chase}} = \{H(z), D_L(z), D_A(z), D_M(z), D_V(z), t(z), q_{\text{dec}}(z)\}\,. \tag{1187}$$

The dimensionless expansion function is

$$E(z) = \frac{H(z)}{H_0}. \tag{1188}$$

The chasing density is reconstructed from the coupled equations rather than assumed to scale independently.

A phenomenological late-time summary may use

$$w^{\text{eff}}_{\phi}(a) \approx w_0 + w_a\,(1-a) \tag{1189}$$

over a restricted redshift range.

The parameters

$$w_0, \qquad w_a \tag{1190}$$

are observational summaries.

They are not fundamental GT1 parameters.

The fundamental quantities remain

$$\left\{V(\phi), A(\phi), \phi_{\rm ini}, \dot{\phi}_{\rm ini}\right\}. \tag{1191}$$

228 Perturbative Observational Tests

A candidate chasing perturbation test vector includes

$$\boldsymbol{\mathcal{O}}^{(\rm pert)}_{\rm chase} = \left\{\delta\phi, \delta p_{\rm nad}, \Phi, \Psi, f_{\rm vis}, f_{\rm scaf}, P_m, C_\ell^{XY}, C_\ell^{\phi\phi}\right\}. \tag{1192}$$

The principal observable channels are:

1. changes to matter–radiation equality through scaffold mass drift;
2. modified metric-potential decay;
3. large-angle integrated gravitational effects;
4. modified visible and scaffold growth;
5. gravitational lensing through the redistributed scaffold;
6. relative visible–scaffold velocity and density modes;
7. transition-generated scalar features;
8. cross-correlation with inherited scaffold memory; and
9. late-time scalar clustering near the horizon or Compton scale.

The background and perturbative tests must use one scalar trajectory.

A model that fits distances with one effective $w(a)$ and growth with an unrelated $\mu(k, a)$ has not tested the GT1 chasing theory.

229 Distinction from a Cosmological Constant

The chasing scalar becomes observationally indistinguishable from a cosmological constant when

$$\boxed{\beta = 0, \qquad \dot{\phi} = 0, \qquad V = V_0, \qquad \delta\phi = 0.} \tag{1193}$$

A genuine distinction requires at least one derived nonzero quantity.

Table 4: Candidate distinctions between scalar chasing and an exact cosmological constant.

Quantity	**Cosmological-constant limit**	**Dynamical chasing possibility**
Intrinsic equation of state	$w = -1$	$w_\phi > -1$ for a canonical rolling field.
Effective dilution	Constant density	w_ϕ^{eff} may vary and may appear below -1 through exchange.
Field motion	$\dot{\phi} = 0$	$\dot{\phi} \neq 0$.
Scaffold mass	Constant	$m_{\text{scaf}} = m_{\text{scaf}}(\phi)$.
Energy exchange	$Q_{\text{scaf}} = 0$	$Q_{\text{scaf}} \neq 0$.
Dark fifth force	Absent	$\mathcal{K}^{(\text{QS})}_{\text{scafscaf}} \neq G$ over part of scale space.
Scalar perturbation	No rolling-field mode	$\delta\phi \neq 0$.
Relative growth	Standard gravitational growth	$f_{\text{scaf}} \neq f_{\text{vis}}$.
Cross-leg state	No required hidden-leg scalar history	One field state propagated through both reversals.
Transition residue	Absent	Potentially nonzero oscillatory or correlated feature.

A deviation is scientifically meaningful only when derived from the same $V(\phi)$ and $A(\phi)$ that reproduce the background.

230 Candidate Chasing Signatures

Candidate chasing signatures include:

$$\Delta \ln m_{\text{scaf}} = \frac{1}{M_{\text{Pl}}} \int \beta(\phi) \mathrm{d}\phi, \tag{1194}$$

$$\Delta f_{\text{scafvis}} = f_{\text{scaf}} - f_{\text{vis}}, \tag{1195}$$

$$\Delta\mathcal{K}^{(\mathrm{QS})}_{\mathrm{scaf}} = \mathcal{K}^{(\mathrm{QS})}_{\mathrm{scafscaf}} - G, \tag{1196}$$

$$w^{\mathrm{eff}}_{\phi} + 1 \neq 0, \tag{1197}$$

and

$$\Delta\mathcal{P}^{(\phi,\mathrm{tr})}_{\mathcal{R}}(k) \neq 0. \tag{1198}$$

The quantities above can test dynamical dark energy or dark-sector coupling within the third leg. They do not by themselves identify the hidden first and second legs.

A candidate historical chasing signature must link at least two channels through the same derived transition and coupling model.

For example,

$$\boxed{\Delta\mathcal{P}^{(\mathrm{tr})}_{\mathcal{R}}(k) \longleftrightarrow \Delta\mathcal{P}_{\mathcal{W}}(k) \longleftrightarrow \Delta f_{\mathrm{scafvis}}(k, z)} \tag{1199}$$

through one transition and coupling model.

An isolated late-time deviation in $w(a)$ would support dynamical dark energy more generally.

It would not by itself identify the Triple Bounce history.

231 Chasing Numerical Program

The chasing calculation should proceed through the following controlled stages.

Stage 1: exact cosmological-constant control

Set

$$\beta = 0, \qquad \dot{\phi} = 0, \qquad V = V_0. \tag{1200}$$

Recover the standard constant-vacuum background and perturbation limit.

Stage 2: uncoupled thawing scalar

Use

$$\beta = 0, \qquad \dot{\phi}_{\mathrm{ini}} \approx 0, \tag{1201}$$

and test several initial field positions.

Stage 3: uncoupled freezing or tracking scalar

Use a variable-slope potential and map its basin of attraction.

Stage 4: weak constant coupling

Use

$$|\beta| \ll 1 \tag{1202}$$

inside the stable scalar-attractor region of the background-dynamics analysis.

Stage 5: effective-minimum tracking

Calculate

$$\phi_{\min}(a), \qquad m_{\rm eff}(a), \qquad \frac{m_{\rm eff}}{H}. \tag{1203}$$

Stage 6: early-time control

Verify

$$\Omega_\phi(a) < \Omega_{\phi,\max}^{\rm early}(a). \tag{1204}$$

Stage 7: full linear perturbations

Integrate

$$\{\delta\phi, \delta_{\rm scaf}, \theta_{\rm scaf}, \delta_{\rm vis}, \theta_{\rm vis}, \Phi, \Psi\}. \tag{1205}$$

Stage 8: effective phantom diagnostic

Compare

$$w_\phi \tag{1206}$$

with

$$w_\phi^{\mathrm{eff}} \tag{1207}$$

and verify kinetic positivity.

Stage 9: variable coupling

Test a derived late-activation coupling satisfying

$$\beta_{\mathrm{early}} \approx 0, \qquad \beta_{\mathrm{late}} \neq 0. \tag{1208}$$

Gate before historical extension

Proceed beyond the third-leg study only after one potential, coupling, initial-condition rule, and perturbation implementation passes the constant-vacuum, stability, early-envelope, distance, growth, and lensing checks. Failure at this gate does not justify adding reversal freedom.

Stage 10: first- and second-leg extension

Select a closed hidden-leg action, introduce the corresponding background, and determine whether a scalar state is well defined throughout both intervals.

Stage 11: finite-width reversals

Specify

$$V(\phi, \Xi_E), \qquad A(\phi, \Xi_E), \qquad T^{\Xi}_{\mu\nu}. \tag{1209}$$

Stage 12: transition perturbations

Calculate the mode-dependent adiabaticity parameter

$$\mathcal{A}_k = \left| \frac{\omega_k'}{\omega_k^2} \right|. \tag{1210}$$

Stage 13: central covariance export

Export the chasing-sector contribution to

$$\mathsf{P}_Z^{C+}(k). \tag{1211}$$

Stage 14: joint likelihood

Compare the exact vacuum, uncoupled scalar, coupled scalar, and cross-leg chasing models under common priors.

232 Chasing-Sector Rejection Criteria

The criteria below are claim-specific. Background, perturbative, stability, distance, growth, and lensing failures test the minimum third-leg scalar realization. Hidden-leg state, reversal, covariance, and historical-residue failures test only the full-history extension. The chasing realization fails or requires revision if any of the following is unavoidable.

1. The exact cosmological-constant control cannot be recovered.
2. The scalar density becomes negative in a regime requiring positive energy.
3. The scalar kinetic term has the wrong sign.
4. The physical scalar gradient speed is negative.
5. The scalar potential is unbounded along the required trajectory without a controlled effective description.
6. The field encounters a singular potential derivative.
7. The conformal factor becomes zero or negative.
8. The coupling function diverges.
9. The scaffold mass becomes zero, negative, or divergent.
10. The scalar effective mass becomes singular.
11. A rapid tachyonic instability develops during a required stable epoch.
12. The theory enters strong coupling below the relevant physical scale.
13. The field dominates the radiation era beyond the allowed envelope.
14. The field disrupts matter–radiation equality.

15. The coupled matter epoch has an unacceptable scalar fraction.
16. The field prevents recovery of a sufficiently long pressureless-matter epoch within the observable third-leg benchmark.
17. No stable accelerated late-time solution exists.
18. The accelerated fixed point exists but cannot be reached from a finite initial-condition basin.
19. The acceleration begins at an unacceptable epoch.
20. The scalar reproduces distances but fails structure growth.
21. The same potential cannot reproduce both early-time control and late-time acceleration.
22. A constant-slope exponential model is retained despite its incompatible early- and late-slope requirements.
23. A variable coupling resolves the early-time problem only through a manually timed switch.
24. The effective minimum is unstable.
25. The field cannot adiabatically track the minimum when tracking is claimed.
26. Minimum tracking causes unacceptable scaffold mass drift.
27. The scalar-mediated force required by the chasing model fails scaffold or lensing observations.
28. The background transfer law lacks a covariant perturbative completion.
29. The scalar perturbations destroy acoustic phase coherence.
30. The scalar generates excessive large-scale curvature evolution.
31. The scalar produces excessive nonadiabatic pressure or isocurvature.
32. The effective phantom-like equation of state is interpreted as a fundamental ghost despite a positive canonical kinetic term.
33. A fundamental phantom kinetic term is introduced without a stable completion.
34. The perturbation equations use a sound speed inconsistent with the canonical action.
35. The quasistatic approximation is used during horizon crossing or rapid reversal without validation.
36. The field is described as persistent across all three legs but is defined only during the observable third leg.
37. The first- and second-leg scalar states require unrelated potentials.
38. The field or its canonical momentum becomes singular at either reversal.

39. The potential or coupling is changed externally without including the transition stress–energy.
40. The scalar state is reset manually at the central transition.
41. The transition produces scalar excitations that are omitted from the central covariance.
42. The chasing component is identified with a moving material shell or cosmic boundary without a covariant derivation.
43. The chasing sector is identified with microwave-background radiation.
44. The model reproduces Λ exactly but claims distinct chasing dynamics without a nonzero observable or historical relation.
45. Every nonzero chasing signature is independently adjustable.
46. The same observations are reproduced by a cosmological constant with fewer assumptions and no surviving distinctive prediction.

Proposition 232.1 (Chasing viability and historical identification)**.** *A chasing realization passes the minimum third-leg gate only if one common action and parameter set recovers the constant-vacuum control, maintains stability, satisfies the early envelope, and reproduces the accepted expansion, growth, lensing, and perturbation observables. That success does not imply hidden-leg persistence. Historical support requires the additional derivation and validation of both reversal transfers and at least one linked residue that generic dynamical dark energy does not naturally reproduce.*

233 Formal Summary of the Chasing Sector

The canonical chasing density and pressure are

$$\boxed{\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V, \qquad p_\phi = \frac{1}{2}\dot{\phi}^2 - V.} \tag{1212}$$

The intrinsic equation of state is

$$\boxed{w_\phi = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V}.} \tag{1213}$$

The coupled scalar equation is

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\frac{\beta}{M_{\rm Pl}}\rho_{\rm scaf}.} \tag{1214}$$

The effective equation of state is

$$\boxed{w_\phi^{\mathrm{eff}} = w_\phi + \frac{\beta \rho_{\mathrm{scaf}} \dot{\phi}}{3 H M_{\mathrm{Pl}} \rho_\phi}.} \tag{1215}$$

The effective potential is

$$\boxed{V_{\mathrm{eff}} = V(\phi) + m_{\mathrm{scaf}}(\phi) n_{\mathrm{scaf}}.} \tag{1216}$$

Its minimum satisfies

$$\boxed{V_{,\phi} + \frac{\beta}{M_{\mathrm{Pl}}} \rho_{\mathrm{scaf}} = 0.} \tag{1217}$$

Its effective mass is

$$\boxed{m_{\mathrm{eff}}^2 = V_{,\phi\phi} + \frac{\rho_{\mathrm{scaf}}}{M_{\mathrm{Pl}}^2} \left[\beta^2 + M_{\mathrm{Pl}} \beta_{,\phi}\right].} \tag{1218}$$

The canonical scalar sound speed is

$$\boxed{c_{s,\phi}^2 = 1.} \tag{1219}$$

The exact cosmological-constant limit is

$$\boxed{\beta = 0, \qquad \dot{\phi} = 0, \qquad V = V_0.} \tag{1220}$$

For the declared scalar-dominated exponential fixed point, the conditional acceleration and local-stability region is

$$\boxed{\lambda^2 < 2, \qquad \lambda^2 + \beta\lambda < 3.} \tag{1221}$$

A variable-slope diagnostic capable of separating steep early behavior from shallow late behavior is

$$\boxed{\lambda_{\mathrm{early}}^2 \gg 1, \qquad \lambda_{\mathrm{late}}^2 < 2.} \tag{1222}$$

The candidate cross-leg bookkeeping state is

$$\boxed{\mathcal{C}_j^{\mathrm{book}} = \left\{\phi_j, a_j^3 \dot{\phi}_j, \rho_{\phi,j}, p_{\phi,j}, V_j, \beta_j, Q_{\phi,j}\right\}.} \tag{1223}$$

The reversal adiabaticity statistic is

$$\boxed{\mathcal{A}_k = \left|\frac{\omega_k'}{\omega_k^2}\right|.} \tag{1224}$$

The chasing burden remains split into two gates:

$$\boxed{\begin{aligned} \mathcal{B}^{(3)}_{\text{chase}} &= \text{constant-vacuum recovery} + \text{early control} + \text{late acceleration} \\ &\qquad + \text{stable perturbations} + \text{conserving scaffold exchange} + \text{observational recovery}, \\ \mathcal{B}^{(123)}_{\text{chase}} &= \mathcal{B}^{(3)}_{\text{chase}} + \text{two derived reversal transfers} + \text{historical identification}. \end{aligned}} \tag{1225}$$

The two decisive chasing questions are

$$\boxed{\begin{array}{l} \text{Third-leg viability: can one declared scalar–scaffold action} \\ \text{recover the accepted expansion, stability, growth, lensing, and} \\ \text{perturbation benchmarks with one parameter ledger?} \\ \text{Historical identification: can a closed extension propagate the} \\ \text{same physical state through both reversals and produce a linked} \\ \text{residue not implied by generic dynamical dark energy?} \end{array}} \tag{1226}$$

234 Chasing-Sector Status

The chasing sector is now divided into four controlled levels.

The exact control is

$$\boxed{\text{constant vacuum} : \beta = 0, \quad \dot{\phi} = 0, \quad V = V_0.} \tag{1227}$$

The minimally dynamical model is

$$\boxed{\text{uncoupled canonical scalar} : \beta = 0, \quad \dot{\phi} \neq 0.} \tag{1228}$$

The interacting third-leg model is

$$\boxed{\text{canonical scalar} + \text{conformally coupled scaffold.}} \tag{1229}$$

The still-open full-history chasing extension is

$$\boxed{\text{interacting scalar + hidden-leg continuation + finite-width reversal response.}} \tag{1230}$$

The required calculation sequence is:

1. reproduce the exact cosmological-constant limit;
2. integrate uncoupled thawing and freezing controls;
3. map the stable scalar-attractor basin;
4. apply the early chasing envelope;
5. calculate coupling-induced scaffold mass drift;
6. calculate the effective minimum and scalar mass;
7. test adiabatic minimum tracking;
8. integrate the complete scalar and scaffold perturbations;
9. compare intrinsic and effective equations of state;
10. verify kinetic and gradient stability;
11. calculate background, growth, lensing, and microwave-background observables with one parameter set;
12. test variable-slope potentials;
13. test a covariantly derived late-activation coupling;
14. construct the first- and second-leg chasing backgrounds;
15. introduce a covariant E transition order parameter;
16. derive the outer- and central-reversal scalar transfer;
17. calculate transition adiabaticity and scalar excitation;
18. export the complete chasing contribution to the Paper II central covariance;
19. identify one linked chasing–scaffold historical null test; and
20. compare the full model with a cosmological constant, uncoupled scalar dark energy, and generic interacting dark energy.

The next part develops the complete interaction and exchange architecture: covariant energy and momentum currents, frame decomposition, visible-sector protection, scaffold–chasing exchange, memory transfer, interaction perturbations, reversal coupling, central energy redistribution, conservation audits, and the exact conditions under which the three Gravity Triune functions remain one closed dynamical system.

235 The Interaction-Architecture Problem

Frozen Paper I requires total stress–energy conservation and permits sector exchange only through explicitly defined interaction terms. Frozen Paper II requires the same currents to remain finite through both nonsingular internal transitions and to generate one regular outgoing background and perturbation state. Paper III therefore separates the selected third-leg exchange from the still-open historical extension.

The minimum third-leg burden is

$$\boxed{\mathcal{B}_{\mathrm{int}}^{(3)} = \left\{ \begin{array}{c} \text{one selected action and one sign convention,} \\ \text{equal-and-opposite scaffold–chasing currents,} \\ \text{background and perturbative completion,} \\ \text{zero direct visible and radiation currents,} \\ \text{general-relativity plus }\Lambda\text{CDM control recovery} \end{array} \right\}.} \tag{1231}$$

The full-history burden is

$$\boxed{\mathcal{B}_{\mathrm{int}}^{(123)} = \mathcal{B}_{\mathrm{int}}^{(3)} + \left\{ \begin{array}{c} \text{a derived \textit{Electro} and transition action,} \\ \text{a derived physical memory sector, if one is needed,} \\ \text{finite outer- and central-transition currents,} \\ \text{a conserving thermal handoff and central covariance,} \\ \text{a linked historical residue} \end{array} \right\}.} \tag{1232}$$

The controlling rule is

$$\boxed{\sum_i Q_i^{\nu} = 0.} \tag{1233}$$

No sector may gain or lose stress–energy without a compensating current. The equation does not require every proposed sector to be fundamental, and it does not determine when matter, antimatter, or particle-like excitations first originated.

236 Selected Third-Leg Action and Exact Current

Derivation basis. The selected current is derived from the same conformally coupled action used in coupled quintessence rather than inserted phenomenologically (Bekenstein, 1993; Amendola, 2000).

The only selected interaction action in the present pass is the minimum third-leg GT1 action,

$$
\begin{aligned}
S^{(3)}_{\mathrm{GT1}} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] \\
+ S_{\mathrm{vis}}[g] + S_{\mathrm{rad}}[g] + S_{\mathrm{scaf}}\left[A^2(\phi)g, \psi_{\mathrm{scaf}}\right].
\end{aligned}
\tag{1234}
$$

Define

$$
\beta(\phi) = M_{\mathrm{Pl}}\frac{\mathrm{d}\ln A}{\mathrm{d}\phi}. \tag{1235}
$$

With metric signature $(-+++)$ and the stress–energy convention used throughout Paper III, variation gives

$$
\boxed{\nabla_\mu T^{\mu\nu}_{\mathrm{scaf}} = Q^\nu_{\mathrm{scaf}} = \frac{\beta}{M_{\mathrm{Pl}}}T_{\mathrm{scaf}}\nabla^\nu\phi.} \tag{1236}
$$

The canonical scalar equation is

$$
\boxed{\Box\phi - V_{,\phi} = -\frac{\beta}{M_{\mathrm{Pl}}}T_{\mathrm{scaf}}.} \tag{1237}
$$

Consequently,

$$
\boxed{\nabla_\mu T^{\mu\nu}_{\phi} = -Q^\nu_{\mathrm{scaf}}.} \tag{1238}
$$

Thus the selected dark-sector exchange obeys

$$
\nabla_\mu\left(T^{\mu\nu}_{\mathrm{scaf}} + T^{\mu\nu}_{\phi}\right) = 0. \tag{1239}
$$

The sign of every background, Euler, growth, and perturbation term must be inherited from equation (1236); it may not be chosen independently in a later equation.

237 Energy and Momentum Frame Ledger

For a normalized timelike four-velocity $u^\mu u_\mu = -1$, define

$$\boxed{Q_i^\mu = Q_i u^\mu + F_i^\mu, \qquad Q_i = -u_\mu Q_i^\mu, \qquad u_\mu F_i^\mu = 0.} \tag{1240}$$

Here $F_i^\mu = h^\mu{}_\nu Q_i^\nu$ and $h^\mu{}_\nu = \delta^\mu{}_\nu + u^\mu u_\nu$. Total conservation requires

$$\sum_i Q_i = 0, \qquad \sum_i F_i^\mu = 0. \tag{1241}$$

The total-energy frame, scaffold rest frame, visible-matter frame, and a timelike scalar-gradient frame are generally distinct at perturbative order. A phenomenological statement such as $Q_i^\mu = Q_i u^\mu$ is therefore incomplete until the reference frame is declared.

238 Homogeneous Exchange and the Exact Control

Using units with $c = 1$ in this part, each homogeneous sector satisfies

$$\dot{\rho}_i + 3H(\rho_i + p_i) = Q_i. \tag{1242}$$

For an ideal pressureless scaffold, $T_{\rm scaf} = -\rho_{\rm scaf}$, and the selected action gives

$$\boxed{Q_{\rm scaf} = \frac{\beta}{M_{\rm Pl}} \rho_{\rm scaf} \dot{\phi}, \qquad Q_\phi = -Q_{\rm scaf}.} \tag{1243}$$

Hence

$$\dot{\rho}_{\rm tot} + 3H(\rho_{\rm tot} + p_{\rm tot}) = 0. \tag{1244}$$

The exact constant-vacuum control is

$$\boxed{A(\phi) = A_0 > 0, \quad \beta = 0, \quad \dot{\phi} = 0, \quad \delta\phi = 0, \quad V(\phi) = \rho_\Lambda.} \tag{1245}$$

Recovery of this control tests the third-leg equations. It does not supply evidence for hidden-leg continuation or either internal transition.

239 Visible- and Radiation-Sector Protection

In the selected third-leg action,

$$\boxed{Q^{\mu}_{\rm vis} = 0, \qquad Q^{\mu}_{\rm rad} = 0.} \tag{1246}$$

Visible matter and ordinary radiation are minimally coupled to the physical metric. The absence of a direct scalar current is an action-level control; environmental metric effects from the dark sector must still be calculated.

A future finite transition action may, in principle, source an outgoing radiative, visible-sector, or precursor stress–energy component. In that case its temporary current must be derived from the same transition action and must possess a named compensating source. Paper III does not assume stable ordinary matter at the central transition, does not assign the origin of matter to a particular leg, and does not infer particle production from the late-time conformal coupling.

For ideal classical radiation, $T_{\rm rad} = 0$. This trace statement is a useful check but is not the reason radiation is protected in minimum GT1; radiation is protected because its action is minimally coupled.

240 Particle Number, Field-Dependent Mass, and Production

Outside an explicitly derived production interval, the scaffold number current is conserved:

$$\nabla_\mu J^{\mu}_{\rm scaf} = 0. \tag{1247}$$

With

$$\rho_{\rm scaf} = m_{\rm scaf}(\phi) n_{\rm scaf}, \qquad m_{\rm scaf}(\phi) = m_{{\rm scaf},0} A(\phi), \tag{1248}$$

number conservation yields the selected background exchange in equation (1243). Therefore

$$Q_{\rm scaf} \neq 0 \quad \not\Rightarrow \quad \nabla_\mu J^{\mu}_{\rm scaf} \neq 0. \tag{1249}$$

If a future transition theory contains scaffold number production, write

$$\nabla_\mu J^{\mu}_{\rm scaf} = \Gamma^{(n)}_{\rm scaf}. \tag{1250}$$

When $\rho_{\rm scaf} = m_{\rm scaf} n_{\rm scaf}$ is the complete scaffold energy,

$$\dot\rho_{\rm scaf} + 3H\rho_{\rm scaf} = n_{\rm scaf}\dot m_{\rm scaf} + m_{\rm scaf}\Gamma^{(n)}_{\rm scaf}. \tag{1251}$$

Any additional internal, thermal, or memory energy must be represented by an additional defined contribution to the stress–energy tensor; it may not be hidden inside a second source term that double counts the same energy.

241 Linear Perturbation of the Selected Current

The first variation of the exact covariant current is

$$\delta Q^{\nu}_{\rm scaf} = \frac{1}{M_{\rm Pl}}\Big[\beta\,\delta T_{\rm scaf}\,\nabla^{\nu}\bar{\phi} + \beta\,\bar{T}_{\rm scaf}\,\delta(\nabla^{\nu}\phi) + \beta_{,\phi}\bar{T}_{\rm scaf}\delta\phi\,\nabla^{\nu}\bar{\phi}\Big]. \tag{1252}$$

The variation $\delta(\nabla^{\nu}\phi)$ includes both the field perturbation and the metric perturbation that raises the index. A component formula for δQ, the momentum-transfer potential, or the scaffold Euler equation is valid only after fixing the gauge, Fourier convention, velocity frame, pressure model, and coupling assumptions.

For constant β, conformal Newtonian gauge, the Fourier convention $\nabla^2 \to -k^2$, and an ideal cold scaffold, the adopted component reduction is

$$\boxed{\theta'_{\rm scaf} = -\left(\mathcal{H} + \frac{\beta}{M_{\rm Pl}}\phi'\right)\theta_{\rm scaf} + k^2\left(\Psi + \frac{\beta}{M_{\rm Pl}}\delta\phi\right).} \tag{1253}$$

The same current produces both the velocity-damping modification and the scaffold-only fifth-force potential. Neither term may be assigned an independent coupling parameter.

Gauge-invariant transfer variables may be constructed only from the same current. For example, when $\rho_i' \neq 0$,

$$\delta Q_{i,\rm intr} = \delta Q_i - \frac{Q_i'}{\rho_i'}\delta\rho_i. \tag{1254}$$

Expressions containing $\delta Q_i/Q_i$ are used only when the background current is nonzero and the gauge prescription is explicit.

242 Interaction Stability and Approximation Boundaries

Benchmark literature. Relativistic perturbative stability is checked because phenomenological dark-sector currents can develop early-time large-scale instabilities (Väliviita et al., 2008).

The minimum conformal coupling does not by itself justify stability claims for a completed historical theory. For any proposed extension, expand the action to quadratic order, eliminate nondynamical constraint variables, and then test the kinetic and gradient matrices of the physical modes:

$$\lambda_A(\mathsf{K}_{\text{phys}}) > 0, \qquad \lambda_A\left(\mathsf{K}_{\text{phys}}^{-1}\mathsf{G}_{\text{phys}}\right) \geq 0. \tag{1255}$$

Testing unreduced matrices before solving the gravitational and fluid constraints can misidentify gauge or constraint variables as physical instabilities. The effective theory must also remain below its state-dependent strong-coupling scale.

A negative interval of the effective scaffold damping coefficient

$$\mathcal{D}_{\text{scaf}} = \mathcal{H} + \frac{\beta}{M_{\text{Pl}}}\phi' \tag{1256}$$

is not automatically fatal; its integrated amplification and the full coupled mode system must be evaluated.

243 Historical Interaction Action as a Derivation Target

Paper III does not select a microscopic *Electro* field, a reversal order parameter, or a physical memory field. A future full-history action may have the schematic organization

$$\begin{aligned} S_{\text{hist}}^{(123)} = S_{\text{GT1}}^{(3)} &+ S_E[g,\psi_E] + S_\Xi[g,\Xi_E] + S_{\mathcal{W}}[g,\mathcal{W}] \\ &+ S_{\text{cross}}[g,\phi,\psi_{\text{scaf}},\psi_E,\Xi_E,\mathcal{W},\ldots]. \end{aligned} \tag{1257}$$

This is a field-content ledger, not an adopted action. Every additional term requires defined degrees of freedom, symmetries, dimensions, kinetic terms, stress–energy, equations of motion, and a controlled third-leg reduction.

After such an action exists, a pairwise current graph may be used as bookkeeping:

$$\mathcal{J}^\mu_{i\leftarrow j} = -\mathcal{J}^\mu_{j\leftarrow i}, \qquad Q_i^\mu = \sum_{j\neq i}\mathcal{J}^\mu_{i\leftarrow j}. \tag{1258}$$

The graph does not prove that any proposed edge exists. In the selected third-leg theory, the only nonzero edge is the scaffold–scalar edge.

244 Memory Bookkeeping and Energy Accounting

The inherited-scaffold hypothesis may eventually require a covariant internal state $\mathcal{W}$. Two cases must remain separate.

If $\mathcal{W}$ is only a bookkeeping variable, it carries no independent stress–energy and its relaxation describes loss of recoverable information, classical phase mixing, or coarse-grained correlation.

If $\mathcal{W}$ is a physical field or internal energetic degree of freedom, its action must generate $T^{\mu\nu}_{\mathcal{W}}$ and

$$\nabla_\mu T^{\mu\nu}_{\mathcal{W}} = Q^\nu_{\mathcal{W}}. \tag{1259}$$

Then total conservation requires the compensating destination of any physical memory energy:

$$Q^\nu_{\mathcal{W}} + Q^\nu_{\mathrm{scaf}} + Q^\nu_\phi + Q^\nu_E + Q^\nu_\Xi + Q^\nu_{\mathrm{other}} = 0. \tag{1260}$$

A phenomenological relaxation law is therefore a derivation template, not a completed energy equation. Loss of observable structure and loss of stress–energy are not the same statement.

245 Outer Transition and Second-Leg Exchange

The outer transition is a finite nonsingular interval $\mathcal{I}_{\mathrm{O}} = [t^-_{\mathrm{O}}, t^+_{\mathrm{O}}]$ during which $\dot{R}_E$ changes sign from positive to negative while the global background may retain $H > 0$.

A future finite-width theory must supply

$$\dot{\rho}_i + 3H(\rho_i + p_i) = Q^{(\mathrm{O})}_i, \qquad \sum_i Q^{(\mathrm{O})}_i = 0, \tag{1261}$$

and the corresponding momentum currents. These equations state the conservation burden; they do not identify the physical force causing the reversal.

During the second leg, the minimum scaffold–scalar current may be continued only if the same action remains valid there. A memory state, if physical, requires its own projected transport equation and stress–energy. Neither retention nor damping may be inferred from the word "scaffold" alone.

The outer transition is not assumed to create a third-leg hot plasma, stable matter, or a new singularity.

246 Central Transition and Thermal Handoff

The central transition is the finite nonsingular interval $\mathcal{I}_{\mathrm{C}} = [t_{\mathrm{C}}^{-}, t_{\mathrm{C}}^{+}]$ during which $\dot{R}_E$ changes sign from negative to positive. Its cause and microphysics remain unresolved.

Consistent with frozen Paper II, a derived outgoing state may contain

$$\left\{\begin{matrix} \text{effective radiation,} & \text{optional particle-like excitations,} \\ \text{candidate electromagnetic modes,} & \text{third-leg matter precursors,} \\ \text{scaffold and chasing-sector states,} & \text{an outgoing perturbation covariance} \end{matrix}\right\}. \tag{1262}$$

This list does not require stable ordinary matter, a pair-rich phase, or a complete conventional particle inventory at the instant of reversal. A separate derived thermal-handoff map must connect the central output to the earliest state for which ordinary species and Einstein–Boltzmann evolution are valid.

For each transition current,

$$\dot{\rho}_i + 3H(\rho_i + p_i) = Q_i^{(\mathrm{C})}, \qquad \sum_i Q_i^{(\mathrm{C})} = 0. \tag{1263}$$

If the transition is short compared with the Hubble time, integrated comoving transfer may be a useful approximation, but expansion work must be retained whenever $H\tau_C$ is not negligible. No production fraction may exceed the energy available in the complete incoming state, and no fraction is an independent fitting parameter unless the action derives it.

247 Finite Transition Operators and Covariance

Let $\boldsymbol{\mathcal{B}}_j^{-}$ and $\boldsymbol{\mathcal{B}}_j^{+}$ denote the complete background states on either side of transition $j \in \{\mathrm{O}, \mathrm{C}\}$. A finite solution defines

$$\boldsymbol{\mathcal{B}}_j^{+} = \mathfrak{T}_j[\boldsymbol{\mathcal{B}}_j^{-}]. \tag{1264}$$

The map is the result of integrating the selected equations; it is not an arbitrary junction reset.

After gauge fixing and elimination of nondynamical constraints, let the physical perturbation state obey

$$\boldsymbol{Y}'_{\mathrm{phys}} = \mathsf{A}_{\mathrm{phys}}(k,\eta)\boldsymbol{Y}_{\mathrm{phys}} + \boldsymbol{S}_{\mathrm{phys}}(k,\eta). \tag{1265}$$

For a regular linear ordinary differential system, the finite-width operator is

$$\mathsf{M}_j(k) = \mathcal{T} \exp\left[\int_{\eta_j^-}^{\eta_j^+} \mathsf{A}_{\mathrm{phys}}(k,\eta)\mathrm{d}\eta\right]. \tag{1266}$$

If the system is differential–algebraic, the gravitational and matter constraints must be solved consistently rather than placed into this exponential as independent dynamical modes.

For a source term $\boldsymbol{N}_j$, the outgoing covariance is

$$\begin{aligned}\mathsf{P}_j^+ = {} & \mathsf{M}_j\mathsf{P}_j^-\mathsf{M}_j^\dagger + \mathsf{P}_{N_j} \\ & + \mathsf{M}_j\mathsf{P}_{Y^- N_j} + \mathsf{P}_{N_j Y^-}\mathsf{M}_j^\dagger.\end{aligned} \tag{1267}$$

Every stochastic source and cross-covariance must follow from a declared statistical model. The result must be Hermitian, positive semidefinite, and compatible with the Einstein constraints.

248 Chronological Full-History Composition

When all leg propagators and transition operators have actually been derived, their chronological composition is

$$\boxed{\mathsf{H}_{123} = \mathsf{U}_3\mathsf{M}_{\mathrm{C}}\mathsf{U}_2\mathsf{M}_{\mathrm{O}}\mathsf{U}_1.} \tag{1268}$$

The first-leg propagator acts first and therefore appears on the right. At the present stage, U_1, U_2, M_{O}, and M_{C} are derivation targets rather than calculated operators.

Interaction-induced entropy and curvature transfer must be propagated through the same ordered system. A background-conserving interaction may still fail by generating excessive isocurvature, anisotropy, non-Gaussianity, or unstable relative modes.

249 Conservation, Constraint, and Entropy Audits

Every selected realization must pass the following sequence:

1. derive every field equation by variation;
2. calculate every sector stress–energy tensor;
3. calculate $Q_i^\nu = \nabla_\mu T_i^{\mu\nu}$;
4. verify $\sum_i Q_i^\nu = 0$ analytically;
5. verify the homogeneous total continuity equation;

6. verify the temporal and spatial perturbative constraints;
7. integrate both finite transitions without leaving the constraint surface;
8. verify any claimed number-production law separately;
9. identify the destination of any physical memory energy; and
10. verify nonnegative coarse-grained entropy production only where an irreversible kinetic or thermodynamic model has been defined.

Define the current residual

$$\epsilon_Q = \frac{\sqrt{|\mathcal{C}_{Q,\mu}\mathcal{C}_Q^\mu|}}{H\rho_{\rm tot} + \epsilon_{\rm floor}}, \qquad \mathcal{C}_Q^\mu = \sum_i Q_i^\mu. \tag{1269}$$

The numerical tolerance must be smaller than the physical exchange or historical residue being claimed. Matching a final Friedmann value after an unconstrained jump is not a valid transition solution.

250 Shared Parameters and Staged Numerical Program

The same $A(\phi)$, $\beta(\phi)$, $V(\phi)$, scaffold mass relation, and initial state must control the background transfer, perturbed current, drag, scaffold response kernel, growth, lensing, and all third-leg observables.

The calculation order is deliberately gated:

1. recover the uncoupled general-relativity plus ΛCDM control;
2. implement the exact selected scaffold–scalar current;
3. verify background and perturbative conservation in two gauges;
4. recover the analytical drag and quasistatic scaffold-response limits;
5. test early-time bounds, distances, growth, lensing, and stability;
6. freeze the viable third-leg parameter ledger;
7. only then select any physical memory, *Electro*, or transition action;
8. derive and integrate the outer transition and second-leg continuation;
9. derive and integrate the central transition and thermal handoff;
10. propagate the complete covariance and perform historical null tests.

A failed third-leg realization may not be rescued merely by adding unconstrained reversal or memory parameters.

251 Interaction Failure and Non-Identification Conditions

The minimum third-leg interaction realization fails if any of the following is unavoidable:

1. the scalar and scaffold currents are not derived from the same action;
2. their energy or momentum currents fail to cancel;
3. the background and perturbative sign conventions disagree;
4. the visible or radiation sector acquires an unexplained direct current;
5. field-dependent mass exchange is confused with particle production;
6. a production claim lacks a number-current and compensating energy source;
7. the constant-vacuum control cannot be recovered;
8. the interaction produces ghosts, gradient instability, uncontrolled antifriction, or unacceptable differential growth;
9. independently calibrated couplings are required for background, growth, force, and lensing calculations; or
10. numerical conservation residuals are comparable to the claimed signal.

The historical extension fails if a finite action cannot generate regular currents through both transitions, if a state is manually reset, if physical memory energy disappears without a destination, if the central handoff violates the Paper II constraints, or if the outgoing covariance is unphysical.

A third-leg model may remain viable while historical identification fails. If all surviving effects are degenerate with generic interacting dark energy and no linked two-reversal residue remains, the result is non-identification of Triple Bounce history rather than automatic failure of the selected third-leg action.

252 Formal Summary of the Exchange Architecture

The selected identities are

$$\boxed{\begin{aligned} \nabla_\mu T^{\mu\nu}_{\mathrm{scaf}} &= \frac{\beta}{M_{\mathrm{Pl}}} T_{\mathrm{scaf}} \nabla^\nu \phi, \\ \nabla_\mu T^{\mu\nu}_{\phi} &= -\frac{\beta}{M_{\mathrm{Pl}}} T_{\mathrm{scaf}} \nabla^\nu \phi, \\ Q^\mu_{\mathrm{vis}} &= Q^\mu_{\mathrm{rad}} = 0, \\ \sum_i Q^\mu_i &= 0. \end{aligned}} \tag{1270}$$

The third-leg and full-history burdens remain distinct:

$$\boxed{\mathcal{B}^{(3)}_{\mathrm{int}} \quad \not\Rightarrow \quad \mathcal{B}^{(123)}_{\mathrm{int}}.} \tag{1271}$$

The exact scaffold–scalar current, its background sign, and its perturbative inheritance are selected-action results. A physical memory current, an *Electro* current, reversal-dependent currents, production currents, thermal-handoff fractions, and the two transition operators remain open derivation targets.

The decisive questions are

$$\boxed{\begin{array}{l} \text{Third-leg closure: does one selected current reproduce the} \\ \text{accepted background, stability, growth, lensing, and perturbation} \\ \text{benchmarks with one parameter ledger?} \\ \text{Historical closure: can one finite action extend that state through} \\ \text{both reversals, conserve every current, and generate a linked} \\ \text{historical residue?} \end{array}} \tag{1272}$$

253 Interaction and Reversal Status

The interaction architecture now has the following status:

$$
\boxed{\begin{aligned}
\text{Selected third-leg action} &: \text{specified,}\\
\text{Scaffold–chasing current and sign} &: \text{analytically audited,}\\
\text{Visible and radiation direct currents} &: \text{zero in minimum GT1,}\\
\text{Particle-number bookkeeping} &: \text{audited,}\\
\text{Perturbative current inheritance} &: \text{specified; full numerical solution pending,}\\
\text{Physical memory action} &: \text{not selected,}\\
\text{Microscopic \textit{Electro} action} &: \text{not selected,}\\
\text{Outer and central transition currents} &: \text{not derived,}\\
\text{Thermal handoff and production history} &: \text{not derived,}\\
\text{Historical interaction identification} &: \text{not established.}
\end{aligned}} \tag{1273}
$$

The next part develops the full linear perturbation system while retaining the selected current and all of the restrictions established in this pass.

254 The Linear Perturbation Burden

Frozen Paper I requires the Gravity Triune to preserve general relativity as the classical binding baseline and to distinguish a third-leg observational benchmark from claims about the origin of matter. Frozen Paper II requires one central covariance, one thermal handoff, and one downstream Einstein–Boltzmann evolution. Paper III therefore audits the perturbations in two stages:

$$
\boxed{\mathcal{B}^{(3)}_{\rm pert} = \left\{\begin{array}{c}\text{constraint-preserving GT1 perturbations,}\\ \text{a derived thermal handoff,}\\ \text{one third-leg transfer and observable map}\end{array}\right\},} \tag{1274}
$$

and

$$
\boxed{\mathcal{B}^{(123)}_{\rm pert} = \mathcal{B}^{(3)}_{\rm pert} + \left\{\begin{array}{c}\text{derived first- and second-leg propagators,}\\ \text{finite outer- and central-transition operators,}\\ \text{a linked historical residue}\end{array}\right\}.} \tag{1275}
$$

A viable third-leg perturbation model does not by itself identify the hidden history. Conversely, an underived memory or transition mode may not be added to rescue a failed third-leg realization.

255 Conventions and Scope

Conformal time and the conformal Hubble rate are

$$\mathrm{d}\eta = \frac{\mathrm{d}t}{a}, \qquad \mathcal{H} = \frac{a'}{a}, \qquad X' = \frac{\mathrm{d}X}{\mathrm{d}\eta}. \tag{1276}$$

The metric signature and Fourier convention are

$$(-,+,+,+), \qquad \nabla^2 \longrightarrow -k^2, \qquad \theta_i = \mathrm{i}k_j v_i^j. \tag{1277}$$

Scalar calculations use conformal Newtonian gauge,

$$\boxed{\mathrm{d}s^2 = a^2(\eta)\left[-(1+2\Psi)\mathrm{d}\eta^2 + (1-2\Phi)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right].} \tag{1278}$$

Every equation in this pass is classified as one of the following:

1. an exact identity of the selected third-leg action;
2. a linear equation under the declared gauge and matter content;
3. a controlled approximation with a stated domain; or
4. an open historical template awaiting an action.

256 Einstein Constraints and Metric Response

Perturbation basis. The Einstein constraints and gauge conventions follow standard gauge-invariant cosmological perturbation theory (Bardeen, 1980; Ma and Bertschinger, 1995; Malik and Wands, 2009).

The temporal and scalar momentum constraints are

$$-k^2\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Psi) = 4\pi G a^2 \delta\rho_{\mathrm{tot}}, \tag{1279}$$

$$k^2(\Phi' + \mathcal{H}\Psi) = 4\pi G a^2 \sum_i (\rho_i + p_i)\theta_i. \tag{1280}$$

The scalar anisotropic-stress equation is

$$\boxed{k^2(\Phi - \Psi) = 12\pi G a^2 \sum_i (\rho_i + p_i)\sigma_i.} \tag{1281}$$

The pressure equation is

$$\Phi'' + \mathcal{H}(\Psi' + 2\Phi') + (2\mathcal{H}' + \mathcal{H}^2)\Psi + \frac{k^2}{3}(\Phi - \Psi) = 4\pi G a^2 \delta p_{\rm tot}. \tag{1282}$$

The constraints determine metric response from the complete stress–energy perturbation. They are not additional evolution equations for independently adjustable potentials.

257 Selected Third-Leg Degrees of Freedom

Before a thermal species description is valid, the minimum selected GT1 sector contains the metric, the scaffold number and velocity perturbations, and the chasing-field perturbation. After the thermal handoff, the state also contains the ordinary photon, baryon, neutrino, and any other derived species variables.

A convenient differential–algebraic state is

$$\boldsymbol{Y} = \Big(\delta_n, \theta_{\rm scaf}, \delta\phi, \pi_\phi, \delta_b, \theta_b, \boldsymbol{F}_\gamma, \boldsymbol{F}_\nu, \ldots; \Phi, \Psi\Big)^{\mathsf{T}}, \tag{1283}$$

where $\pi_\phi \equiv \delta\phi'$. The semicolon separates dynamical variables from metric variables fixed by constraints. A numerical code may instead eliminate Φ and Ψ and evolve a reduced physical state.

258 Scaffold Number and Energy Perturbations

The selected scaffold action may be represented by conserved particle number with field-dependent mass,

$$\rho_{\rm scaf} = m_{\rm scaf}(\phi) n_{\rm scaf}, \qquad \frac{\mathrm{d}\ln m_{\rm scaf}}{\mathrm{d}\phi} = \frac{\beta(\phi)}{M_{\rm Pl}}. \tag{1284}$$

In conformal Newtonian gauge, number conservation gives

$$\boxed{\delta_n' = -\theta_{\rm scaf} + 3\Phi'.} \tag{1285}$$

The scaffold energy contrast is not identical to the number contrast:

$$\boxed{\delta_{\rm scaf} = \delta_n + \frac{\beta}{M_{\rm Pl}}\delta\phi.} \tag{1286}$$

For field-dependent β,

$$\delta'_{\rm scaf} = -\theta_{\rm scaf} + 3\Phi' + \frac{\beta}{M_{\rm Pl}}\delta\phi' + \frac{\beta_{,\phi}\phi'}{M_{\rm Pl}}\delta\phi. \tag{1287}$$

This identity is a required consistency check. It prevents the scalar mass perturbation from being counted both inside $\delta_{\rm scaf}$ and as an independent density source.

259 Scaffold Euler Equation

The same conformal coupling that produces the background current gives

$$\boxed{\theta'_{\rm scaf} = -\left(\mathcal{H} + \frac{\beta}{M_{\rm Pl}}\phi'\right)\theta_{\rm scaf} + k^2\left(\Psi + \frac{\beta}{M_{\rm Pl}}\delta\phi\right).} \tag{1288}$$

The two interaction effects are therefore linked:

$$\text{mass-drift term} \longleftrightarrow \frac{\beta\phi'}{M_{\rm Pl}}, \qquad \text{scalar force} \longleftrightarrow \frac{\beta\delta\phi}{M_{\rm Pl}}. \tag{1289}$$

Neither term may be inserted with a sign chosen independently from the selected covariant current.

260 Chasing-Field Perturbation

At fixed scaffold number density, define

$$V_{\rm eff}(\phi, n_{\rm scaf}) = V(\phi) + m_{\rm scaf}(\phi)n_{\rm scaf}. \tag{1290}$$

The background equation is

$$\phi'' + 2\mathcal{H}\phi' + a^2 V_{{\rm eff},\phi} = 0. \tag{1291}$$

The linear field equation is

$$\boxed{\begin{aligned}\delta\phi'' + 2\mathcal{H}\delta\phi' + \left(k^2 + a^2 m^2_{\phi,n}\right)\delta\phi = \phi'(\Psi' + 3\Phi') - 2a^2 V_{{\rm eff},\phi}\Psi \\ - a^2\frac{\beta}{M_{\rm Pl}}\rho_{\rm scaf}\delta_n.\end{aligned}} \tag{1292}$$

where the fixed-number effective mass is

$$\boxed{m^2_{\phi,n} = V_{,\phi\phi} + \frac{\rho_{\rm scaf}}{M^2_{\rm Pl}}\left(\beta^2 + M_{\rm Pl}\beta_{,\phi}\right).} \tag{1293}$$

The source in equation (1292) is δ_n. Replacing it by $\delta_{\rm scaf}$ without simultaneously changing the mass term would double count part of the field-dependent mass response.

261 Scalar Stress–Energy Perturbations

For the canonical chasing field,

$$\delta\rho_\phi = \frac{1}{a^2}\left(\phi'\delta\phi' - \phi'^2\Psi\right) + V_{,\phi}\delta\phi, \tag{1294}$$

$$\delta p_\phi = \frac{1}{a^2}\left(\phi'\delta\phi' - \phi'^2\Psi\right) - V_{,\phi}\delta\phi, \tag{1295}$$

and

$$\delta q_\phi = -\frac{\phi'}{a^2}\delta\phi. \tag{1296}$$

The canonical field has

$$\sigma_\phi = 0, \qquad c_{s,\phi}^2 = 1. \tag{1297}$$

These properties do not eliminate relative entropy generated by the scaffold–chasing interaction.

262 Visible Matter and Radiation Protection

In the minimum third-leg action,

$$Q^\mu_{\rm vis} = Q^\mu_{\rm rad} = 0. \tag{1298}$$

After the thermal handoff and outside production intervals, ideal pressureless visible matter satisfies

$$\boxed{\delta'_{\rm vis} = -\theta_{\rm vis} + 3\Phi', \qquad \theta'_{\rm vis} = -\mathcal{H}\theta_{\rm vis} + k^2\Psi.} \tag{1299}$$

A perfect radiation fluid obeys

$$\delta'_{\rm rad} = -\frac{4}{3}\theta_{\rm rad} + 4\Phi', \qquad \theta'_{\rm rad} = k^2\left(\frac{\delta_{\rm rad}}{4} - \sigma_{\rm rad}\right) + k^2\Psi. \tag{1300}$$

The actual third-leg calculation must use photon and neutrino Boltzmann hierarchies, baryon–photon scattering, recombination, and the same thermal history required in Paper II.

The optical-depth convention is

$$\kappa(\eta) = \int_{\eta}^{\eta_0} a n_e \sigma_T \mathrm{d}\eta', \qquad \kappa' = -a n_e \sigma_T, \qquad g(\eta) = -\kappa' e^{-\kappa}. \tag{1301}$$

263 Gauge-Invariant Modes

The curvature perturbation on uniform total-density hypersurfaces and the comoving curvature perturbation are

$$\zeta = -\Phi - \frac{\mathcal{H}}{\rho'_{\rm tot}}\delta\rho_{\rm tot}, \qquad \mathcal{R} = -\Phi - \frac{\mathcal{H}}{\rho_{\rm tot} + p_{\rm tot}}\delta q_{\rm tot}. \tag{1302}$$

For independently defined sectors,

$$S_{ij} = 3(\zeta_i - \zeta_j). \tag{1303}$$

On large scales, the exact total conservation law gives the schematic relation

$$\mathcal{R}' = -\frac{\mathcal{H}}{\rho_{\rm tot} + p_{\rm tot}}\delta p_{\rm nad} + \mathcal{O}\left(\frac{k^2}{\mathcal{H}^2}\right), \tag{1304}$$

provided every interaction is included in the total stress–energy. Interaction effects enter through the derived nonadiabatic pressure and relative modes; an extra arbitrary source is not added to the total conservation law.

264 Quasistatic Scaffold Response

Approximation basis. The quasistatic response is treated as a controlled subhorizon approximation rather than an exact all-scale identity (Bellini and Sawicki, 2014).

The quasistatic diagnostic is restricted to

$$k^2 \gg \mathcal{H}^2, \qquad |\delta\phi''|, |\mathcal{H}\delta\phi'| \ll k^2|\delta\phi|. \tag{1305}$$

Then the fixed-number field equation becomes

$$\boxed{\left(k^2 + a^2 m_{\phi,n}^2\right)\delta\phi \simeq -a^2\frac{\beta}{M_{\rm Pl}}\rho_{\rm scaf}\delta_n.} \tag{1306}$$

The corresponding scaffold-force response relative to the number source is

$$\boxed{\mu_{\mathrm{force}}^{(n)}(k,a) = 1 + 2\beta^2 \frac{k^2}{k^2 + a^2 m_{\phi,n}^2}.} \tag{1307}$$

This is not a universal Newton constant and it is not a photon-lensing kernel. If it is rewritten in terms of δ_{scaf}, the conversion in equation (1286) must be retained.

265 Metric Lensing and Gravitational Slip

Visible particles and photons remain minimally coupled to $g_{\mu\nu}$. The Weyl potential is

$$\Phi_W = \frac{\Phi + \Psi}{2}. \tag{1308}$$

Lensing must be computed from the Einstein constraints using the total stress–energy perturbation. The scaffold fifth force may alter the scaffold distribution and therefore lensing indirectly, but

$$\boxed{\mu_{\mathrm{force}}^{(n)} \text{ is not inserted directly into } \Phi + \Psi.} \tag{1309}$$

For the minimum canonical scalar and ideal cold scaffold, intrinsic anisotropic stress vanishes. Standard free-streaming species may still produce $\Phi \neq \Psi$. The exact ΛCDM control recovers the standard metric and lensing transfer.

266 Constraint-Preserving Evolution

Let the reduced evolution be

$$\boldsymbol{y}' = \mathsf{A}_{\mathrm{phys}}(k,\eta)\boldsymbol{y}, \tag{1310}$$

and let the Einstein and algebraic identities be

$$\mathsf{C}(k,\eta)\boldsymbol{Y} = 0. \tag{1311}$$

A valid implementation must monitor at least

$$\epsilon_{00} = \frac{|\mathcal{C}_{00}|}{\mathcal{N}_{00}}, \qquad \epsilon_{0i} = \frac{|\mathcal{C}_{0i}|}{\mathcal{N}_{0i}}, \tag{1312}$$

$$\epsilon_{nE} = \left|\delta_{\mathrm{scaf}} - \delta_n - \frac{\beta}{M_{\mathrm{Pl}}}\delta\phi\right|, \qquad \epsilon_Q = \frac{|\sum_i \delta Q_i^\mu|}{\mathcal{N}_Q}. \tag{1313}$$

The normalizations must remain finite and must be declared by the numerical implementation. Constraint drift invalidates the solution even when an individual power spectrum appears acceptable.

267 Paper II Central Covariance and Thermal Handoff

Paper III uses the same independent-mode vector defined in frozen Paper II,

$$\boldsymbol{Z}_{\boldsymbol{k}}^{+} = \left(\mathcal{R}_{\boldsymbol{k}}, S_{1,\boldsymbol{k}}, \ldots, h_{+,\boldsymbol{k}}, h_{\times,\boldsymbol{k}}, V_{1,\boldsymbol{k}}, \ldots\right)^{\mathsf{T}}, \tag{1314}$$

with covariance

$$\boxed{\left\langle \boldsymbol{Z}_{\boldsymbol{k}}^{+} \boldsymbol{Z}_{\boldsymbol{k}'}^{+\dagger} \right\rangle = (2\pi)^3 \delta_D(\boldsymbol{k} - \boldsymbol{k}') \mathsf{P}_Z^{+}(k).} \tag{1315}$$

The covariance must be Hermitian and positive semidefinite:

$$\mathsf{P}_Z^{+} = \mathsf{P}_Z^{+\dagger}, \qquad \mathsf{P}_Z^{+} \succeq 0. \tag{1316}$$

This central covariance does not assert that a standard plasma or stable ordinary matter exists instantaneously at the central transition. Frozen Paper II defines the full thermal handoff as

$$\mathcal{H}_{\mathrm{th}} : \mathcal{I}_{\mathrm{C}}^{+} \longrightarrow \mathcal{I}_{\mathrm{th}}. \tag{1317}$$

For a linear stochastic implementation of that same map, Paper III may write

$$\boxed{\boldsymbol{Y}_{\mathrm{th}} = \mathsf{H}_{\mathrm{th}}(k) \boldsymbol{Z}^{+} + \boldsymbol{N}_{\mathrm{th}}(k),} \tag{1318}$$

where H_{th} is the linearized transfer operator, not a replacement definition of $\mathcal{H}_{\mathrm{th}}$. If $\boldsymbol{N}_{\mathrm{th}}$ is stochastic, its auto- and cross-covariances must be specified. The handoff cannot be replaced by an independently chosen inflationary initial spectrum.

268 Constraint-Compatible Covariance Transfer

For a deterministic third-leg transfer,

$$\boldsymbol{Y}(\eta, k) = \mathsf{U}_3(\eta, \eta_{\mathrm{th}}; k) \boldsymbol{Y}_{\mathrm{th}}(k). \tag{1319}$$

The covariance propagates as

$$\boxed{\mathsf{P}_Y(\eta,k)=\mathsf{U}_3\mathsf{P}_{Y,\mathrm{th}}\mathsf{U}_3^\dagger.} \tag{1320}$$

For constrained variables, the initial covariance must lie on the constraint surface:

$$\boxed{\mathsf{C}\mathsf{P}_Y=0,\qquad \mathsf{P}_Y\mathsf{C}^\dagger=0.} \tag{1321}$$

The safest construction is to assign covariance only to independent physical modes and derive the constrained metric components from them.

269 Acoustic and Matter Transfer

Computational basis. Acoustic and matter transfer requires an Einstein–Boltzmann implementation comparable to established line-of-sight and Boltzmann solvers (Seljak and Zaldarriaga, 1996; Lewis et al., 2000; Blas et al., 2011; Hu and Dodelson, 2002).

The regular adiabatic control must produce coherent photon–baryon oscillations. Schematically,

$$\Theta_0+\Phi=A(k)\cos[kr_s(\eta)]+B(k)\sin[kr_s(\eta)]+\text{driven response}, \tag{1322}$$

where

$$r_s(\eta)=\int_{\eta_{\mathrm{th}}}^{\eta}c_s(\tilde{\eta})\mathrm{d}\tilde{\eta}. \tag{1323}$$

A historical contribution may modulate the spectrum but may not destroy the observed common phase. The matter spectrum generated from the same central covariance is

$$\boxed{P_m(k,z)=\frac{2\pi^2}{k^3}\sum_{A,B}\mathsf{P}^+_{Z,AB}(k)T_{mA}(k,z)T^*_{mB}(k,z).} \tag{1324}$$

All cross-covariances are retained. The matter spectrum, cosmic microwave background, and lensing field may not be initialized from separate unrelated primordial inputs.

270 Vector and Tensor Controls

A canonical scalar has no independent linear vector mode. In the absence of a derived transverse source, vector metric perturbations decay in the standard way. Memory- or reversal-generated vector modes remain open historical extensions.

For the minimum GT1 action, tensor modes satisfy

$$\boxed{h_\lambda'' + 2\mathcal{H}h_\lambda' + k^2 h_\lambda = \frac{2a^2}{M_{\rm Pl}^2}\Pi_\lambda^{\rm TT}.} \tag{1325}$$

Thus the selected action has constant Planck mass and luminal tensor propagation at the equation level. A complete prediction still requires the tensor source, initial covariance, compact-source physics, and observational transfer.

271 Historical Perturbations Remain Open

Possible historical variables include

$$\{\delta\Xi_E, \delta\mathcal{W}_A, \delta E_A, \text{transition-source modes}\}. \tag{1326}$$

They are not active degrees of freedom in the selected GT1 action. Their admissible future form is

$$\boldsymbol{X}_{\rm hist}' = \mathsf{A}_{\rm hist}\boldsymbol{X}_{\rm hist} + \mathsf{S}_{\rm hist}\boldsymbol{y}, \tag{1327}$$

only after an action defines their kinetic terms, stress–energy, constraints, stability, and currents. A scale-by-scale transfer matrix chosen by hand is not a derivation.

272 Numerical Implementation Sequence

The reproducible sequence is:

1. reproduce the general-relativistic plus ΛCDM control;
2. implement the uncoupled canonical scalar;
3. add the action-derived scaffold coupling;
4. verify the number–energy identity and current residuals;
5. solve the full linear system without quasistatic assumptions;

6. recover the quasistatic force kernel only inside its domain;
7. introduce the Paper II central covariance and thermal handoff;
8. propagate one covariance through photons, baryons, neutrinos, scaffolding, and chasing;
9. compute the cosmic microwave background, matter, and lensing observables from that shared transfer;
10. perform convergence, gauge, ablation, and parameter-recovery tests;
11. freeze the viable third-leg parameter ledger; and
12. only then add one derived historical sector at a time.

273 Perturbative Failure Conditions

The minimum third-leg realization fails if any of the following occurs:

1. the Einstein constraints or total-current sum drift under evolution;
2. the number and energy perturbations violate equation (1286);
3. a ghost, gradient instability, singular kinetic coefficient, or ill-posed mode appears in the solved domain;
4. the full solution does not recover the declared quasistatic limit where that approximation applies;
5. the photon-lensing calculation requires direct insertion of the scaffold-force kernel;
6. no common central covariance fits the acoustic phase, matter transfer, and lensing observables;
7. the standard ΛCDM perturbation control is not recovered as $\beta \to 0$, $\delta\phi \to 0$, and $V \to \rho_\Lambda$; or
8. viable perturbations require a background outside the accepted third-leg basin.

The historical extension fails separately if no finite, stable action produces the required memory or reversal transfer. A viable third-leg model with no identifiable historical residue remains a viable dark sector model, but it does not identify Triple Bounce history.

274 Formal Summary of the Linear Perturbation Audit

The selected third-leg scalar core is

$$\boxed{\begin{cases} \delta_n' = -\theta_{\rm scaf} + 3\Phi', \\ \delta_{\rm scaf} = \delta_n + \dfrac{\beta}{M_{\rm Pl}}\delta\phi, \\ \theta_{\rm scaf}' = -\left(\mathcal{H} + \dfrac{\beta\phi'}{M_{\rm Pl}}\right)\theta_{\rm scaf} + k^2\left(\Psi + \dfrac{\beta}{M_{\rm Pl}}\delta\phi\right), \\ \delta\phi'' + 2\mathcal{H}\delta\phi' + \left(k^2 + a^2 m_{\phi,n}^2\right)\delta\phi = \phi'(\Psi' + 3\Phi') - 2a^2 V_{{\rm eff},\phi}\Psi - a^2\dfrac{\beta}{M_{\rm Pl}}\rho_{\rm scaf}\delta_n. \end{cases}} \tag{1328}$$

The metric is determined by the Einstein constraints, and the statistical input is the Paper II covariance $\mathsf{P}_Z^{+}(k)$ followed by the derived thermal handoff. The controlled hierarchy is

$$\boxed{\begin{aligned} &\text{selected-action identities} \longrightarrow \text{constraint-preserving full evolution} \longrightarrow \text{controlled approximations} \\ &\longrightarrow \text{shared observable transfer} \longrightarrow \text{historical extension.} \end{aligned}} \tag{1329}$$

275 Perturbation-System Status

The current status is

$$\boxed{\begin{aligned} \text{Metric and Fourier conventions}: &\ \text{frozen}, \\ \text{Einstein constraints}: &\ \text{audited}, \\ \text{Scaffold number–energy relation}: &\ \text{audited}, \\ \text{Scaffold Euler and scalar-field equations}: &\ \text{sign-consistent with Pass 6}, \\ \text{Quasistatic force kernel}: &\ \text{conditional diagnostic}, \\ \text{Photon-lensing rule}: &\ \text{protected}, \\ \text{Paper II central covariance}: &\ \text{identically inherited}, \\ \text{Thermal handoff}: &\ \text{required; not derived}, \\ \text{Full numerical perturbation solution}: &\ \text{not completed}, \\ \text{Memory and reversal perturbations}: &\ \text{not selected}, \\ \text{Historical identification}: &\ \text{not established.} \end{aligned}} \tag{1330}$$

The next part develops the unified observational and identification program using this audited perturbation architecture.

276 Unified Observation and Inference Burden

Frozen Paper I requires empirical anchors, model definitions, candidate mechanisms, and derived predictions to remain distinct. Frozen Paper II requires one finite parameter ledger, one shared background, the same central covariance, a declared calibration–validation split, explicit complexity control, and separate tests of observational recovery and historical identification. Paper III adopts those requirements without weakening them.

The minimum observational burden is

$$\boxed{\mathcal{B}^{(3)}_{\mathrm{obs}} = \left\{ \begin{array}{c} \text{one action and one background,} \\ \text{one Paper II covariance and thermal handoff,} \\ \text{one parameter and nuisance ledger,} \\ \text{one joint forward model,} \\ \text{calibration, locked validation, and prediction} \end{array} \right\}.} \tag{1331}$$

The stronger historical burden is

$$\boxed{\mathcal{B}^{(123)}_{\mathrm{obs}} = \mathcal{B}^{(3)}_{\mathrm{obs}} + \left\{ \begin{array}{c} \text{derived hidden-leg and transition dynamics,} \\ \text{a linked cross-observable residue,} \\ \text{independent validation against alternatives} \end{array} \right\}.} \tag{1332}$$

A successful third-leg fit is therefore not, by itself, evidence that the hidden first and second legs occurred.

277 One Shared Forward Model

All predicted observables must descend from the same frozen model version,

$$\boxed{\boldsymbol{d}_{\mathrm{th}} = \mathcal{F}\Big[S^{(3)}_{\mathrm{GT1}}, \boldsymbol{\vartheta}, \mathsf{P}^{+}_{Z}(k), \mathsf{H}_{\mathrm{th}}, \boldsymbol{\nu}\Big],} \tag{1333}$$

where $\boldsymbol{\vartheta}$ denotes physical model parameters, $\boldsymbol{\nu}$ denotes nuisance parameters, $\mathsf{P}^{+}_{Z}(k)$ is the central covariance inherited from Paper II, and H_{th} is the linearized representation of the separately derived full handoff $\mathcal{H}_{\mathrm{th}}$. The forward map includes background evolution, perturbation transfer, recombination, nonlinear corrections when justified, and instrument or survey response.

No channel may use a separately tuned background, primordial spectrum, interaction sign, scaffold abundance, or chasing history. In particular,

$$\boxed{\boldsymbol{\vartheta}_{\rm CMB} = \boldsymbol{\vartheta}_{\rm BAO} = \boldsymbol{\vartheta}_{\rm growth} = \boldsymbol{\vartheta}_{\rm lensing} = \boldsymbol{\vartheta}_{\rm local} = \boldsymbol{\vartheta}_{\rm GW}.} \tag{1334}$$

Paper IV packet quantities are excluded from this forward model unless a later integrated audit derives their place without modifying the frozen Paper III equations.

278 Finite Parameter Ledger

The ledger must classify every quantity as fundamental, derived, effective, calibration, or nuisance. A compact third-leg ledger may be written as

$$\boldsymbol{\vartheta} = (\boldsymbol{\vartheta}_{\rm std}, \boldsymbol{\vartheta}_{\rm GT1}, \boldsymbol{\vartheta}_{\rm init}), \qquad \boldsymbol{\nu} = (\boldsymbol{\nu}_{\rm cal}, \boldsymbol{\nu}_{\rm fg}, \boldsymbol{\nu}_{\rm sel}, \boldsymbol{\nu}_{\rm nl}). \tag{1335}$$

The public ledger must record, for each entry:

1. its symbol, units, physical stage, and defining equation;
2. whether it is fitted, fixed, derived, or marginalized;
3. its prior and theoretical admissibility range;
4. the calibration data allowed to constrain it;
5. the validation and held-out observables it predicts; and
6. every observable and nuisance model it affects.

Whole free functions are not counted as single parameters. A potential, coupling, transition profile, covariance feature, or memory kernel must be assigned a finite predeclared representation before inference.

279 Calibration, Validation, and Prediction

The scientific statuses are

$$\boxed{\text{calibration} \neq \text{fit} \neq \text{locked validation} \neq \text{prediction} \neq \text{historical identification.}} \tag{1336}$$

Calibration selects a finite model version and estimates its permitted parameters. Locked validation evaluates the unchanged version on data not used to choose its form. A prediction is a derived observable not used to construct, tune, or rescue the model. Historical identification requires the additional criteria stated below.

Once validation data are inspected, any change to the action, potential, coupling, transition law, covariance feature, parameter range, or nuisance model defines a new model version. Previously inspected data cannot remain held out for that new version.

280 Background and Thermal Observables

Observational basis. The background test set combines distance, expansion, and acoustic-ruler information rather than fitting any one probe in isolation (Riess et al., 1998; Perlmutter et al., 1999; Eisenstein et al., 2005; Alam et al., 2021; DESI Collaboration et al., 2025a).

The shared background determines

$$D_C(z) = \int_0^z \frac{c\,\mathrm{d}z'}{H(z')}, \qquad D_L(z) = (1+z)D_M(z), \qquad D_A(z) = \frac{D_M(z)}{1+z}, \tag{1337}$$

with the curvature dependence contained in D_M. The same $H(z)$ must be used for supernova distances, chronometers, strong-lensing time delays, standard sirens, and every acoustic-scale calculation.

The third-leg thermal tests include:

1. a derived hot-state handoff and nonnegative species densities;
2. successful light-element and recombination baselines;
3. a sound horizon and drag scale calculated from the same background;
4. baryon acoustic oscillation distances using that same sound scale;
5. no hidden alteration of ordinary photon propagation; and
6. consistency of the age and expansion history across all probes.

These are downstream third-leg benchmarks. They do not determine when matter, antimatter, or particle-like excitations first originated in the complete Triple Bounce history.

281 Microwave-Background and Primordial Tests

Observational basis. The microwave-background target is the full temperature, polarization, lensing, and acoustic structure recovered by standard Boltzmann evolution (Hu and Dodelson, 2002; Planck Collaboration et al., 2020; Seljak and Zaldarriaga, 1996; Lewis et al., 2000).

The angular spectra must be generated from the Paper II covariance,

$$C_\ell^{XY} = 4\pi \int \frac{\mathrm{d}k}{k} \sum_{A,B} \mathsf{P}^{+}_{Z,AB}(k)\Delta^X_{\ell,A}(k)\Delta^{Y*}_{\ell,B}(k), \tag{1338}$$

using transfer functions from the same GT1 background and thermal history. The minimum tests are temperature and polarization spectra, acoustic phase coherence, peak heights, damping, lensing, isocurvature, non-Gaussianity, statistical anisotropy, tensor and vector output, and any off-diagonal covariance predicted by the selected model.

Foregrounds, calibration, beam response, noise, and survey masks belong inside the likelihood. A residual is not a cosmological signature until those terms have been marginalized and the result survives a declared validation test.

282 Growth, Lensing, and Nonlinear Structure

Observational basis. Growth and lensing must be computed jointly from the shared metric and matter system (Bartelmann and Schneider, 2001; Ma and Bertschinger, 1995; Bellini and Sawicki, 2014).

The same perturbation solution must generate the matter and velocity observables,

$$\boldsymbol{d}_{\mathrm{LSS}} = \begin{cases} P_{ij}(k,z), \\ f\sigma_8(z), \\ C_\ell^{\gamma\gamma}, \\ C_\ell^{g\gamma}, \\ C_\ell^{\kappa\kappa}, \\ \text{cluster and halo statistics} \end{cases} \tag{1339}$$

where all visible–scaffold cross-covariances are retained. The scaffold fifth-force kernel may affect scaffold motion, but it may not be inserted directly into photon lensing. Lensing remains a prediction of the metric potentials determined by the Einstein equations.

Galaxy bias, intrinsic alignments, baryonic feedback, mass calibration, selection functions, and nonlinear emulators are nuisance or modeling components, not independent evidence for the Gravity Triune. A historical-memory claim additionally requires a derived memory variable and a linked effect that cannot be reproduced by ordinary initial conditions, bias freedom, or late nonlinear physics.

283 Local, Compact-System, and Gravitational-Wave Tests

Observational basis. Local, compact-system, and wave constraints are independent gates on the same parameter region (Will, 2014; Damour and Esposito-Farèse, 1996; LIGO Scientific Collaboration and Virgo Collaboration, 2016; LIGO Scientific Collaboration et al., 2017).

The selected visible-sector metric coupling supplies a general- relativistic control, but realistic local viability still requires solutions in the relevant environments. The unified test set includes:

1. equivalence-principle and post-Newtonian recovery;
2. limits on visible fifth forces and environmental scalar profiles;
3. stellar, compact-object, and binary solutions;
4. scaffold self-interaction and dark fifth-force constraints;
5. gravitational-wave speed, damping, polarizations, and source dynamics;
6. standard-siren consistency with the electromagnetic distance scale; and
7. absence of a local or strong-field pathology used to obtain the cosmological fit.

Standard tensor propagation in the minimum action does not complete the binary or source-amplitude calculation.

284 Joint Likelihood and Cross-Covariance

Inference basis. The joint-likelihood and model-comparison discipline follows established information-criterion and Bayesian-evidence methods (Akaike, 1974; Schwarz, 1978; Kass and Raftery, 1995; Trotta, 2008).

Let the observed data vector be

$$\boldsymbol{d} = (\boldsymbol{d}_{\rm bg}, \boldsymbol{d}_{\rm CMB}, \boldsymbol{d}_{\rm BAO}, \boldsymbol{d}_{\rm LSS}, \boldsymbol{d}_{\rm local}, \boldsymbol{d}_{\rm GW})\,. \tag{1340}$$

For a Gaussian block, the log-likelihood is

$$-2\ln\mathcal{L} = (\boldsymbol{d}-\boldsymbol{d}_{\rm th})^{\mathsf{T}}\,\mathsf{C}^{-1}\,(\boldsymbol{d}-\boldsymbol{d}_{\rm th}) + \ln\det\mathsf{C} + \text{constant}, \tag{1341}$$

with non-Gaussian likelihoods used where required. Shared sky area, calibration, source populations, lensing fields, and sample selection can create off-diagonal covariance. Multiplying probe likelihoods as if they were independent is not permitted when those correlations are material.

The posterior is

$$p(\boldsymbol{\vartheta}, \boldsymbol{\nu} \mid \boldsymbol{d}, \mathcal{M}) \propto \mathcal{L}(\boldsymbol{d} \mid \boldsymbol{\vartheta}, \boldsymbol{\nu}, \mathcal{M}) \pi(\boldsymbol{\vartheta}, \boldsymbol{\nu} \mid \mathcal{M}), \tag{1342}$$

subject to hard theoretical priors from positivity, stability, constraint preservation, and the declared control limits.

285 Benchmark Models and Complexity Control

Inference basis. Nested controls, complexity penalties, and prior-volume effects are retained explicitly in model comparison (Akaike, 1974; Schwarz, 1978; Kass and Raftery, 1995; Trotta, 2008).

The minimum comparison set is

$$\begin{cases} \mathcal{M}_0 : \text{general relativity plus } \Lambda\text{CDM}, \\ \mathcal{M}_1 : \text{uncoupled canonical chasing control}, \\ \mathcal{M}_2 : \text{third-leg GT1 with the selected finite parameterization}, \\ \mathcal{M}_3 : \text{full-history GT1, only after finite transition and memory dynamics are derived.} \end{cases} \tag{1343}$$

Model evidence,

$$Z_{\mathcal{M}} = \int \mathcal{L}(\boldsymbol{d} \mid \boldsymbol{\theta}, \mathcal{M}) \pi(\boldsymbol{\theta} \mid \mathcal{M}) \mathrm{d}\boldsymbol{\theta}, \tag{1344}$$

may be supplemented by information criteria and predictive checks, but no single scalar score substitutes for residual analysis, stability, parameter recovery, and validation. Functional flexibility, feature searches, transition freedom, and post-selection tuning must be counted.

286 Historical Signatures and Identification Standard

A historical signature must be produced by the same action and transition solution that supplies the accepted third-leg background. It must not be pasted onto the primordial spectrum after the fit.

The candidate classes inherited from Paper II include finite-transition oscillations, two-transition interference, scale-dependent structural retention, linked phase or topological statistics, weak directional or parity residues, and cross-correlations among curvature, structure, magnetic, vector, or tensor observables. Their existence and shapes remain to be derived.

Historical identification requires all of the following:

$$\boxed{\begin{aligned}\mathcal{I}_{123} = 1 \Longrightarrow\ & \text{action-derived feature} \wedge \text{cross-observable linkage} \\ & \wedge \text{locked validation} \wedge \text{complexity control} \\ & \wedge \text{superiority to relevant inflationary, bounce, feature, and late-time alternatives.}\end{aligned}} \tag{1345}$$

One anomaly, one fitted oscillation, one low-multipole residual, or one numerical correspondence is insufficient. Failure to identify a unique historical residue does not automatically falsify a viable third-leg GT1 realization; it falsifies or weakens the stronger historical claim.

287 Null Tests, Ablations, and Parameter Recovery

The inference pipeline must pass synthetic-data and structural tests before scientific interpretation:

1. recover the known input parameters from simulated data;
2. verify calibrated posterior coverage;
3. recover $\mathcal{M}_0$ when the data are generated by the control model;
4. set the scaffold–chasing coupling to zero;
5. remove the chasing dynamics while retaining the scaffold;
6. remove historical memory and transition features;
7. remove individual data channels and nuisance families;
8. test feature significance under the full look-elsewhere search; and
9. verify that the result is not driven by one prior boundary or one unmodeled residual.

Every ablation must recompute the shared background and transfer functions rather than changing one spectrum by hand.

288 Decision Logic and Claim-Specific Falsification

The allowed scientific outcomes are

$$\boxed{\begin{cases} \text{A: third-leg GT1 fails theoretical or observational viability,} \\ \text{B: third-leg GT1 is viable but not historically identifying,} \\ \text{C: a full-history realization is a validated historical candidate,} \\ \text{D: historical identification is supported against the declared alternatives.} \end{cases}} \tag{1346}$$

Paper III is falsified as a selected third-leg realization if no stable, constraint-preserving parameter region jointly recovers the required background, thermal, microwave-background, growth, lensing, local, and gravitational-wave controls. The historical extension fails if its transition or memory dynamics are inconsistent, if its residue is not derived, if it disappears under locked validation, or if a less structured alternative explains the same result after complexity control.

A good background fit cannot excuse a failed perturbation or local test. A fitted feature cannot excuse an unstable action. Additional historical or Paper IV parameters may not be introduced solely to rescue either failure.

289 Reproducible Numerical Workflow

The minimum implementation order is:

1. freeze the action, conventions, finite parameterization, and data split;
2. solve the shared background and verify conservation and stability;
3. evolve the Pass 7 perturbation system and Einstein residuals;
4. apply the Paper II central covariance and thermal handoff;
5. generate all linear observables through one Einstein–Boltzmann pipeline;
6. add declared nonlinear and instrument models without changing the theory;
7. validate the likelihood with synthetic parameter-recovery tests;
8. fit only the declared calibration data;
9. freeze the posterior and model version;
10. evaluate validation and held-out observables;
11. run null tests, ablations, posterior-predictive checks, and model comparison; and

12. classify the result using the decision logic above.

The code, parameter ledger, priors, data assignments, covariance choices, and numerical tolerances must be public enough for independent reproduction.

290 Observation and Identification Status

The observation and inference audit establishes the inference architecture, not an observational result. The current status is

$$\boxed{\begin{aligned} \text{Shared forward-model requirement}: &\ \text{frozen}, \\ \text{Paper II covariance inheritance}: &\ \text{frozen}, \\ \text{Finite parameter-ledger rules}: &\ \text{specified}, \\ \text{Probe and nuisance hierarchy}: &\ \text{specified}, \\ \text{Calibration–validation discipline}: &\ \text{specified}, \\ \text{Historical-identification standard}: &\ \text{specified}, \\ \text{Numerical joint inference}: &\ \text{not completed}, \\ \text{Third-leg observational viability}: &\ \text{not established}, \\ \text{Historical identification}: &\ \text{not established}. \end{aligned}} \tag{1347}$$

This status preserves the intended role of Paper III: it provides the equations, controls, and reproducible decision structure that the cosmology community can derive, implement, test, and reject or refine.

291 The Closure Question

The preceding analyses have constructed a candidate effective realization of the Gravity Triune and specified its theoretical and observational burdens.

The remaining question is not whether each sector can be described individually.

It is whether all sectors form one closed theory.

The closure question is

$$\boxed{\begin{array}{l} \text{Does one finite covariant model determine binding, scaffolding,} \\ \text{chasing, their interactions, their perturbations, both} \\ \text{reversals, the central outgoing state, and all third-leg} \\ \text{observables without adding independent functions at each stage?} \end{array}} \tag{1348}$$

Closure requires more than writing formally covariant equations.

The theory must specify:

$$\begin{cases} \text{fundamental or effective degrees of freedom,} \\ \text{one action or equivalent closed field equations,} \\ \text{initial and transition data,} \\ \text{constraint equations,} \\ \text{sector interaction currents,} \\ \text{perturbation dynamics,} \\ \text{observable projection,} \\ \text{parameter and prior structure,} \\ \text{reduction limits,} \\ \text{failure conditions.} \end{cases} \tag{1349}$$

A theory is not closed when any required observable is produced by an independent phenomenological function that is not linked to the common dynamics.

292 Two Distinct Closure Targets

Paper III distinguishes two theories that must not be conflated.

The first is the minimal third-leg Gravity Triune realization:

$$\boxed{\mathcal{G}_{\mathrm{GT1}}^{(3)} = \text{general relativity} + \text{canonical chasing scalar} + \text{conformally coupled cold scaffold.}} \tag{1350}$$

Its purpose is to determine whether the binding, scaffolding, and chasing functions can form a viable interacting third-leg cosmology.

The second is the complete historical realization:

$$\boxed{\mathcal{G}_{\mathrm{GT1}}^{(123)} = \mathcal{G}_{\mathrm{GT1}}^{(3)} + \text{first and second legs} + \text{two finite reversals} + \text{historical transfer.}} \tag{1351}$$

The third-leg theory can be viable even when the historical extension is not identified.

Conversely, a historical interpretation cannot be accepted unless the third-leg theory is already viable.

The logical ordering is

$$\boxed{\text{third-leg closure} \longrightarrow \text{third-leg viability} \longrightarrow \text{historical closure} \longrightarrow \text{historical identification.}} \tag{1352}$$

293 Minimum Complete Third-Leg Model

The minimum complete third-leg action is

$$\begin{aligned} S_{\mathrm{GT1}}^{(3)} = & \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] \\ & + S_{\mathrm{vis}}\left[g_{\mu\nu},\psi_{\mathrm{vis}}\right] + S_{\mathrm{rad}}\left[g_{\mu\nu},\psi_{\mathrm{rad}}\right] \\ & + S_{\mathrm{scaf}}\left[A^2(\phi)g_{\mu\nu},\psi_{\mathrm{scaf}}\right]. \end{aligned} \tag{1353}$$

The corresponding metric equation is

$$\boxed{M_{\mathrm{Pl}}^2 G_{\mu\nu} = T_{\mu\nu}^{\mathrm{vis}} + T_{\mu\nu}^{\mathrm{rad}} + T_{\mu\nu}^{\mathrm{scaf}} + T_{\mu\nu}^{\phi}.} \tag{1354}$$

The scalar equation is

$$\boxed{\Box\phi - V_{,\phi} = -\frac{\beta}{M_{\mathrm{Pl}}}T_{\mathrm{scaf}},} \tag{1355}$$

where

$$\beta(\phi) = M_{\mathrm{Pl}}\frac{\mathrm{d}\ln A}{\mathrm{d}\phi}. \tag{1356}$$

The scaffold current is

$$\boxed{\nabla_\mu T_{\mathrm{scaf}}^{\mu\nu} = \frac{\beta}{M_{\mathrm{Pl}}}T_{\mathrm{scaf}}\nabla^\nu\phi.} \tag{1357}$$

The scalar current is the opposite:

$$\nabla_\mu T_{\phi}^{\mu\nu} = -\frac{\beta}{M_{\mathrm{Pl}}}T_{\mathrm{scaf}}\nabla^\nu\phi. \tag{1358}$$

Visible matter and radiation satisfy

$$\nabla_\mu T_{\mathrm{vis}}^{\mu\nu} = 0, \qquad \nabla_\mu T_{\mathrm{rad}}^{\mu\nu} = 0 \tag{1359}$$

outside production intervals.

This action is the minimum GT1 theory that can be tested without introducing historical memory or reversal microphysics.

294 Minimum Complete Historical Extension

A complete historical realization requires additional covariant degrees of freedom.

A representative extension is

$$\begin{aligned} S_{\mathrm{GT1}}^{(123)} = S_{\mathrm{GT1}}^{(3)} &+ S_{\Xi}\left[g_{\mu\nu}, \Xi_E\right] \\ &+ S_{\mathcal{W}}\left[g_{\mu\nu}, \mathcal{W}_{\mu\nu}, \phi, \Xi_E, \psi_{\mathrm{scaf}}\right] \\ &+ S_E\left[g_{\mu\nu}, E_A, \Xi_E\right] + S_{\mathrm{tr}}\left[\phi, \Xi_E, \mathcal{W}_{\mu\nu}, E_A, \psi_{\mathrm{scaf}}\right]. \end{aligned} \tag{1360}$$

The transition order parameter

$$\Xi_E \tag{1361}$$

must determine the finite outer and central reversal dynamics.

The memory state

$$\mathcal{W}_{\mu\nu} \tag{1362}$$

is required only when persistent historical structure is claimed.

The E degrees of freedom are represented collectively by

$$E_A. \tag{1363}$$

The complete stress–energy tensor is

$$\begin{aligned} T_{\mu\nu}^{\mathrm{tot}} = T_{\mu\nu}^{\mathrm{vis}} + T_{\mu\nu}^{\mathrm{rad}} + T_{\mu\nu}^{\mathrm{scaf}} + T_{\mu\nu}^{\phi} \\ + T_{\mu\nu}^{\Xi} + T_{\mu\nu}^{\mathcal{W}} + T_{\mu\nu}^{E} + T_{\mu\nu}^{\mathrm{other}}. \end{aligned} \tag{1364}$$

It must satisfy

$$\boxed{\nabla_\mu T_{\mathrm{tot}}^{\mu\nu} = 0} \tag{1365}$$

through all three legs and both reversals.

295 Mandatory and Optional Ingredients

The components of the framework have different logical status.

Table 5: Mandatory and optional ingredients in the GT1 realization.

Ingredient	**Status**	**Reason**
Physical metric $g_{\mu\nu}$	Mandatory	Required for visible motion, lensing, causal structure, and metric binding.
Einstein–Hilbert binding	Mandatory in GT1	Defines the conservative metric-binding baseline.
Visible minimal coupling	Mandatory in minimal GT1	Protects visible geodesic motion and local gravity.
Radiation minimal coupling	Mandatory outside production intervals	Preserves ordinary photon and relativistic propagation.
Scaffold sector	Mandatory	Carries the dark-matter-like clustering and structural role.
Chasing sector	Mandatory	Carries the dynamical dark-energy-like role.
Scaffold–chasing interaction	Optional for the exact control; mandatory for interacting GT1	Produces mass drift, dark fifth force, and common sector dynamics.
Canonical chasing scalar	Selected realization	Provides the simplest stable dynamical chasing baseline.
Transition order parameter Ξ_E	Mandatory for full historical closure	A finite reversal cannot remain an externally imposed time rule.
Memory state $\mathcal{W}_{\mu\nu}$	Optional for third-leg viability; mandatory for explicit historical memory	A perfect cold fluid does not uniquely encode prior-leg ancestry.
Elastic scaffold	Optional	One possible realization of a memory-bearing medium.
Particle-production sector	Conditional	Required when the central transition produces new particle populations.

Ingredient	Status	Reason
Modified binding gravity	Not required in GT1	Introduced only if metric binding fails a specific requirement.
Additional chasing fields	Optional	Permitted only if one scalar cannot satisfy the closure conditions.
Historical observable	Not required for third-leg viability; required for historical identification	Absence prevents identification but does not automatically falsify the third-leg theory.

The model should retain the minimum number of sectors capable of satisfying the required physical functions.

296 Action Closure

Action closure requires that every dynamical quantity appearing in the field equations be obtained from one common action or an explicitly declared effective variational principle.

Define the action-closure indicator

$$\mathcal{C}_{\mathrm{act}} = \mathbb{I}\left[\text{all dynamical equations descend from one declared action}\right]. \tag{1366}$$

Action closure requires

$$\boxed{\mathcal{C}_{\mathrm{act}} = 1.} \tag{1367}$$

The indicator vanishes when:

- the scalar potential is changed by an external time function;
- the scaffold coupling is altered manually at a reversal;
- memory damping is introduced without an effective dissipative closure;
- radiation is generated without a transition interaction;
- a fifth force is inserted without the corresponding scalar equation; or
- an observable transfer function is supplied independently of the field dynamics.

An effective theory may contain phenomenological coefficients.

Those coefficients must still belong to a declared operator expansion or constitutive system with a stated domain of validity.

297 Dynamical Closure

Dynamical closure requires enough equations to determine every independent state variable from admissible initial data.

Let the complete state be

$$\boldsymbol{Y} = \left(g_{\mu\nu}, \phi, \psi_{\mathrm{scaf}}, \Xi_E, \mathcal{W}_{\mu\nu}, E_A, \psi_{\mathrm{vis}}, \psi_{\mathrm{rad}}\right). \tag{1368}$$

Let the complete field equations be

$$\mathcal{E}_A\left[\boldsymbol{Y}\right] = 0. \tag{1369}$$

Dynamical closure requires:

$$\mathrm{rank}\left(\frac{\delta \mathcal{E}_A}{\delta Y_B}\right) = N_{\mathrm{phys}} \tag{1370}$$

after gauge freedom and constraints are removed.

Define

$$\mathcal{C}_{\mathrm{dyn}} = \mathbb{I}\left[\text{the physical initial-value problem is determined}\right]. \tag{1371}$$

A model is dynamically underdetermined when it requires arbitrary functions such as

$$\{Q_i(t), \mu(k,t), \Gamma_{\mathrm{mem}}(k,t), T_{\mathrm{O}}(k), T_{\mathrm{C}}(k)\} \tag{1372}$$

without deriving them from a finite parameterization or common action.

298 Conservation Closure

Conservation closure requires

$$\boxed{\sum_i Q_i^{\mu} = 0} \tag{1373}$$

at the background, perturbative, and transition levels.

Define

$$\mathcal{C}_{\text{cons}} = \mathbb{I}\left[\sum_i Q_i^\mu = 0 \text{ analytically and numerically}\right]. \tag{1374}$$

The closure test includes:

$$\begin{cases} \text{background energy exchange,} \\ \text{linear energy transfer,} \\ \text{linear momentum transfer,} \\ \text{particle-number production,} \\ \text{memory relaxation,} \\ \text{transition energy redistribution,} \\ \text{radiative losses,} \\ \text{entropy production.} \end{cases} \tag{1375}$$

A model can satisfy the background continuity equation while violating momentum conservation at the perturbative level.

Therefore,

$$\boxed{\text{background conservation} \not\Longrightarrow \text{full conservation closure.}} \tag{1376}$$

299 Constraint and Initial-Data Closure

A valid initial state must satisfy all gravitational and sectoral constraints.

Let

$$\boldsymbol{\mathcal{C}} = (\mathcal{C}_F, \mathcal{C}_{00}, \mathcal{C}_{0i}, \mathcal{C}_n, \mathcal{C}_\phi, \mathcal{C}_\Xi, \mathcal{C}_\mathcal{W}). \tag{1377}$$

Initial-data closure requires

$$\boxed{\boldsymbol{\mathcal{C}}[\boldsymbol{Y}_{\text{ini}}] = 0.} \tag{1378}$$

The central outgoing state must be generated by

$$\boldsymbol{Y}_{C^+} = \mathfrak{T}_C[\boldsymbol{Y}_{C^-}; \boldsymbol{\theta}_C]. \tag{1379}$$

Its covariance must satisfy

$$\mathsf{P}_{C+} = \mathsf{P}_{C+}^{\dagger}, \qquad \mathsf{P}_{C+} \succeq 0. \tag{1380}$$

Define

$$\mathcal{C}_{\text{ini}} = \mathbb{I}\left[\text{initial data satisfy all constraints and possess a valid covariance}\right]. \tag{1381}$$

Selecting the post-transition densities and perturbations independently does not satisfy initial-data closure.

300 Perturbative Closure

Perturbative closure requires a complete evolution equation for every physical scalar, vector, and tensor mode.

The linear state satisfies

$$\boldsymbol{X}' = \mathsf{A}(k,\eta)\boldsymbol{X} + \boldsymbol{S}(k,\eta). \tag{1382}$$

Define

$$\mathcal{C}_{\text{pert}} = \mathbb{I}\left[\mathsf{A} \text{ and } \boldsymbol{S} \text{ are fully specified for all claimed modes}\right]. \tag{1383}$$

Closure requires:

$$\begin{cases} \text{Einstein constraints,} \\ \text{visible and radiation hierarchies,} \\ \text{scaffold number and velocity perturbations,} \\ \text{chasing-field perturbations,} \\ \text{interaction-current perturbations,} \\ \text{memory perturbations,} \\ \text{transition-field perturbations,} \\ \text{vector modes,} \\ \text{tensor modes.} \end{cases} \tag{1384}$$

A background memory density without a perturbation theory is not a closed cosmological component.

A background energy-transfer function without δQ_i and F_i^{μ} is not a closed interaction model.

301 Stability Closure

After all nondynamical constraints are eliminated, the physical quadratic action has the form

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\left[\dot{\boldsymbol{q}}^{\mathsf{T}}\mathsf{K}\dot{\boldsymbol{q}} - a^{-2}\partial_i\boldsymbol{q}^{\mathsf{T}}\mathsf{G}\partial_i\boldsymbol{q} - \boldsymbol{q}^{\mathsf{T}}\mathsf{M}^2\boldsymbol{q}\right]. \tag{1385}$$

Stability closure requires

$$\mathsf{K} \succ 0, \tag{1386}$$

$$\lambda_A\left(\mathsf{K}^{-1}\mathsf{G}\right) \geq 0, \tag{1387}$$

and

$$E_{\mathrm{phys}} \ll \Lambda_{\mathrm{sc}} \tag{1388}$$

through every claimed regime.

Define

$$\mathcal{C}_{\mathrm{stab}} = \mathbb{I}\left[\text{all physical sectors are stable and within the effective cutoff}\right]. \tag{1389}$$

The closure condition is not limited to the observable third leg.

A full historical model must remain controlled through:

$$\{\mathcal{I}_1, \mathcal{I}_{\mathrm{O}}, \mathcal{I}_2, \mathcal{I}_{\mathrm{C}}, \mathcal{I}_3\}. \tag{1390}$$

302 Observational Closure

Closure basis. Observational closure requires a common parameter region across microwave-background, distance, growth, lensing, local, and gravitational-wave data (Planck Collaboration et al., 2020; Alam et al., 2021; DESI Collaboration et al., 2025a; Will, 2014; LIGO Scientific Collaboration et al., 2017).

Observational closure requires one common map

$$\boxed{\boldsymbol{\mathcal{O}} = \mathfrak{P}\left[S, \boldsymbol{Y}_{\mathrm{ini}}, \boldsymbol{\theta}\right]} \tag{1391}$$

from the action, initial data, and finite parameter vector to every observable.

Define

$$\mathcal{C}_{\rm obs} = \mathbb{I}\left[\text{all observables descend from one shared solution}\right]. \tag{1392}$$

The observable set includes

$$\begin{cases} H(z), D_A, D_L, \\ C_\ell^{XY}, r_s, r_d, \\ P_m, f\sigma_8, \\ \Phi + \Psi, C_\ell^{\kappa\kappa}, \\ \text{halo and cluster statistics,} \\ \text{local and compact-object observables,} \\ \text{gravitational-wave observables.} \end{cases} \tag{1393}$$

Observational closure fails when a separate phenomenological parameter history is fitted to each probe.

303 Historical Identification Closure

Historical identification requires more than full dynamical closure.

It requires at least one nondegenerate observable relation that depends on the ordered two-reversal history.

Define the historical residue operator

$$\mathfrak{R}_{\rm hist} = \mathfrak{P} \circ \left(\mathsf{H}_{123} - \mathsf{H}_3\right), \tag{1394}$$

where

$$\mathsf{H}_3 \tag{1395}$$

denotes a matched third-leg-only model with the same accessible post-central background state.

Historical distinguishability requires

$$\boxed{\mathfrak{R}_{\rm hist} \neq 0} \tag{1396}$$

for at least one observable combination after all allowed third-leg parameters are refitted.

Define

$$\mathcal{C}_{\rm id} = \mathbb{I}\left[\text{one shared two-reversal relation remains observationally identifiable}\right]. \tag{1397}$$

If

$$\mathcal{C}_{\rm id} = 0, \tag{1398}$$

the model may remain physically viable but historically unconfirmed.

304 Complete Closure Product

Define the third-leg closure product

$$\boxed{\mathcal{C}_3 = \mathcal{C}_{\rm act}\mathcal{C}_{\rm dyn}\mathcal{C}_{\rm cons}\mathcal{C}_{\rm ini}\mathcal{C}_{\rm pert}\mathcal{C}_{\rm stab}\mathcal{C}_{\rm obs}.} \tag{1399}$$

A closed third-leg realization requires

$$\mathcal{C}_3 = 1. \tag{1400}$$

Define the historical closure product

$$\boxed{\mathcal{C}_{123} = \mathcal{C}_3\mathcal{C}_\Xi\mathcal{C}_\mathcal{W}\mathcal{C}_{\rm O}\mathcal{C}_{\rm C}\mathcal{C}_{\rm id}.} \tag{1401}$$

Here:

$$\{\mathcal{C}_\Xi, \mathcal{C}_\mathcal{W}, \mathcal{C}_{\rm O}, \mathcal{C}_{\rm C}\} \tag{1402}$$

measure closure of the transition, memory, outer-reversal, and central-reversal sectors.

A historically closed and identified realization requires

$$\mathcal{C}_{123} = 1. \tag{1403}$$

The product notation is organizational.

It does not imply that all failures possess equal scientific weight.

305 Claim Hierarchy

Claims should be graded according to the strongest completed closure level.

Table 6: Claim hierarchy for Paper III.

Claim level	**Required result**	**Permitted statement**
Conceptual organization	The three functions are defined without contradiction.	The Gravity Triune is a role-based framework.
Covariant candidate	One action or closed system is written.	A covariant GT1 candidate can be formulated.
Mathematical closure	All field, current, constraint, and perturbation equations are complete.	The tested realization is mathematically closed.
Theoretical admissibility	Closure plus stability, positivity, and effective-control tests.	A theoretically admissible parameter region exists.
Third-leg viability	Background, perturbative, local, compact, and wave tests are passed.	The realization is compatible with the tested third-leg observations.
Generic nonstandard support	Data favor a scalar, coupling, memory medium, or modified dark sector.	Evidence supports nonstandard dark-sector dynamics.
Historical suggestiveness	A cross-leg-like residue is present, but alternatives remain viable.	The data motivate a historical interpretation.
Historical identification	A linked two-reversal prediction is supported and credible alternatives are disfavored.	The tested three-leg realization is empirically supported within the declared comparison class.
Fundamental theory	A microscopic completion, ultraviolet consistency, and derivation of all effective sectors are supplied.	The framework has a candidate fundamental completion.

Paper III presently develops the first several levels.

It does not establish the final levels by formal construction alone.

306 Necessary Conditions

The following are necessary conditions for third-leg GT1 viability:

$$\mathcal{C}_{\mathrm{act}} = 1, \tag{1404}$$

$$\mathcal{C}_{\mathrm{cons}} = 1, \tag{1405}$$

$$\mathcal{C}_{\mathrm{stab}} = 1, \tag{1406}$$

and

$$\mathcal{M}_{\mathrm{GT1}} \longrightarrow \mathcal{M}_{\mathrm{GR}+\Lambda+\mathrm{CDM}} \tag{1407}$$

through a continuous control limit.

The background must possess a trajectory satisfying

$$\{\text{radiation era}, \text{matter era}, \text{late acceleration}\}\,. \tag{1408}$$

The perturbations must satisfy

$$\begin{cases} \text{acceptable acoustic phase,} \\ \text{acceptable matter growth,} \\ \text{acceptable lensing,} \\ \text{acceptable entropy content,} \\ \text{acceptable tensor propagation.} \end{cases} \tag{1409}$$

Visible matter must retain

$$u^{\mu}_{\mathrm{vis}} \nabla_{\mu} u^{\nu}_{\mathrm{vis}} = 0 \tag{1410}$$

in the minimal model.

These conditions are necessary but not individually sufficient.

307 Sufficient Conditions Are Not Yet Established

No compact analytical set of sufficient conditions has yet been derived for full GT1 viability.

For example,

$$\text{stable accelerated fixed point} \tag{1411}$$

does not guarantee:

$$\{\text{correct equality}, \text{correct acoustics}, \text{correct growth}, \text{correct lensing}, \text{local viability}\}\,. \tag{1412}$$

Similarly,

$$\text{acceptable } H(z) \tag{1413}$$

does not guarantee acceptable perturbations.

A sufficient viability result would require demonstrating the existence of a nonempty parameter region

$$\mathcal{P}_{\text{viable}} \subset \mathcal{P}_{\text{GT1}} \tag{1414}$$

such that every point satisfies all hard theoretical conditions and all required observational likelihood thresholds.

Formally,

$$\boxed{\mathcal{P}_{\text{viable}} = \{\boldsymbol{\theta} : \mathbb{I}_{\text{th}}(\boldsymbol{\theta}) = 1, \quad \mathcal{L}_{\text{joint}}(\boldsymbol{\theta}) \text{ acceptable}\} \neq \varnothing.} \tag{1415}$$

Paper III defines how this region must be calculated.

It does not assume that the region is nonempty.

308 Conditional Obstruction I: No Double Counting of Binding

In general relativity, gravitational binding is represented through spacetime geometry.

There is no unique local covariant gravitational stress–energy tensor that can simply be added to the matter source.

Therefore, the GT1 binding function is

$$\boxed{\mathfrak{G}_{\text{bind}} = \text{metric response in the Einstein equation.}} \tag{1416}$$

The model must not simultaneously include:

$$G_{\mu\nu} \tag{1417}$$

as the geometric binding law and an independently postulated

$$T_{\mu\nu}^{\text{bind}} \tag{1418}$$

representing the same gravitational effect.

A modified-gravity contribution may be moved to the right-hand side:

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{bind,eff}}\right). \tag{1419}$$

Such a representation is bookkeeping.

It does not establish an independent binding substance.

309 Conditional Obstruction II: A Perfect Cold Fluid Does Not Prove Historical Memory

A perfect cold scaffold is specified locally by

$$\{\rho_{\text{scaf}}, u^{\mu}_{\text{scaf}}\}. \tag{1420}$$

These variables do not uniquely identify the path through which the state was produced.

Therefore,

$$\boxed{\text{cold-dark-matter-like clustering} \not\Longrightarrow \text{cross-leg historical memory.}} \tag{1421}$$

A historical memory claim requires at least one additional state variable, nonlocal functional, topological invariant, or transition correlation.

Possible examples include

$$\left\{\mathcal{W}_{\mu\nu}, \varphi^{I}, \mathcal{Q}_{\text{top}}, \mathcal{K}_{\text{path}}\right\}. \tag{1422}$$

The additional state must possess:

$$\{\text{an action}, \text{an evolution law}, \text{stress–energy}, \text{transition transfer}, \text{observable projection}\}. \quad (1423)$$

Without these elements, the memory interpretation remains descriptive.

310 Conditional Obstruction III: Explicit Time Dependence

Suppose the scalar potential is written as

$$V = V(\phi, t). \quad (1424)$$

Then the scalar sector exchanges energy with the external time-dependent structure.

Unless that structure is represented by a dynamical field,

$$\frac{\partial V}{\partial t} \quad (1425)$$

has no compensating sectoral current.

The same problem occurs for

$$A = A(\phi, t), \quad (1426)$$

or

$$T_{\mathrm{O}} = T_{\mathrm{O}}(t), \qquad T_{\mathrm{C}} = T_{\mathrm{C}}(t) \quad (1427)$$

when time is used as an external switching rule.

Closure requires replacing the external clock with a covariant state:

$$t \longrightarrow \Xi_E. \quad (1428)$$

Then

$$V = V(\phi, \Xi_E), \qquad A = A(\phi, \Xi_E), \quad (1429)$$

and the Ξ_E stress–energy carries the compensating exchange.

311 Conditional Obstruction IV: Constant-Slope Early–Late Tension

For a constant-slope exponential potential,

$$V(\phi) = V_0 \exp\left(-\lambda \frac{\phi}{M_{\rm Pl}}\right), \tag{1430}$$

the radiation-scaling scalar fraction is

$$\Omega_\phi^{(R)} = \frac{4}{\lambda^2}. \tag{1431}$$

A small early scalar fraction requires

$$\lambda^2 \gg 1. \tag{1432}$$

The scalar-dominated accelerated fixed point requires

$$\lambda^2 < 2. \tag{1433}$$

One constant value of λ cannot satisfy both conditions when the field occupies both fixed points.

Therefore,

$$\boxed{\text{the simplest constant-slope scaling-to-acceleration pathway is excluded.}} \tag{1434}$$

This does not exclude:

$$\left\{\begin{matrix} \text{thawing initial conditions, variable slope, moving effective minimum,} \\ \text{multiple fields, transition excitation} \end{matrix}\right\}. \tag{1435}$$

312 Conditional Obstruction V: Chasing Cannot Remain Purely Verbal

The phrase

$$\text{“chasing continues outward”} \tag{1436}$$

is not itself covariant.

A coordinate-independent chasing statement must be expressed through quantities such as

$$\left\{\rho_\phi, p_\phi, w_\phi, Q^\mu_\phi, \nabla_\mu\phi\nabla^\mu\phi, q, \epsilon_H\right\}. \tag{1437}$$

For example, active acceleration support may be defined by

$$\rho_\phi + 3p_\phi < 0. \tag{1438}$$

Cross-leg persistence may be defined by the continuous field state

$$\left(\phi, a^3\dot{\phi}\right) \tag{1439}$$

through both reversals.

The chasing sector must not be treated as:

$$\{\text{a material boundary}, \text{a shell at the cosmic edge}, \text{microwave radiation}, \text{the } \textit{Electro} \text{ front itself}\}. \tag{1440}$$

313 Conditional Obstruction VI: Background Fitting Is Insufficient

Two models can possess nearly identical

$$H(z) \tag{1441}$$

while producing different:

$$\left\{\Phi, \Psi, P_m, f_{\rm vis}, f_{\rm scaf}, C_\ell^{XY}, C_\ell^{\phi\phi}\right\}. \tag{1442}$$

Therefore,

$$\boxed{\text{successful background fitting} \not\Longrightarrow \text{cosmological viability.}} \tag{1443}$$

Likewise, fitting a late-time effective equation of state

$$w_{\rm eff}(z) \tag{1444}$$

does not test:

$$\left\{\begin{matrix}\text{scaffold mass drift}, & \text{dark fifth force}, & \text{interaction perturbations}, \\ \text{early scalar density}, & \text{transition covariance} & \end{matrix}\right\}. \tag{1445}$$

The full Einstein–Boltzmann and local-gravity calculations are mandatory.

314 Parameter-Flow Closure

The common parameter flow begins with

$$\{V(\phi), A(\phi), S_\Xi, S_\mathcal{W}, \boldsymbol{Y}_{\rm ini}\}. \tag{1446}$$

These determine the background:

$$\{\phi(a), \rho_{\rm scaf}(a), m_{\rm scaf}(a), H(a), m_{\rm eff}(a)\}. \tag{1447}$$

The background determines the perturbation matrix:

$$\mathsf{A}(k, a). \tag{1448}$$

The perturbation system determines:

$$\{T_{AB}(k, z), \Phi, \Psi, \delta_{\rm vis}, \delta_{\rm scaf}, \delta\phi\}. \tag{1449}$$

These determine observables:

$$\left\{C_\ell^{XY}, P_m, f\sigma_8, C_\ell^{\kappa\kappa}, \text{halo statistics}, \text{local observables}\right\}. \tag{1450}$$

The complete chain is

$$\boxed{S \longrightarrow \boldsymbol{Y}_{\rm bg} \longrightarrow \mathsf{A} \longrightarrow \mathsf{U} \longrightarrow \boldsymbol{\mathcal{O}}.} \tag{1451}$$

Parameter-flow closure fails when any arrow is replaced by an independently fitted phenomenological function.

315 Functional Freedom

A theory with arbitrary functions

$$\{V(\phi), A(\phi), \Gamma_{\text{mem}}(k,t), T_{\text{O}}(k), T_{\text{C}}(k), \mathcal{P}_{\text{source}}(k)\} \tag{1452}$$

can possess enough flexibility to imitate many observable histories.

Functional freedom must therefore be controlled.

A finite basis may be written as

$$V(\phi) = \sum_{n=0}^{N_V} v_n \mathcal{B}_n^{(V)}(\phi), \tag{1453}$$

$$\ln A(\phi) = \sum_{n=0}^{N_A} a_n \mathcal{B}_n^{(A)}(\phi), \tag{1454}$$

and

$$\Gamma_{\text{mem}}(k,t) = \sum_{m,n} g_{mn} \mathcal{B}_m^{(k)}(k) \mathcal{B}_n^{(t)}(t). \tag{1455}$$

The basis order must remain finite and justified.

Alternatively, a microscopic model may determine the functions directly.

Define the functional parameter count

$$N_{\text{func}} = N_V + N_A + N_{\mathcal{W}} + N_{\text{O}} + N_{\text{C}}. \tag{1456}$$

Increasing N_{func} may improve fit while reducing predictivity.

316 Predictivity Budget

Let

$$N_{\text{obs}} \tag{1457}$$

be the effective number of independent observable combinations.

Let

$$N_{\text{id}} \tag{1458}$$

be the number of identifiable physical and nuisance parameter directions.

A simple organizational predictivity budget is

$$\mathcal{B}_{\text{pred}} = N_{\text{obs}} - N_{\text{id}}. \tag{1459}$$

A positive value does not prove predictive success.

It indicates that the model is not trivially saturated by identifiable parameters.

A stronger criterion is the existence of an out-of-sample relation

$$\mathcal{O}_B = \mathcal{F}_B\left[\widehat{\boldsymbol{\theta}}\left(\mathcal{O}_A\right)\right] \tag{1460}$$

that was not used to calibrate $\widehat{\boldsymbol{\theta}}$.

For historical identification, the preferred predictive structure is

$$\boxed{\text{one transition parameter set} \longrightarrow \text{two or more linked observable channels.}} \tag{1461}$$

317 Theory-Completion Gates

The theory must pass the following gates in order.

317.1 Gate T1: action specification

Specify the complete action for the tested realization.

317.2 Gate T2: field equations

Derive all background and perturbation equations from the action.

317.3 Gate T3: conservation

Verify

$$\sum_i Q_i^\mu = 0. \tag{1462}$$

317.4 Gate T4: constraint preservation

Show that the gravitational and sectoral constraints are preserved.

317.5 Gate T5: theoretical health

Demonstrate:

$$\{\text{no ghosts}, \text{no uncontrolled gradient instability}, \text{acceptable cutoff}, \text{regular transitions}\}. \tag{1463}$$

317.6 Gate T6: control limit

Recover general relativity with cold dark matter and a cosmological-constant-like limit.

317.7 Gate T7: historical completion

For the full framework, derive the first and second legs and both transition operators.

Failure at an earlier gate prevents meaningful interpretation of later observational fits.

318 Computation Gates

After the theory gates, the calculation must pass the following computational gates.

318.1 Gate C1: homogeneous controls

Reproduce the uncoupled and exact-vacuum controls.

318.2 Gate C2: background basin

Demonstrate a finite initial-condition basin yielding an acceptable third-leg sequence.

318.3 Gate C3: linear perturbations

Integrate the complete Einstein–Boltzmann system.

318.4 Gate C4: gauge agreement

Verify equality of gauge-invariant outputs in independent gauge implementations.

318.5 Gate C5: quasistatic validation

Map the domain in which the approximate dark-force equations reproduce the full perturbation solution.

318.6 Gate C6: nonlinear structure

Simulate visible and scaffold structure formation with the common force law.

318.7 Gate C7: local and compact systems

Propagate the same coupling and scalar range into weak- and strong-field calculations.

318.8 Gate C8: finite reversals

Construct

$$\mathsf{M}_{\mathrm{O}}(k), \qquad \mathsf{M}_{\mathrm{C}}(k) \tag{1464}$$

from finite transition evolution.

318.9 Gate C9: covariance export

Produce one positive-semidefinite central covariance for all third-leg observables.

319 Observation Gates

The observational gates are:

319.1 Gate O1: background

Fit distances, chronometers, equality, and the acceleration history.

319.2 Gate O2: acoustic observables

Recover microwave-background and baryon acoustic phases with one sound history.

319.3 Gate O3: growth and lensing

Fit visible growth, total matter growth, lensing, and their cross-correlations.

319.4 Gate O4: nonlinear structure

Fit halo, cluster, and offset statistics.

319.5 Gate O5: local gravity

Pass equivalence-principle, post-Newtonian, and environmental tests.

319.6 Gate O6: compact and wave systems

Pass compact-object, binary, scalar-radiation, and tensor-wave tests.

319.7 Gate O7: common parameter region

Demonstrate a nonempty joint viable region.

319.8 Gate O8: historical null test

For historical identification, demonstrate a linked residue that survives alternative-model comparison.

320 Cross-Paper Consistency Gates

Paper III must remain consistent with Papers I and II.

The first cross-paper gate is architectural consistency:

$$\text{Paper III fields} \longleftrightarrow \text{Paper I three-leg architecture.} \tag{1465}$$

The second is transition consistency:

$$\{\mathsf{M}_{\mathrm{O}}, \mathsf{M}_{\mathrm{C}}\}_{\text{Paper III}} = \{\mathcal{T}_{\mathrm{O}}, \mathcal{T}_{\mathrm{C}}\}_{\text{Paper II}} \tag{1466}$$

after notation is reconciled.

The third is covariance consistency:

$$\mathsf{P}_Z^{C+} \text{ must be identical across Papers II and III.} \tag{1467}$$

The fourth is chronology:

$$\boxed{\mathsf{H}_{123} = \mathsf{U}_3 \mathsf{M}_\mathrm{C} \mathsf{U}_2 \mathsf{M}_\mathrm{O} \mathsf{U}_1.} \tag{1468}$$

The fifth is calibration status.

Quantities adopted in Paper I must not be relabeled as Paper III predictions unless newly derived.

The sixth is claims consistency.

A limitation stated in Paper II may not silently become a result in Paper III.

321 Claim-Specific Falsification Map

Different failures possess different scopes.

A failure of the canonical chasing scalar may falsify

$$\mathcal{G}_\mathrm{GT1}^\mathrm{canonical} \tag{1469}$$

without falsifying every possible chasing realization.

A failure of the conformally coupled cold scaffold may falsify

$$\mathcal{G}_\mathrm{GT1}^\mathrm{conformal} \tag{1470}$$

without excluding every interacting scaffold.

A failure of the memory tensor may falsify

$$\mathcal{G}_\mathrm{GT1}^\mathcal{W} \tag{1471}$$

without excluding every possible historical invariant.

A failure of every covariant and stable realization capable of implementing the required three functions would threaten the broader Gravity Triune architecture.

The falsification hierarchy is

$$\boxed{\text{parameter point} \subset \text{specific model} \subset \text{model class} \subset \text{role architecture.}} \tag{1472}$$

The manuscript should always state which level is being rejected and which neighboring claim families remain undecided. A result may propagate across claim families only through a formally derived dependency.

322 Underdetermination Map

The framework remains underdetermined wherever more than one inequivalent model can perform the same declared role.

Current underdetermined elements include:

$$\begin{cases} \text{the exact scalar potential,} \\ \text{the microscopic scaffold identity,} \\ \text{the coupling function,} \\ \text{the memory carrier,} \\ \text{the transition order parameter,} \\ \text{the } \textit{Electro} \text{ covariant field content,} \\ \text{the central particle-production mechanism,} \\ \text{the origin of matter–antimatter asymmetry,} \\ \text{the historical residue template.} \end{cases} \tag{1473}$$

Underdetermination is not identical to inconsistency.

It means that the present framework defines a research program or model class rather than one unique completed theory.

Define the unresolved-theory vector

$$\boldsymbol{U} = (U_V, U_A, U_{\mathrm{scaf}}, U_{\mathcal{W}}, U_\Xi, U_E, U_{\mathrm{prod}}, U_{\mathrm{hist}})\,. \tag{1474}$$

A fully specified realization requires

$$\boldsymbol{U} = \boldsymbol{0}. \tag{1475}$$

Paper III reduces the underdetermination but does not eliminate it.

323 Graded Outcome Structure

The final outcome should use graded categories.

Table 7: Graded outcomes for the GT1 closure analysis.

Outcome	**Condition**	**Meaning**
Internally inconsistent	Field equations, currents, constraints, or definitions contradict one another.	The tested realization is rejected.
Mathematically incomplete	Required fields, currents, perturbations, or transition laws are missing.	The model cannot yet be tested as a closed theory.
Theoretically inadmissible	Closure exists, but ghosts, instabilities, singularities, or cutoff failures are unavoidable.	The tested realization is rejected.
Underdetermined	Multiple free functions remain without a finite predictive parameterization.	The framework remains a model class rather than a unique theory.
Conditionally viable	A theoretically admissible region exists, but one or more required observational calculations remain incomplete.	The realization remains a candidate.
Third-leg viable	One common parameter region passes background, perturbative, local, and strong-field tests.	The tested interacting Gravity Triune realization is observationally allowed.
Observationally degenerate	The model is viable but indistinguishable from a simpler standard or generic interacting model.	No distinct support is obtained.
Generic new physics supported	A nonstandard scalar, coupling, or scaffold effect is favored.	Evidence supports a broader nonstandard dark-sector class.
Historically suggestive	A residue compatible with two reversals appears, but alternatives remain competitive.	Historical interpretation remains provisional.

Outcome	Condition	Meaning
Historically identified within the tested class	One linked shared-parameter residue survives global significance, systematics, prediction, and alternative comparison.	The tested three-leg history is supported within the declared model space.

The manuscript should avoid compressing these distinct outcomes into a binary success-or-failure claim.

324 Priority Research Sequence

The highest-priority work is ordered by dependency.

324.1 Priority 1: complete the minimum third-leg action

Fix one explicit

$$V(\phi), \qquad A(\phi). \tag{1476}$$

324.2 Priority 2: solve the background basin

Determine whether a finite basin yields:

$$\text{radiation} \longrightarrow \text{matter} \longrightarrow \text{late acceleration}. \tag{1477}$$

324.3 Priority 3: implement the full linear system

Construct the Einstein–Boltzmann calculation with action-derived dark interaction.

324.4 Priority 4: test growth and lensing jointly

Determine whether one $\beta(\phi)$ fits:

$$\{P_m, f\sigma_8, \Phi + \Psi, C_\ell^{\kappa\kappa}\}. \tag{1478}$$

324.5 Priority 5: verify local and compact recovery

Propagate the same scalar range and coupling into local systems.

324.6 Priority 6: establish third-leg viability

Do not add historical memory before the minimum coupled model is tested.

324.7 Priority 7: specify a covariant transition field

Construct

$$S_{\Xi} + S_E + S_{\mathrm{tr}}. \tag{1479}$$

324.8 Priority 8: derive the central hot state

Calculate radiation, scaffold, visible, and scalar outputs.

324.9 Priority 9: add historical memory only when required

Introduce $\mathcal{W}_{\mu\nu}$ only if a precise retention claim is retained.

324.10 Priority 10: derive one linked historical prediction

Use one transition parameter set to predict at least two channels.

325 Minimum Numerical Products

Before the GT1 realization can be described as viable, the following numerical products are required.

1. A background phase portrait.
2. A finite initial-condition basin.
3. The histories

$$\left\{H, \phi, \dot{\phi}, \rho_{\mathrm{scaf}}, m_{\mathrm{scaf}}, m_{\mathrm{eff}}, Q_{\mathrm{scaf}}\right\}. \tag{1480}$$

4. A complete linear transfer matrix.
5. Microwave-background temperature and polarization spectra.

6. The baryon acoustic ruler and matter transfer function.
7. Visible, scaffold, and total matter growth.
8. Metric lensing from the same scaffold solution.
9. A map of the quasistatic approximation error.
10. A stability map over the cosmological trajectory.
11. A local weak-field solution.
12. Post-Newtonian quantities.
13. At least one compact-object solution.
14. Tensor-wave propagation and source predictions.
15. A joint viable-parameter search.
16. For the historical model, finite outer and central transition solutions.
17. A positive-semidefinite central covariance.
18. For the historical model, at least one linked observable template.

Analytical consistency arguments remain valuable.

They do not replace these numerical products.

326 Minimum Publication Claims

At the present derivational stage, Paper III may claim that:

1. the Gravity Triune can be translated into a role-based covariant effective framework;
2. binding can be represented conservatively through the general-relativistic metric response;
3. scaffolding can be represented by a cold sector conformally coupled to a canonical scalar;
4. chasing can be represented by the canonical scalar stress–energy;
5. the same coupling generates scaffold mass drift, energy exchange, velocity drag, and a dark-sector fifth force;
6. visible matter can remain minimally coupled and geodesic;
7. the exact cosmological-constant and cold-scaffold control limit exists;
8. the background, perturbative, local, and historical closure conditions can be stated explicitly;

9. a perfect cold scaffold alone does not establish historical memory;
10. a full historical model requires covariant transition and memory dynamics;
11. viability and historical identification are logically distinct; and
12. the proposed realization is falsifiable through a finite hierarchy of theoretical and observational tests.

These are structural and conditional results.

They are not empirical confirmation of the Gravity Triune.

327 Explicit Nonclaims

Paper III does not establish that:

1. binding, scaffolding, and chasing are three fundamental substances;
2. general relativity is replaced;
3. dark matter has been microscopically identified;
4. dark energy is a material shell moving outward;
5. the chasing field is uniquely determined;
6. the scaffold must be elastic;
7. the first-leg web has been shown to survive both reversals;
8. the outer or central transition has been derived from fundamental microphysics;
9. the central transition produces the observed baryon asymmetry;
10. electromagnetism alone generates matter–antimatter asymmetry;
11. the hot third-leg particle spectrum has been calculated;
12. the early thermal history has been numerically reproduced;
13. the complete microwave-background spectra have been fitted;
14. the matter power spectrum has been fitted;
15. the local or compact-system parameter region has been established;
16. a historical residue has been detected;
17. a pre-third-leg history is required by current observations;

18. the full three-leg model is more probable than ΛCDM or other alternatives;
19. a microscopic quantum-gravity completion has been supplied; or
20. Paper IV's mapped geometry has been justified.

These exclusions protect the distinction between formal architecture, candidate realization, completed calculation, and empirical result.

328 Integrated Claim-Specific Rejection Conditions

The specific frozen GT1 realization is rejected or requires a new version if any unavoidable condition below is established. Each item must be assigned to minimum third-leg GT1, the historical extension, or historical identification before the verdict is reported.

1. No stable canonical-scalar and conformal-scaffold parameter region exists.
2. The required coupling violates visible-sector gravitational limits.
3. The scaffold fifth force cannot fit growth and lensing simultaneously.
4. The model cannot preserve a sufficiently standard radiation era.
5. The model cannot preserve a sufficiently long matter era.
6. No stable accelerated late trajectory exists.
7. The exact control limit cannot be recovered.
8. The perturbation system contains an unavoidable ghost.
9. The perturbation system contains an unavoidable rapid gradient instability.
10. The effective theory becomes strongly coupled below the required scale.
11. The interaction current cannot be made covariantly conserving.
12. The Einstein constraints cannot be preserved.
13. The central initial covariance cannot remain positive semidefinite.
14. The acoustic phase cannot be reproduced.
15. The microwave-background and baryon acoustic rulers require incompatible histories.
16. The same parameter region cannot fit matter growth and lensing.
17. The local and cosmological scalar ranges are incompatible.
18. Compact-object solutions are unavoidably unstable.

19. The tensor propagation law conflicts with the minimal gravitational action.
20. No nonempty joint viable parameter region exists.
21. The historical extension requires singular reversals.
22. The transition field must be externally prescribed rather than dynamically closed.
23. Historical memory requires an unphysical or unstable medium.
24. Every apparent historical residue can be absorbed into freely chosen third-leg initial conditions.
25. The full model has no predictive restriction beyond independently fitted effective functions.

Failure of the historical conditions does not automatically reject the minimum third-leg interacting theory, the Gravity Triune role architecture, or the Triple Bounce directional history.

Failure of the minimum third-leg theory rejects the selected frozen GT1 realization. It does not prove inflation necessary or decide Paper II's independent horizon and flatness counterclaims.

329 Paper III Outcome Logic

The logical outcome of Paper III may be written as

$$\mathcal{O}_{\text{Paper III}} = \mathcal{F}\left(\mathcal{C}_3, \mathcal{V}_{\text{GT1}}, \mathcal{C}_{123}, \mathcal{I}_{\text{hist}}\right). \tag{1481}$$

The principal cases are:

$$\mathcal{C}_3 = 0 \quad \Longrightarrow \quad \text{mathematically incomplete.} \tag{1482}$$

$$\mathcal{C}_3 = 1, \quad \mathcal{V}_{\text{GT1}} = 0 \quad \Longrightarrow \quad \text{closed but nonviable.} \tag{1483}$$

$$\mathcal{C}_3 = 1, \quad \mathcal{V}_{\text{GT1}} = 1 \quad \Longrightarrow \quad \text{third-leg viable.} \tag{1484}$$

$$\mathcal{V}_{\text{GT1}} = 1, \quad \mathcal{C}_{123} = 0 \quad \Longrightarrow \quad \text{viable but historically incomplete.} \tag{1485}$$

$$\mathcal{C}_{123} = 1, \quad \mathcal{I}_{\text{hist}} = 0 \quad \Longrightarrow \quad \text{historically closed but observationally unidentified.} \tag{1486}$$

$$\mathcal{C}_{123} = 1, \quad \mathcal{I}_{\text{hist}} = 1 \quad \Longrightarrow \quad \text{historically supported within the tested comparison class.} \tag{1487}$$

The present manuscript should not select the final case before the required calculations are performed.

330 Current Scientific Status of the Gravity Triune

At the current stage, the Gravity Triune has been translated from a role-based conceptual proposal into a candidate covariant effective research program.

The minimum GT1 realization possesses:

$$\begin{cases} \text{a definite metric-binding baseline,} \\ \text{a canonical chasing realization,} \\ \text{a conformally coupled scaffold realization,} \\ \text{an action-derived exchange current,} \\ \text{a continuous standard control limit,} \\ \text{explicit background equations,} \\ \text{explicit perturbation equations,} \\ \text{local and strong-field recovery conditions,} \\ \text{transition and memory closure requirements,} \\ \text{observational and falsification criteria.} \end{cases} \tag{1488}$$

It does not yet possess:

$$\begin{cases} \text{a demonstrated nonempty viable parameter region,} \\ \text{a completed Einstein–Boltzmann implementation,} \\ \text{a completed nonlinear simulation suite,} \\ \text{a derived finite reversal solution,} \\ \text{a derived central hot-state spectrum,} \\ \text{a demonstrated historical residue,} \\ \text{empirical preference over benchmark models.} \end{cases} \tag{1489}$$

The appropriate present status is therefore

$$\boxed{\begin{array}{l}\text{covariantly formulable,}\\ \text{structurally testable,}\\ \text{partially dynamically specified,}\\ \text{not yet numerically validated,}\\ \text{not yet historically identified.}\end{array}} \tag{1490}$$

331 Formal Summary of the Closure Analysis

The minimum third-leg action is

$$\boxed{S^{(3)}_{\mathrm{GT1}} = S_{\mathrm{EH}} + S_{\mathrm{vis}}[g] + S_{\mathrm{rad}}[g] + S_{\phi}[g,\phi] + S_{\mathrm{scaf}}[A^2(\phi)g, \psi_{\mathrm{scaf}}].} \tag{1491}$$

The minimum interaction current is

$$\boxed{Q^{\nu}_{\mathrm{scaf}} = \frac{\beta}{M_{\mathrm{Pl}}} T_{\mathrm{scaf}} \nabla^{\nu}\phi, \qquad Q^{\nu}_{\phi} = -Q^{\nu}_{\mathrm{scaf}}.} \tag{1492}$$

The third-leg closure product is

$$\boxed{\mathcal{C}_3 = \mathcal{C}_{\mathrm{act}}\mathcal{C}_{\mathrm{dyn}}\mathcal{C}_{\mathrm{cons}}\mathcal{C}_{\mathrm{ini}}\mathcal{C}_{\mathrm{pert}}\mathcal{C}_{\mathrm{stab}}\mathcal{C}_{\mathrm{obs}}.} \tag{1493}$$

The historical closure product is

$$\boxed{\mathcal{C}_{123} = \mathcal{C}_3\mathcal{C}_{\Xi}\mathcal{C}_{\mathcal{W}}\mathcal{C}_{\mathrm{O}}\mathcal{C}_{\mathrm{C}}\mathcal{C}_{\mathrm{id}}.} \tag{1494}$$

The parameter-flow closure is

$$\boxed{S \longrightarrow \boldsymbol{Y}_{\mathrm{bg}} \longrightarrow \mathsf{A} \longrightarrow \mathsf{U} \longrightarrow \boldsymbol{\mathcal{O}}.} \tag{1495}$$

The physical viability region is

$$\boxed{\mathcal{P}_{\mathrm{viable}} = \{\boldsymbol{\theta} : \mathbb{I}_{\mathrm{th}} = 1, \quad \mathcal{L}_{\mathrm{joint}} \text{ acceptable}\}.} \tag{1496}$$

A viable realization requires

$$\boxed{\mathcal{P}_{\mathrm{viable}} \neq \varnothing.} \tag{1497}$$

Historical distinguishability requires

$$\boxed{\mathfrak{R}_{\text{hist}} = \mathfrak{P} \circ (\mathsf{H}_{123} - \mathsf{H}_3) \neq 0} \tag{1498}$$

after allowed third-leg parameters are refitted.

The complete logical distinction is

$$\boxed{\begin{aligned}&\text{covariant formulation} \neq \text{mathematical closure}\\ &\neq \text{theoretical admissibility} \neq \text{third-leg viability}\\ &\neq \text{generic evidence for new physics} \neq \text{historical identification.}\end{aligned}} \tag{1499}$$

The decisive closure question is

$$\boxed{\begin{aligned}&\text{Can the minimum GT1 action produce a nonempty stable parameter}\\ &\text{region that passes the complete third-leg background,}\\ &\text{perturbative, lensing, local, compact, and tensor tests, and can}\\ &\text{a subsequently completed transition and memory sector generate}\\ &\text{one linked two-reversal prediction that cannot be absorbed into}\\ &\text{freely chosen third-leg initial conditions?}\end{aligned}} \tag{1500}$$

332 Closure Status

The integrated closure analysis establishes the following.

1. The minimum third-leg GT1 theory can be stated through one covariant action.
2. Binding is represented through the general-relativistic metric response rather than an independent binding substance.
3. Scaffolding and chasing can exchange energy and momentum through one action-derived conformal current.
4. Visible matter can remain minimally coupled.
5. The exact constant-vacuum and cold-scaffold control limit is available.
6. The third-leg model possesses explicit background and perturbation equations.
7. The local, compact, tensor, and observational recovery conditions are defined.
8. The historical model requires additional transition and memory degrees of freedom.

9. A perfect cold scaffold does not by itself prove cross-leg ancestry.
10. Externally switched reversal laws do not constitute a closed theory.
11. The simplest constant-slope scaling-to-acceleration pathway contains a conditional early–late obstruction.
12. A successful background does not establish perturbative viability.
13. A viable third-leg interaction does not establish the Triple Bounce history.
14. A historical residue must survive refitting of all allowed third-leg initial conditions.
15. The current theoretical status remains conditional until the required numerical products are generated.

333 Integrated Discussion

Paper III asked whether the three proposed Gravity Triune functions can be translated from role-based language into a covariant and falsifiable effective framework.

The three functions are:

$$\boxed{\mathfrak{G}_{\rm bind} = \text{binding}, \qquad \mathfrak{G}_{\rm scaf} = \text{scaffolding}, \qquad \mathfrak{G}_{\rm chase} = \text{chasing}.} \tag{1501}$$

The selected GT1 realization identifies them as:

$$\boxed{\begin{aligned} \mathfrak{G}_{\rm bind} &= \text{the general-relativistic metric response}, \\ \mathfrak{G}_{\rm scaf} &= \text{a cold gravitating sector conformally coupled to } \phi, \\ \mathfrak{G}_{\rm chase} &= \text{a canonical scalar field } \phi. \end{aligned}} \tag{1502}$$

This translation resolves one central ambiguity.

Binding is not introduced as a separate material stress–energy tensor in addition to the Einstein tensor.

Instead,

$$\boxed{\begin{array}{l} \text{binding is the conservative geometric response of spacetime} \\ \text{to the complete stress–energy source.} \end{array}} \tag{1503}$$

Scaffolding and chasing are represented as dynamical source sectors.

Their interaction is generated by one conformal factor,

$$A(\phi), \tag{1504}$$

rather than by independently selected background and perturbative exchange functions.

The minimum third-leg action is

$$\begin{aligned} S_{\mathrm{GT1}}^{(3)} = &\int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] \\ &+ S_{\mathrm{vis}}[g,\psi_{\mathrm{vis}}] + S_{\mathrm{rad}}[g,\psi_{\mathrm{rad}}] + S_{\mathrm{scaf}}\left[A^2(\phi)g,\psi_{\mathrm{scaf}}\right]. \end{aligned} \tag{1505}$$

The interaction current is

$$\boxed{Q^\nu_{\mathrm{scaf}} = \frac{\beta}{M_{\mathrm{Pl}}}T_{\mathrm{scaf}}\nabla^\nu\phi, \qquad Q^\nu_\phi = -Q^\nu_{\mathrm{scaf}},} \tag{1506}$$

where

$$\beta(\phi) = M_{\mathrm{Pl}}\frac{\mathrm{d}\ln A}{\mathrm{d}\phi}. \tag{1507}$$

The same coupling determines:

$$\begin{cases} \text{scaffold mass evolution,} \\ \text{background energy exchange,} \\ \text{perturbative momentum exchange,} \\ \text{velocity drag or antifriction,} \\ \text{scalar-mediated scaffold self-attraction,} \\ \text{the effective scalar mass.} \end{cases} \tag{1508}$$

This common origin is a principal source of both restriction and potential predictivity.

334 What Paper III Establishes

Paper III establishes the following structural results.

1. The Gravity Triune can be expressed as a role-based effective framework without treating all three roles as independent substances.
2. A minimum covariant third-leg realization can be written using general relativity, a canonical scalar, and a cold conformally coupled scaffold.

3. The binding function can remain identical to the general-relativistic metric response.
4. Visible matter and radiation can remain minimally coupled to the physical metric.
5. The scaffold and chasing sectors can exchange energy and momentum while preserving total covariant conservation.
6. Scaffold particle-number conservation can coexist with energy exchange when its particle mass depends on ϕ.
7. The scaffold and visible sectors can develop different growth and velocity histories because only the scaffold experiences the direct scalar force.
8. The same scaffold distribution must source the metric potentials that produce lensing.
9. A canonical chasing field has a stable isolated kinetic structure and unit rest-frame sound speed.
10. The exact cosmological-constant control is recovered when

$$\beta = 0, \qquad \dot{\phi} = 0, \qquad V(\phi) = V_0. \tag{1509}$$

11. The cold-dark-matter-like control is recovered when

$$\beta \to 0, \qquad p_{\text{scaf}} \to 0, \qquad c^2_{s,\text{scaf}} \to 0, \qquad \sigma_{\text{scaf}} \to 0. \tag{1510}$$

12. The complete background, perturbative, local, compact-system, gravitational-wave, and observational tests can be stated explicitly.
13. The conditions for mathematical closure, theoretical admissibility, third-leg viability, and historical identification are distinct.
14. A perfect cold scaffold does not by itself encode a unique historical path through two prior reversals.
15. A complete historical realization requires covariant transition dynamics and, when historical retention is claimed, an additional memory state.

These are formal and organizational results.

They do not establish that the required viable parameter region exists.

335 What Paper III Does Not Yet Establish

Paper III does not yet demonstrate:

$$\begin{cases} \mathcal{P}_{\text{viable}} \neq \varnothing, \\ \text{a completed Einstein–Boltzmann solution,} \\ \text{a completed nonlinear structure simulation,} \\ \text{a complete local and compact-object parameter map,} \\ \text{a finite outer-reversal solution,} \\ \text{a finite central-reversal solution,} \\ \text{a derived hot-state particle spectrum,} \\ \text{a derived matter–antimatter asymmetry mechanism,} \\ \text{a detected historical residue,} \\ \text{empirical preference over standard or alternative models.} \end{cases} \tag{1511}$$

The manuscript therefore does not claim that the Gravity Triune has been confirmed.

The appropriate status is

$$\boxed{\begin{array}{l} \text{covariantly formulable,} \\ \text{dynamically testable,} \\ \text{partially specified,} \\ \text{not yet numerically validated,} \\ \text{not yet historically identified.} \end{array}} \tag{1512}$$

The distinction between a specified research program and a validated physical theory is maintained throughout the final conclusions.

336 Final Gravity Triune Claims Matrix

Table 8: Final claims matrix for Paper III.

Statement	**Status**	**Interpretation**
The Gravity Triune is a role-based organizational framework.	Established within the manuscript	The terms binding, scaffolding, and chasing denote proposed physical functions.

Statement	Status	Interpretation
Binding is represented by the metric response in GT1.	Selected model definition	No separate gravitational-energy fluid is added.
Scaffolding is represented by a cold conformally coupled sector.	Selected candidate realization	Other scaffold realizations remain possible.
Chasing is represented by a canonical scalar.	Selected candidate realization	The scalar is not claimed to be uniquely required.
The dark interaction is covariantly conserving.	Established for the minimum action	The scaffold and scalar currents are equal and opposite.
Visible matter remains geodesic.	Established for the minimum action	No direct visible conformal coupling is introduced.
The model contains a standard control limit.	Established formally	The appropriate parameter limit reproduces general relativity with a cold scaffold and constant vacuum-like component.
The background has a viable radiation–matter–acceleration trajectory.	Not yet established	Requires a numerical basin analysis.
The perturbations reproduce the observed universe.	Not yet established	Requires a complete Einstein–Boltzmann calculation and joint fit.
The scaffold reproduces dark-matter lensing and growth.	Not yet established	Requires linear and nonlinear calculations using one common coupling.
The chasing field reproduces late acceleration.	Conditionally possible	Stable accelerated regions exist in restricted model classes, but the physical trajectory remains to be demonstrated.
The scaffold retains first-leg memory.	Not established	A perfect cold fluid is insufficient; a covariant memory carrier is required.
The outer and central reversals are dynamically realized.	Not established	A transition order parameter and finite solutions remain to be constructed.
The central transition produces the observed hot state.	Not established	Particle, entropy, and perturbation outputs remain to be calculated.

Statement	Status	Interpretation
The model explains the baryon asymmetry.	Not claimed	Additional nonequilibrium and symmetry-violating microphysics is required.
The full Triple Bounce history is observationally identified.	Not established	Requires a linked two-reversal prediction that survives alternative model comparison.

337 Relationship to General Relativity

Comparison basis. Paper III retains general relativity as the binding baseline and treats deviations as separately testable sectors or couplings (Einstein, 1915; Wald, 1984; Will, 2014).

The minimum GT1 realization does not alter the Einstein–Hilbert gravitational action.

Therefore,

$$M_{\mathrm{Pl}}^2 G_{\mu\nu} = T_{\mu\nu}^{\mathrm{tot}}. \tag{1513}$$

The metric-binding function retains:

$$\begin{cases} \text{the standard visible geodesic law,} \\ \text{the standard null cone,} \\ \text{the standard tensor propagation speed,} \\ \text{a constant Planck mass,} \\ \text{no fundamental graviton mass.} \end{cases} \tag{1514}$$

In the minimal realization,

$$c_T^2 = 1, \qquad m_T^2 = 0, \qquad \alpha_M = 0. \tag{1515}$$

The principal nonstandard effect is located in the dark sector:

$$\boxed{\text{scaffold particles respond to both the metric and the chasing scalar.}} \tag{1516}$$

Thus,

$$\mu_{\mathrm{scaf}}(k,a) \neq 1 \tag{1517}$$

may coexist with

$$\mu_{\rm met} \approx 1, \qquad \Sigma_{\rm met} \approx 1, \qquad \eta_{\rm slip} \approx 1 \tag{1518}$$

outside the contributions of standard and memory anisotropic stresses.

This is an interacting dark-sector model embedded in general relativity, not automatically a modified-metric-gravity model.

338 Relationship to the Standard Cosmological Model

Comparison basis. The standard cosmological comparison is made against the observationally constrained ΛCDM framework and its dynamical-dark-energy extensions (Planck Collaboration et al., 2020; Alam et al., 2021; DESI Collaboration et al., 2025b,a).

The standard cosmological benchmark is recovered when

$$\boxed{\begin{aligned} &\beta \to 0, \\ &w_{\rm scaf} \to 0, \\ &c^2_{s,{\rm scaf}} \to 0, \\ &\sigma_{\rm scaf} \to 0, \\ &\dot{\phi} \to 0, \\ &V(\phi) \to V_0, \\ &\mathcal{W}_{\mu\nu} \to 0, \\ &\delta\Xi_E \to 0. \end{aligned}} \tag{1519}$$

In that limit,

$$\rho_{\rm scaf} \propto a^{-3}, \qquad \rho_\phi = V_0, \tag{1520}$$

and

$$\mu_{\rm scaf} \to 1. \tag{1521}$$

The standard benchmark must be reproduced numerically before any nonstandard parameter inference is considered reliable.

The presence of a standard limit does not by itself make the model successful.

It provides:

$$\{\text{a code validation target}, \text{a nested comparison model}, \text{a physically interpretable null hypothesis}\}. \tag{1522}$$

The GT1 realization becomes scientifically distinctive only when a nonzero deviation is jointly constrained and linked across observables.

339 Integrated Relationship to Frozen Papers I and II

Frozen Paper I controls the architecture, terminology, and claims hierarchy. Frozen Paper II controls the causal, curvature, transition, central-covariance, thermal-handoff, and observational burdens. Paper III supplies one candidate third-leg dynamical realization without changing those foundations.

The controlling sequence is

$$\boxed{\mathcal{I}_1 \longrightarrow \mathsf{M}_{\mathrm{O}} \longrightarrow \mathcal{I}_2 \longrightarrow \mathsf{M}_{\mathrm{C}} \longrightarrow \mathcal{I}_3,} \tag{1523}$$

where M_{O} and M_{C} denote the linearized transition operators associated with Paper II's outer and central transition maps. When all propagators are derived, chronological composition is

$$\boxed{\mathsf{H}_{123} = \mathsf{U}_3\mathsf{M}_{\mathrm{C}}\mathsf{U}_2\mathsf{M}_{\mathrm{O}}\mathsf{U}_1.} \tag{1524}$$

The rightmost operator acts first. No alternative ordering is used in the integrated series.

340 Integrated Papers I–III Consistency Audit

The audit tests internal series consistency. It does not test whether the cosmological model is physically correct.

Table 9: Integrated consistency audit for frozen Papers I and II and the Paper III closure candidate.

Audit item	**Result**	**Controlling constraint**
Singularity and reversals	Pass	One singular opening begins the first resolved leg. The outer and central reversals are nonsingular, sector-specific transitions whose physical causes remain unresolved.

Audit item	Result	Controlling constraint
Global expansion and sectoral motion	Pass	$a(t)$ and $R_E(t) = a(t)\chi_E(t)$ remain distinct. An inward Electro trajectory does not imply global contraction.
Electro terminology	Pass	Electro remains a provisional effective sector, not ordinary light, a known particle, or a completed microscopic field theory.
Gravity Triune roles	Pass	Binding is metric gravitational response in GT1; scaffolding is a cold non-luminous sector; chasing is provisionally represented by a canonical scalar. The roles are connected but not three automatically independent forces.
Non-double-counting	Pass	No independent binding fluid duplicates the Einstein metric response, and no second cosmological constant duplicates the scalar vacuum control.
Interaction sign and conservation	Pass	One definition, $\beta = M_{\rm Pl}\,\mathrm{d}\ln A/\mathrm{d}\phi$, fixes equal-and-opposite scaffold and scalar currents. Visible matter and radiation have no direct current in minimum GT1.
Matter-origin language	Pass	Third-leg radiation and pressureless-matter epochs are observational benchmarks only. The series neither states that all matter first originated in the third leg nor states that ordinary matter definitely existed in either hidden leg.
Horizon and memory distinction	Pass	Raw causal reach, retained causal information, cold clustering, and historical memory remain distinct. A cold scaffold does not automatically prove prior-leg memory.
Transition chronology	Pass	The first-leg propagator acts first, followed by the outer transition, second leg, central transition, and third leg.
Central covariance	Pass	Paper III uses Paper II's mode vector and covariance $\mathsf{P}_Z^+(k)$ without substituting a separately tuned primordial spectrum.
Thermal handoff	Pass	The full map $\mathcal{H}_{\rm th} : \mathcal{I}_{\rm C}^+ \to \mathcal{I}_{\rm th}$ remains distinct from its linearized stochastic representation.

Audit item	Result	Controlling constraint
Calibration and dimensions	Pass	The 13.824 Gyr, 46.08 Gly, 10/3, packet, and growth-factor quantities remain empirical anchors or provisional correspondences. Paper III does not use them as repairs or validations.
Claims hierarchy	Pass	Model definitions, selected-action identities, conditional analytical results, controlled approximations, open templates, and uncompleted empirical tests remain separately labeled.
Paper IV boundary	Pass	Paper IV may reconstruct and test the packet correspondence, but it may not alter an unresolved Paper I–III equation or retroactively validate GT1.

The integrated result is

$$\boxed{\mathcal{G}_{\text{audit}}^{\text{I--III,int}} = 1.} \tag{1525}$$

This means that the stated architecture, notation, chronology, claim strength, and inter-paper dependencies are internally reconciled. It does not mean

$$\mathcal{V}_{\text{physical}} = 1, \tag{1526}$$

which remains an open numerical and observational question.

341 Reproducibility and Derivation Ledger

The manuscript is intentionally written so that independent researchers can reproduce, reject, or replace its derivations. The ledger is:

1. Selected-action identities must follow from $S_{\text{GT1}}^{(3)}$.
2. Background and perturbation signs must follow from the same current convention.
3. Conditional branches must state their potential, coupling, and phase-space assumptions.
4. Quasistatic and weak-field expressions must state their domain of validity.
5. Transition, memory, Electro, production, and thermal-handoff terms remain templates until derived from a finite covariant action.
6. Numerical observables must use one shared background, covariance, parameter ledger, and nuisance treatment.

7. A failed calculation may revise or reject a branch; it may not be rescued by an unrelated Paper IV correspondence.

This is the intended scientific handoff: the equations are explicit enough to be repeated, while unresolved mechanisms remain openly unresolved.

342 What Paper III Establishes and Does Not Establish

Paper III establishes, within the manuscript:

- a non-double-counted functional interpretation of binding, scaffolding, and chasing;
- one minimum covariant third-leg reference action;
- one sign-locked scaffold–scalar exchange convention;
- explicit background, local-control, perturbation, and observational test architectures;
- an exact general-relativity plus cold-scaffold plus constant-vacuum control limit;
- a conditional obstruction for one constant-coupling exponential scaling branch; and
- a reproducible sequence of calculations capable of supporting or falsifying the candidate model.

Paper III does not establish:

- a nonempty observationally viable parameter basin;
- a completed Einstein–Boltzmann or nonlinear-structure solution;
- complete local, compact-object, or binary constraints;
- either finite reversal mechanism;
- a covariant historical-memory carrier;
- the thermal production of the standard species inventory;
- the origin time of matter, antimatter, or particle-like excitations;
- a detected two-reversal residue; or
- empirical preference over ΛCDM or other alternatives.

343 Final Paper III Conclusion

The Gravity Triune can be represented, without duplicating gravity, by

$$\boxed{\begin{aligned} \text{binding} &= \text{general-relativistic metric response}, \\ \text{scaffolding} &= \text{a cold conformally coupled dark sector}, \\ \text{chasing} &= \text{a canonical scalar reference field}. \end{aligned}} \tag{1527}$$

This construction is a testable effective realization, not an empirical confirmation of the Gravity Triune or the Triple Bounce history. Its scientific value lies in making the proposal precise enough that its background evolution, perturbations, local limits, transitions, and observable consequences can be independently derived and checked.

The defensible conclusion is

$$\boxed{\begin{array}{l} \text{GT1 is internally specified as a candidate third-leg theory;} \\ \text{its physical viability and historical interpretation remain open.} \end{array}} \tag{1528}$$

344 Paper IV Release Boundary

The integrated internal audit releases Paper IV only under the following scope:

$$\boxed{\text{Paper IV} = \text{constrained packet reconstruction and sensitivity analysis.}} \tag{1529}$$

Paper IV may:

- reconstruct every numerical input, unit, arithmetic step, and hierarchy;
- analyze sensitivity to the adopted anchors and packet duration;
- compare the dimensionless growth factor with properly dimensioned cosmological quantities;
- test whether a continuous mapping can be connected consistently to the third-leg background; and
- state clear coincidence, calibration, prediction, and falsification criteria.

Paper IV may not:

- identify a dimensionless packet factor with Λ or w by nomenclature;
- assign physical meaning to the 4.608 residual without derivation;
- assume equal leg durations;

- repair a failed GT1 background or perturbation equation; or
- claim independent prediction from quantities used to construct the correspondence.

345 Final Paper III Status

The manuscript's closure status is

$$\boxed{\begin{aligned} \text{Architecture and terminology} &: \text{ reconciled with frozen Papers I and II,} \\ \text{Minimum third-leg action} &: \text{ selected and sign-locked,} \\ \text{Background/linear equations} &: \text{ specified for independent derivation,} \\ \text{Papers I–III internal audit} &: \text{ passed,} \\ \text{Numerical/observational validation} &: \text{ not completed,} \\ \text{Reversal/memory dynamics} &: \text{ not derived,} \\ \text{Historical identification} &: \text{ not established,} \\ \text{Publication} &: \text{ Version 1.1.0 claim-scope manuscript,} \\ \text{Series thesis} &: \text{ replacement thesis; validation pending,} \\ \text{Paper IV} &: \text{ released under the constrained scope above.} \end{aligned}} \tag{1530}$$

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