Third-Leg Expansion and Packet Reconstruction

Dimensional, Sensitivity, and Cosmological Tests of the 10/3 Mapping

Paper IV - Version 1.1.0
Replacement Thesis and Claim-Scope Edition

Justin B. Fine
Principal Investigator, Ontomics

July 27, 2026

Abstract

Paper IV closes the four-paper Triple Bounce series with a constrained numerical reconstruction of the third-leg packet architecture. The analysis fixes the dimensional dictionary, audits the provenance of the 64 Myr node and 320 Myr packet, reconstructs the central $N_h = 720$ mapping, and tests the symmetric integer-node frame $678 \leq N_h \leq 762$. The central inputs imply $S = 10/3$, $N_p = 43.2$, and $g_5 = S^{1/N_p} = 1.028261735$. Rebinning tests show that g_5 depends on the selected packet duration, whereas the invariant endpoint quantity is $\ln S/T_3$. The endpoint mapping fixes only $\int H dt = \ln S$ and therefore admits infinitely many continuous histories with the same packet factor. Controlled linear, constant-ϵ_H, quadratic-tilt, oscillatory, and rejection shapes are evaluated through H, $\dot{H}$, q, j, effective equation of state, conformal reach, comoving-Hubble-radius evolution, and curvature evolution. The observational benchmark $w = -1.028 \pm 0.031$ is a dark-energy-sector fit, not the total cosmic equation of state and not an identity with g_5. For a flat matter–radiation–dark-energy decomposition, $w_{\mathrm{DE}} = (2q - 1 - \Omega_r)/(3\Omega_{\mathrm{DE}})$. In the minimum flat GT1 realization inherited from Paper III, positive standard-sector densities and a canonical scalar require $\dot{H} \leq 0$ for the total source. The endpoint stretch supplies only $\ln(10/3) = 1.20397$ e-folds and does not, by itself, test or decide Paper II's full three-leg replacement thesis. The proposed replacement of the single-direction reconstruction and separate inflationary stage belongs to the complete causal, geometric, transition, and perturbation history, not to the endpoint packet factor alone. The final Paper IV result is therefore a reproducible and numerically stable correspondence: the packet construction yields a dimensionless factor close to the magnitude 1.028, while its physical relation to negative pressure, dark energy, or the chasing sector remains unestablished.

Replacement-Thesis Boundary and Packet Claim Scope

The series-level thesis is that the complete outward–inward–outward history replaces the standard single-direction historical reconstruction and removes the necessity of a separate primordial inflationary stage. Paper IV neither withdraws nor validates that thesis. It tests a distinct packet-reconstruction claim family.

Define

$$\begin{aligned} Q_{\mathrm{arith}} &: \text{the declared anchors imply the displayed packet identities,} && (1)\\ Q_{\mathrm{phys}} &: \text{a covariant mechanism selects the packet scale and connects it to observables,} && (2)\\ T_{\mathrm{replace}} &: \text{the complete three-leg history replaces the separate inflationary stage.} && (3) \end{aligned}$$

Then

$$\boxed{\mathrm{Fail}(Q_{\mathrm{phys}}) \not\Rightarrow \mathrm{Fail}(Q_{\mathrm{arith}}) \not\Rightarrow \mathrm{Fail}(T_{\mathrm{replace}}),} \tag{4}$$

and

$$\boxed{\mathrm{Pass}(Q_{\mathrm{arith}}) \not\Rightarrow \mathrm{Pass}(Q_{\mathrm{phys}}) \not\Rightarrow \mathrm{Pass}(T_{\mathrm{replace}}).} \tag{5}$$

The 1.20397 endpoint e-fold quantity is a property of the third-leg endpoint reconstruction. It is not the complete conformal reach, curvature transfer, or perturbation history of the first leg, second leg, and both reversals. Therefore it cannot be used either to validate or to falsify the full replacement thesis. Paper IV's negative verdicts are restricted to packet selection, interpolation, dimensional interpretation, and dynamical linkage.

Contents

1 Authority Chain and Release Boundary

Papers I–III, Version 1.1.0, are controlling [1, 2, 3]. Paper IV may reconstruct arithmetic, units, hierarchy, sensitivity, continuous mappings, and comparison targets. It may not revise the one-singularity architecture, repair an unresolved causal or dynamical equation, or convert numerical proximity into physical identity.

$$\boxed{\begin{aligned}\text{Paper I architecture} &\rightarrow \text{Paper II causal and geometric tests}\\ &\rightarrow \text{Paper III covariant dynamics}\\ &\rightarrow \text{Paper IV constrained reconstruction}\end{aligned}} \tag{6}$$

The final claim status is therefore

$$\boxed{\text{internally defined arithmetic correspondence} \neq \text{independent cosmological prediction.}} \tag{7}$$

2 Cross-Paper Mathematical Interface and Due-Diligence Gate

The purpose of this section is not to reproduce three completed papers or to introduce a second theory. It fixes the minimum equation chain that any packet interpretation must survive. Papers I–III provide the architecture, causal and curvature burdens, and covariant Gravity Triune dynamics, respectively [1, 2, 3]. Paper IV supplies only a reconstruction and a proposed map into that inherited system.

Paper	**Controlling role**	**Constraint on Paper IV**
I	Architecture, terminology, one-singularity history, and claim-strength hierarchy	Packet quantities remain provisional signatures; inflation, horizon, flatness, dark matter, and dark energy remain physical tests rather than verbal identifications.
II	Continuous background, causal history, curvature transfer, transition matching, perturbation transfer, and observational burden	A numerical packet match cannot replace the conformal-time, curvature, covariance, or thermal-handoff calculation.
III	Covariant binding, scaffolding, chasing, interaction, perturbation, growth, and lensing dynamics	A packet-derived $H(a)$ must enter the same GT1 background and perturbation system; it may not independently retune growth or lensing.
IV	Dimensional reconstruction, sensitivity analysis, continuum map, and comparison	Correspondence may be reported; dynamical identity requires an action-derived map and held-out observational tests.

2.1 Conventions

Unless c is displayed explicitly, the compact covariant equations in this section use $c = 1$, metric signature $(-,+,+,+)$, and $u^\mu u_\mu = -1$. The packet distance dictionary later restores c explicitly. This prevents a distance in gigalight-years from being silently treated as a duration in gigayears.

3 General-Relativistic Foundation

The minimum spacetime foundation is the Einstein equation [4]

$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu},} \tag{8}$$

with

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \tag{9}$$
$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}, \tag{10}$$
$$R = g^{\mu\nu} R_{\mu\nu}. \tag{11}$$

For a perfect fluid,

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}. \tag{12}$$

The contracted Bianchi identity requires

$$\nabla_\mu G^{\mu\nu} = 0, \tag{13}$$

and, for constant Λ, implies total covariant conservation,

$$\boxed{\nabla_\mu T^{\mu\nu} = 0.} \tag{14}$$

This is the conservation firewall: a packet prescription cannot create net stress–energy merely because a geometric multiplier was defined.

Freely falling trajectories satisfy the geodesic equation

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma^{\alpha}_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0. \tag{15}$$

The Raychaudhuri equation for a timelike congruence is [6]

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu + \nabla_\mu a^\mu. \tag{16}$$

It links expansion, shear, vorticity, acceleration, and curvature. Paper IV does not replace this focusing relation with packet arithmetic.

4 Background Cosmology and Cosmographic Diagnostics

For a homogeneous and isotropic third-leg background,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]. \tag{17}$$

The Friedmann and acceleration equations are [5]

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \tag{18}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{19}$$

Total conservation gives

$$\boxed{\dot{\rho} + 3H(\rho + p) = 0,} \tag{20}$$

or a sum of interacting-sector equations whose exchange currents cancel. Define

$$\rho_c = \frac{3H^2}{8\pi G}, \tag{21}$$

$$\Omega_i = \frac{\rho_i}{\rho_c}, \tag{22}$$

$$\Omega_k = -\frac{k}{a^2 H^2}, \tag{23}$$

$$\Omega_\Lambda = \frac{\Lambda}{3H^2}. \tag{24}$$

Under the displayed convention,

$$1 = \Omega_m + \Omega_r + \Omega_{\text{other}} + \Omega_\Lambda + \Omega_k. \tag{25}$$

The useful kinematical diagnostics are

$$q = -\frac{\ddot{a}}{aH^2}, \tag{26}$$

$$j = \frac{\dddot{a}}{aH^3}, \tag{27}$$

$$\epsilon_H = -\frac{\dot{H}}{H^2} = 1 + q. \tag{28}$$

The jerk j helps distinguish expansion histories that share the same present H and q [10]. For a spatially flat standard Friedmann interpretation of the total effective source,

$$\boxed{w_{\text{eff}} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -1 + \frac{2}{3}\epsilon_H.} \tag{29}$$

This conditional relation, not the numerical size of g_p, is the pressure bridge used in Paper IV.

5 Inflation, Horizon, and Flatness Benchmarks

The number of e-folds is

$$N(t_i, t_f) = \ln\left(\frac{a_f}{a_i}\right) = \int_{t_i}^{t_f} H\, dt. \tag{30}$$

Accelerated expansion requires

$$\ddot{a} > 0 \quad \Longleftrightarrow \quad q < 0 \quad \Longleftrightarrow \quad \epsilon_H < 1. \tag{31}$$

The inflationary horizon benchmark is stronger than positive endpoint growth:

$$\boxed{\frac{d}{dt}\left(\frac{1}{aH}\right) < 0.} \tag{32}$$

The relevant causal quantities are

$$\eta(t) = \int^{t} \frac{dt'}{a(t')}, \tag{33}$$

$$\chi_{\rm ph}(t) = \int_{t_i}^{t} \frac{dt'}{a(t')}, \tag{34}$$

$$R_H(t) = \frac{1}{H(t)}, \tag{35}$$

$$r_H^{\rm com}(t) = \frac{1}{a(t)H(t)}. \tag{36}$$

Near-flatness is tracked by

$$\Omega_k = -\frac{k}{a^2H^2}. \tag{37}$$

For fixed k,

$$\boxed{\frac{d\ln|\Omega_k|}{dN} = 2(\epsilon_H - 1) = 2q.} \tag{38}$$

Therefore the packet-interpolated history must be evaluated for conformal reach and curvature evolution. A ratio near 10/3 does not by itself solve either problem. The observational inflation and flatness benchmarks remain those inherited from Paper II and the standard literature [12].

6 Bounce-Definition Firewall

A standard nonsingular bounce of the global scale factor requires

$$H(t_b) = 0, \qquad \dot{H}(t_b) > 0. \tag{39}$$

The Triple Bounce framework instead reserves two internal reversals for an effective Electro trajectory $R_E(t)$:

$$\dot{R}_E(t_r) = 0, \qquad \ddot{R}_E(t_r) \neq 0, \tag{40}$$

while

$$H(t_r) > 0 \tag{41}$$

may remain possible. Paper IV must not convert a sectoral reversal into an unannounced contraction of the entire FLRW scale factor.

7 Perturbation, Growth, and Observable Interface

The scalar metric perturbations may be written in Newtonian gauge as

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)d\boldsymbol{x}^2\right]. \tag{42}$$

A compact late-time growth interface is

$$\frac{d^2\delta_m}{d(\ln a)^2} + \left[2 + \frac{d\ln H}{d\ln a}\right]\frac{d\delta_m}{d\ln a} - \frac{3}{2}\Omega_m(a)\mu(k,a)\delta_m = 0, \tag{43}$$

where $\mu = 1$ in the simplest general-relativistic pressureless limit. Interactions or additional degrees of freedom must be derived rather than hidden inside a fit function. Define

$$D(a) = \frac{\delta_m(a)}{\delta_m(a_{\mathrm{ref}})}, \tag{44}$$

$$f(a) = \frac{d\ln D}{d\ln a}, \tag{45}$$

$$P_m(k,z) = P_{\mathrm{prim}}(k)T_m^2(k)D^2(z). \tag{46}$$

These quantities are propagated through one perturbation system [7]. The lensing sector is controlled by the Weyl combination

$$\Phi_W = \frac{\Phi + \Psi}{2}, \tag{47}$$

not by inserting a scaffold force enhancement directly into the photon kernel [8].

For the cosmic microwave background,

$$\frac{\Delta T}{T}(\hat{\boldsymbol{n}}) = \sum_{\ell m} a_{\ell m}Y_{\ell m}(\hat{\boldsymbol{n}}), \tag{48}$$

$$C_\ell = \langle |a_{\ell m}|^2 \rangle. \tag{49}$$

The baryon acoustic scale is tied to the sound horizon,

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)}\, dz, \tag{50}$$

and observational comparisons use combinations such as $D_M(z)/r_d$ and $D_H(z)/r_d$, with $D_H(z) \equiv 1/H(z)$ in the present $c = 1$ convention. A packet map that changes $H(z)$ therefore changes distances, sound-horizon calibration, growth, and lensing together.

8 Paper III GT1 Interface: Scaffolding and Chasing

The selected Paper III reference realization is summarized by

$$\begin{aligned} S^{(3)}_{\mathrm{GT1}} = \int d^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] \\ + S_{\mathrm{vis}}[g] + S_{\mathrm{rad}}[g] + S_{\mathrm{scf}}[A^2(\phi)g, \psi_{\mathrm{scf}}]. \end{aligned} \tag{51}$$

With

$$\beta(\phi) = M_{\mathrm{Pl}}\frac{d\ln A}{d\phi}, \tag{52}$$

its cold-scaffold background exchange is

$$\dot\rho_{\mathrm{scf}} + 3H\rho_{\mathrm{scf}} = \frac{\beta}{M_{\mathrm{Pl}}}\rho_{\mathrm{scf}}\dot\phi, \tag{53}$$

$$\ddot\phi + 3H\dot\phi + V_{,\phi} = -\frac{\beta}{M_{\mathrm{Pl}}}\rho_{\mathrm{scf}}. \tag{54}$$

Visible matter and radiation have no direct GT1 current in the minimum realization. Paper IV may propose a reconstructed $H(a)$ or endpoint map, but it cannot alter these exchange signs, introduce a second chasing field, or tune photons and scaffold growth independently.

9 Packet-to-Continuum Reconstruction Gate

The endpoint data determine only a total logarithmic growth,

$$\mathcal{N}_{\mathrm{end}}(T_3) \equiv \ln\left[\frac{a_p(T_3)}{a_i}\right] = \ln S. \tag{55}$$

A general smooth interpolation can be written as

$$\boxed{a_p(t) = a_i \exp\left[\ln S\, F\left(\frac{t}{T_3}\right)\right],} \tag{56}$$

where

$$F(0) = 0, \qquad F(1) = 1. \tag{57}$$

Writing $x = t/T_3$ gives

$$H_p(t) = \frac{\ln S}{T_3} F'(x), \tag{58}$$

$$\epsilon_{H,p}(t) = -\frac{F''(x)}{\ln S\,[F'(x)]^2}, \tag{59}$$

$$q_p(t) = \epsilon_{H,p}(t) - 1. \tag{60}$$

The endpoint-equivalent exponential is the special control $F(x) = x$. It yields

$$\dot{H}_p = 0, \qquad \epsilon_{H,p} = 0, \qquad q_p = -1, \qquad w_{\mathrm{eff},p} = -1 \tag{61}$$

under the flat standard-Friedmann interpretation. Therefore the constant-rate control does *not* derive $w = -1.028$. It derives the cosmological-constant kinematics $w_{\mathrm{eff}} = -1$. Any departure must come from the shape $F(x)$ or from a separately derived dynamical sector.

The local sensitivity of the packet factor is

$$\boxed{\frac{\partial \ln g_p}{\partial N_h} = \frac{1}{N_p N_h}, \qquad \frac{\delta g_p}{g_p} \simeq \frac{\delta N_h}{N_p N_h}.} \tag{62}$$

This analytic compression explains the narrow g_p band across the wide integer-node frame.

10 Endpoint Degeneracy and the Shape-Selection Burden

The packet reconstruction fixes the endpoint condition

$$\ln\left(\frac{a_f}{a_i}\right) = \ln S, \tag{63}$$

but it does not select a unique time history. Let $f(x)$ be any positive integrable function on $0 \le x \le 1$ satisfying

$$\int_0^1 f(x)\,dx = 1. \tag{64}$$

Then

$$F(x) = \int_0^x f(u)\,du \tag{65}$$

obeys $F(0) = 0$, $F(1) = 1$, and $F'(x) > 0$. Every such F produces the same total stretch through

$$a_F(t) = a_i S^{F(t/T_3)}. \tag{66}$$

Therefore the endpoint arithmetic determines only the mean logarithmic rate

$$\boxed{\langle H \rangle = \frac{1}{T_3}\int_0^{T_3} H(t)\,dt = \frac{\ln S}{T_3},} \tag{67}$$

not $\dot{H}$, acceleration, pressure, jerk, conformal reach, or curvature transfer. A physical interpretation requires an action, constitutive law, or matching condition that selects one member of this infinite family without using the desired observational answer as the selector.

11 Derivative Dictionary for a General Shape

Write

$$x = \frac{t}{T_3}, \qquad L = \ln S. \tag{68}$$

For

$$a_F(t) = a_i \exp[LF(x)], \tag{69}$$

the complete background derivative dictionary is

$$H_F = \frac{L}{T_3}F', \tag{70}$$

$$\dot{H}_F = \frac{L}{T_3^2}F'', \tag{71}$$

$$\ddot{H}_F = \frac{L}{T_3^3}F''', \tag{72}$$

$$\epsilon_{H,F} = -\frac{F''}{L(F')^2}, \tag{73}$$

$$q_F = -1 - \frac{F''}{L(F')^2}, \tag{74}$$

$$j_F = 1 + \frac{3F''}{L(F')^2} + \frac{F'''}{L^2(F')^3}. \tag{75}$$

Under a flat standard-Friedmann interpretation of the *total* source,

$$w_{\mathrm{tot,F}} = -1 - \frac{2}{3}\frac{F''}{L(F')^2} = -1 + \frac{2}{3}\epsilon_{H,F}. \tag{76}$$

Expansion requires $F' > 0$. Acceleration and a shrinking comoving Hubble radius require

$$\epsilon_{H,F} < 1 \quad \Longleftrightarrow \quad q_F < 0. \tag{77}$$

The endpoint ratio of the comoving Hubble radius is

$$\boxed{\frac{(aH)_f^{-1}}{(aH)_i^{-1}} = \frac{F'(0)}{SF'(1)},} \tag{78}$$

and, for fixed spatial curvature k,

$$\boxed{\frac{|\Omega_{k,f}|}{|\Omega_{k,i}|} = \left[\frac{F'(0)}{SF'(1)}\right]^2.} \tag{79}$$

The dimensionless conformal-reach functional is

$$\mathcal{I}_F(S) = \int_0^1 S^{-F(x)}\,dx, \qquad \Delta\chi_F = \frac{cT_3}{a_i}\mathcal{I}_F. \tag{80}$$

None of these quantities is fixed by S alone.

12 Controlled Shape Families

12.1 Linear endpoint control

$$F_0(x) = x. \tag{81}$$

This gives constant H, $\epsilon_H = 0$, $q = -1$, $j = 1$, and $w_{\text{tot}} = -1$. It is the endpoint-equivalent exponential control, not a reconstructed third-leg history.

12.2 Constant-ϵ_H Family

For constant $\epsilon \neq 0$, the normalized solution is

$$\boxed{F_\epsilon(x) = \frac{\ln[1 + (S^\epsilon - 1)x]}{\epsilon \ln S},} \tag{82}$$

with the smooth limit $F_0 = x$. It satisfies

$$\epsilon_H = \epsilon, \qquad q = \epsilon - 1, \qquad j = 1 - 3\epsilon + 2\epsilon^2, \tag{83}$$

and

$$\frac{(aH)_f^{-1}}{(aH)_i^{-1}} = S^{\epsilon-1}, \qquad \frac{|\Omega_{k,f}|}{|\Omega_{k,i}|} = S^{2(\epsilon-1)}. \tag{84}$$

This family is useful because it isolates the derivative assumption from the endpoint normalization.

12.3 Quadratic-tilt control

$$F_\lambda(x) = x + \lambda x(1-x), \qquad |\lambda| < 1. \tag{85}$$

Then

$$F'_\lambda = 1 + \lambda(1-2x), \tag{86}$$
$$F''_\lambda = -2\lambda, \tag{87}$$
$$\epsilon_{H,\lambda} = \frac{2\lambda}{L[1+\lambda(1-2x)]^2}. \tag{88}$$

Choosing $\lambda = -0.021L$ makes the *total-fluid* midpoint diagnostic equal to $w_{\rm tot}(1/2) = -1.028$. This is retained only as a stress control. It is not the correct reconstruction of the observational dark-energy parameter and is incompatible with the minimum flat GT1 total-source inequality derived below.

12.4 Bounded ripple control

$$F_{A,n}(x) = x + \frac{A}{2\pi n}\sin(2\pi n x), \qquad |A| < 1, \quad n \in \mathbb{N}. \tag{89}$$

It preserves the endpoints and keeps $H > 0$, because

$$F'_{A,n} = 1 + A\cos(2\pi n x) > 0. \tag{90}$$

However,

$$F''_{A,n} = -2\pi n A \sin(2\pi n x), \tag{91}$$
$$F'''_{A,n} = -(2\pi n)^2 A \cos(2\pi n x). \tag{92}$$

Thus the jerk burden grows as n^2 while the packet factor remains unchanged. Endpoint agreement therefore cannot justify an oscillation frequency or amplitude.

12.5 Smoothstep rejection control

$$F_{\rm ss}(x) = 3x^2 - 2x^3. \tag{93}$$

Although it has the correct endpoints, $F'_{\rm ss}(0) = F'_{\rm ss}(1) = 0$. The corresponding H vanishes at both boundaries and ϵ_H, q, and j become singular there. It is rejected as a global third-leg background and retained only to demonstrate that endpoint smoothness is not sufficient.

13 Correct Dark-Energy-Sector Reconstruction

The historical benchmark

$$w_{\mathrm{DE}} = -1.028 \pm 0.031 \tag{94}$$

is a constant dark-energy-sector parameter in a specified cosmological fit, not the total cosmic equation of state [11]. The earlier developmental equation

$$\frac{\dot{H}}{H^2} = 0.042 \tag{95}$$

is mathematically correct only for the hypothetical identification $w_{\mathrm{tot}} = -1.028$. It is not the correct physical translation of the cited w_{DE} constraint.

For a spatially flat matter–radiation–dark-energy decomposition,

$$w_{\mathrm{tot}} = \frac{\Omega_r}{3} + \Omega_{\mathrm{DE}} w_{\mathrm{DE}}, \tag{96}$$

and

$$q = \frac{1}{2}\left[1 + \Omega_r + 3\Omega_{\mathrm{DE}} w_{\mathrm{DE}}\right]. \tag{97}$$

Therefore

$$\boxed{w_{\mathrm{DE}}(a) = \frac{2q(a) - 1 - \Omega_r(a)}{3\Omega_{\mathrm{DE}}(a)}.} \tag{98}$$

The required reconstruction chain is

$$\boxed{g_p \not\to w_{\mathrm{DE}}; \qquad F(x) \to H, q \to \Omega_r, \Omega_m, \Omega_{\mathrm{DE}} \to w_{\mathrm{DE}}.} \tag{99}$$

As an illustrative reference-point calculation only, taking $\Omega_{\mathrm{DE}} = 0.70$, $\Omega_r \simeq 0$, and $w_{\mathrm{DE}} = -1.028$ gives

$$q_* = -0.5794, \tag{100}$$
$$\epsilon_{H,*} = 0.4206, \tag{101}$$
$$w_{\mathrm{tot},*} = -0.7196. \tag{102}$$

Hence the fitted dark-energy value does not require $\dot{H} > 0$ for the total universe. The background can have decreasing H while the inferred dark-energy sector lies slightly below -1.

The central packet factor remains

$$g_5 = 1.028261735189, \tag{103}$$

whose magnitude differs from 1.028 by approximately 0.0255%. That is a reproducible numerical correspondence, but it supplies neither the negative sign nor the sector decomposition.

14 Minimum GT1 Compatibility Gate

For the minimum Paper III action, the scalar is canonical and visible matter, radiation, and cold scaffold have nonnegative densities. In a spatially flat background, the total Raychaudhuri/Friedmann relation gives

$$\boxed{\dot{H} = -\frac{1}{2M_{\rm Pl}^2}\left(\rho_{\rm vis} + \rho_{\rm scf} + \frac{4}{3}\rho_{\rm rad} + \dot{\phi}^2\right) \le 0.} \tag{104}$$

The scalar–scaffold coupling redistributes energy between sectors but does not reverse the sign of the total $\rho + p$ under these assumptions. Consequently,

$$\epsilon_H \ge 0, \qquad w_{\rm tot} \ge -1, \qquad F'' \le 0 \tag{105}$$

for a flat minimum-GT1 solution. The constant-total-phantom control $\epsilon = -0.042$, the negative-λ quadratic midpoint match, and any ripple segment with $F'' > 0$ are therefore rejection controls rather than admissible minimum-GT1 backgrounds.

This does not automatically forbid an *effective fitted* $w_{\rm DE} < -1$. In an interacting dark-sector model, forcing the data into a noninteracting constant-w template can assign exchange effects to an effective dark-energy equation of state. The admissible test is whether the full GT1 solution, analyzed with the same inference pipeline as the comparison model, produces such an effective fit while preserving total stability and the Paper III growth and lensing constraints.

A shape may be promoted from kinematic control to GT1 candidate only after it is generated by the coupled system

$$3M_{\rm Pl}^2 H^2 = \rho_{\rm vis} + \rho_{\rm rad} + \rho_{\rm scf} + \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{106}$$

$$\dot{\rho}_{\rm scf} + 3H\rho_{\rm scf} = \frac{\beta}{M_{\rm Pl}}\rho_{\rm scf}\dot{\phi}, \tag{107}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\frac{\beta}{M_{\rm Pl}}\rho_{\rm scf}, \tag{108}$$

with declared $A(\phi)$, $V(\phi)$, initial data, and no packet-specific retuning of observables.

15 Horizon, Flatness, and E-Fold Verdict

The endpoint stretch contains

$$\boxed{N_{\rm endpoint} = \ln S = \ln(10/3) = 1.203972804.} \tag{109}$$

This is the complete logarithmic growth supplied by the central third-leg endpoint reconstruction. It is far below the conventional inflationary e-fold benchmark, but this endpoint quantity is not the full first-leg, second-leg, and transition history. It therefore neither validates nor falsifies Paper II's replacement thesis and cannot substitute for Paper II's full causal and curvature calculation [12, 2].

For the constant-ϵ family, the endpoint tests are analytic:

$$\frac{(aH)_f^{-1}}{(aH)_i^{-1}} = S^{\epsilon-1}, \qquad \frac{|\Omega_{k,f}|}{|\Omega_{k,i}|} = S^{2(\epsilon-1)}. \tag{110}$$

Control	ϵ_H	q	$\mathcal{I}_F$	$r_{H,f}^{\mathrm{com}}/r_{H,i}^{\mathrm{com}}$	$\lvert\Omega_{k,f}\rvert/\lvert\Omega_{k,i}\rvert$
Linear endpoint	0	-1.0000	0.581408	0.300000	0.090000
Composition-conditioned reference	0.4206	-0.5794	0.552969	0.497788	0.247792
Total-phantom stress control	-0.042	-1.0420	0.584292	0.285207	0.081343

All three controls shrink the comoving Hubble radius because $\epsilon_H < 1$, but only by factors of order two to four over the full mapping. The curvature suppression is similarly modest. The total-phantom row is additionally incompatible with the flat minimum-GT1 gate and is displayed only as a mathematical stress control.

The conformal-reach values differ even though every control has the same S. This directly demonstrates why a packet endpoint cannot settle the horizon question. Paper II's conformal-time and transition-matching calculations remain controlling.

16 Shape-Selection Decision Rule

A shape function is not physically selected because it reproduces g_5, passes through a preferred w value, or resembles a familiar constant. Promotion requires all of the following:

1. $F(0) = 0$, $F(1) = 1$, and $F' > 0$ over the active interval;
2. compatibility with the Paper III GT1 background equations and total conservation;
3. no ghost, gradient, or total-null-energy pathology in the selected realization;
4. acceptable q, j, conformal reach, and curvature transfer without endpoint-only tuning;
5. propagation through the same growth, lensing, cosmic microwave background, baryon acoustic oscillation, and distance pipeline;
6. fixed calibration inputs and held-out validation data;
7. survival against simpler controls, including $F = x$ and standard Λ cold dark matter.

Any later dynamical study would use this gate to test GT1-generated histories, but that work lies outside Paper IV's constrained reconstruction scope.

17 Distance Taxonomy and Unit Firewall

For a reconstructed $H(z)$, define the line-of-sight comoving distance

$$\chi(z) = \int_0^z \frac{dz'}{H(z')}. \tag{111}$$

The transverse comoving distance is $D_M = S_k[\chi]$, with the curvature-dependent function S_k. The luminosity and angular-diameter distances are

$$d_L(z) = (1 + z)D_M(z), \tag{112}$$

$$d_A(z) = \frac{D_M(z)}{1 + z}. \tag{113}$$

Paper IV separately labels elapsed cosmic time, lookback time, proper distance, comoving distance, particle horizon, Hubble radius, luminosity distance, angular-diameter distance, and the adopted model distance D_h [9]. The symbol D_h is not allowed to become a time variable merely because one light-year is the distance traveled by light in one year.

18 Equation-Tier Disposition

The broader equation inventory is useful as a due-diligence checklist, but only equations that test the series' core claims are activated in Paper IV.

Tier	Disposition	Reason
GR and differential geometry	Main text plus notation appendix	Required for spacetime, conservation, geodesics, focusing, and the Paper II causal/curvature tests.
Background cosmology	Main text	Required for expansion, acceleration, density closure, inflation, dark-energy diagnostics, and packet-to-continuum mapping.
Perturbations and observables	Compact interface	Required to prevent a background-only success from breaking growth, lensing, the CMB, or the BAO scale.
Triune dynamics	Inherited GT1 interface	Paper III already selects the reference action and coupling signs; Paper IV must not invent a competing action.
Grand-unification machinery	Deferred	A second Hamiltonian, gauge algebra, symmetry-breaking chain, partition function, or renormalization program would exceed Paper IV's release boundary.
Information theory	Future-extension gate only	Entropy or information measures require specified probability distributions, coarse graining, gauge behavior, and observable consequences.

The conceptual order parameter

$$\mathcal{O}(a) = \frac{\rho_{\text{organized}}}{\rho_{\text{total}}} \tag{114}$$

is retained only as a future-extension placeholder. Before activation, $\rho_{\text{organized}}$ must be defined covariantly, distinguished from ordinary structure formation, and shown not to double-count scaffold density or thermodynamic entropy.

19 Claim-Status Ledger

Statement	Status	Permission
$T_3 = 13.824\,\mathrm{Gyr}$	Adopted anchor	May be used in declared calculations; not predicted here.
$N_T = 216$	Defined node representation	Arithmetic bookkeeping.
$N_h = 720$	Central calibrated row	Reference row, not held-out prediction.
$S = 10/3$	Arithmetic consequence	Exact given the selected anchors.
$g_p = 1.028261735189$	Derived packet factor	Exact endpoint-distribution result.
$S \simeq \pi$ at $N_h = 679$	Exploratory correspondence	May be reported with multiple-testing caution.
$\lvert g_p\rvert \simeq \lvert w\rvert$ near 1.028	Exploratory correspondence	May motivate a derivative-based test; no sign or mechanism inferred.
$w_{\text{eff}}(t)$	Conditional derivation target	Must be obtained from H and $\dot{H}$ under stated field equations.
Observable shifts	Prediction target	Must be fixed before held-out CMB, BAO, supernova, growth, and lensing comparison.

20 Controls, Ablations, and Claim-Specific Failure Logic

The minimum controls are

$$S = 1, \quad g_p = 1 \qquad \text{identity/no-growth control,} \tag{115}$$

$$N_h = 720 \qquad \text{central calibrated row,} \tag{116}$$

$$678 \le N_h \le 762 \qquad \text{wide stress frame.} \tag{117}$$

The required ablations are:

1. hold S fixed and vary Δt_p;
2. hold Δt_p fixed and vary N_h;
3. compare continuous interpolation with packet-time sampling;
4. vary the shape function $F(x)$ while fixing $F(0) = 0$ and $F(1) = 1$;
5. propagate every accepted history through the same distance, growth, lensing, and CMB/BAO interface;
6. compare calibration data with held-out validation data.

The physical packet-selection claim is rejected if the result depends only on endpoint normalization, if no action or constitutive law selects $F(x)$, if the inferred history violates conservation or stability, or if a background match damages the Paper III structure-growth and lensing targets. This verdict does not reject the exact packet arithmetic or the independent Triple Bounce replacement thesis. Numerical resemblance remains reportable as a correspondence even when the stronger physical interpretation fails.

21 Source-Provenance Audit and Arithmetic Correction

The developmental packet records contain several distinct strata: user-specified anchors, exact arithmetic consequences, assistant-generated rebinning layers, and speculative physical identifications [13, 14]. Paper IV must preserve that history without treating every developmental sentence as a scientific result.

21.1 Provenance classes

Each inherited item is assigned one of five statuses:

1. **adopted anchor**: fixed for reconstruction but not independently derived here;
2. **exact consequence**: follows algebraically from adopted anchors;
3. **coarse-graining choice**: selects a representation and requires independent physical motivation;
4. **exploratory correspondence**: a numerical proximity with no established dynamical identity;
5. **rejected or quarantined developmental claim**: dimensionally, arithmetically, or physically unsupported.

Item	Inherited form	Status	Paper IV disposition
Base node	64 Myr	adopted anchor	Retain as Δt_n; do not claim an independent cosmological derivation.
Third-leg epoch	13.824 Gyr	adopted anchor	Retain as $T_3 = 216\Delta t_n$.
Central horizon scale	46.08 Gyr and 46.08 Gly used interchangeably	dimensional conflict	Retain only $D_h = 46.08$ Gly as the distance anchor; use 46.08 Gyr only as a light-travel-time equivalent if explicitly labeled.
Five-node packet	320 Myr	coarse-graining choice	Retain provisionally as $\Delta t_p = 5\Delta t_n$.
Packet factor	1.02829	exact consequence of selected binning	Retain as $g_5 = S^{5/216}$; not a binning-invariant observable.
Negative-pressure match	$g_5 \leftrightarrow -w$	exploratory correspondence	Retain numerical resemblance only; derive w_{eff} from H and $\dot{H}$.
2.304 Myr micro-fold	$T_3/6000$	arbitrary rebinning	Quarantine; no unique "finest" subdivision exists.
2.304 Gyr mega-interval	36-node interval	excluded	Remove from the active hierarchy.

21.2 Correction of the developmental tripling claim

One developmental passage states

$$3(216) = 720. \tag{118}$$

That equality is false. The correct multiplication is

$$\boxed{3(216) = 648.} \tag{119}$$

Consequently, a strict tripling of the 216-node epoch would give

$$648(64\,\mathrm{Mly}) = 41.472\,\mathrm{Gly}, \tag{120}$$

not 46.08 Gly. The central horizon row instead satisfies

$$\boxed{720 = \frac{10}{3}(216), \qquad 720(64\,\mathrm{Mly}) = 46.08\,\mathrm{Gly}.} \tag{121}$$

The 720-node construction is therefore a 10/3 mapping, not a repeated-tripling identity. This correction removes a serious arithmetic ambiguity while leaving the central reconstruction itself intact.

21.3 Length–time firewall

The same node count can label either a temporal duration or a light-travel distance only when the conversion by c is explicit:

$$720(64\,\mathrm{Myr}) = 46.08\,\mathrm{Gyr}, \tag{122}$$

$$720(64\,\mathrm{Mly}) = 46.08\,\mathrm{Gly}. \tag{123}$$

Paper IV uses the second expression for the central horizon distance. The dimensionless stretch is therefore

$$S = \frac{D_h}{cT_3}, \tag{124}$$

not the unqualified quotient of a distance by a time. In units with $c = 1$, this distinction remains conceptually active even when the numerical values share the same digits.

21.4 Correction of the micro-fold uniqueness claim

The developmental micro-fold is defined by first selecting 6000 equal subdivisions:

$$\Delta t_\mu = \frac{T_3}{6000} = 2.304\,\mathrm{Myr}, \qquad g_\mu = S^{1/6000}. \tag{125}$$

This is a valid arithmetic rebinning, but it is not the "finest equal subdivision." For every positive integer M, one may define

$$\Delta t_M = \frac{T_3}{M}, \qquad g_M = S^{1/M}, \tag{126}$$

and obtain $(g_M)^M = S$. Unless an independent mechanism selects $M = 6000$, the 2.304 Myr layer carries no additional physical information. It remains in the provenance ledger only to document the developmental path.

21.5 Look-elsewhere caution

The wide node scan, the π proximity, and the packet-duration scan create many opportunities for numerical resemblance. Any claimed correspondence must therefore record whether the compared value was specified before or noticed after scanning. The central $N_h = 720$, $m = 5$ construction is treated as the declared reconstruction; all neighboring constants and alternative bin widths are controls, not rescue parameters.

22 Dimensional Dictionary

The base temporal node is

$$\Delta t_n = 64\,\mathrm{Myr}. \tag{127}$$

The fixed third-leg epoch contains

$$N_T = 216, \qquad T_3 = N_T \Delta t_n = 13.824\,\mathrm{Gyr}. \tag{128}$$

The distance represented by one light-travel node is

$$\Delta D_n = c\Delta t_n = 64\,\mathrm{Mly}. \tag{129}$$

For an integer horizon-node count N_h, define

$$D_h(N_h) = N_h \Delta D_n, \tag{130}$$

$$S(N_h) = \frac{D_h}{cT_3} = \frac{N_h}{216}. \tag{131}$$

For a general packet containing $m > 0$ nodes, define

$$\Delta t_p^{(m)} = m\Delta t_n, \tag{132}$$

$$N_p^{(m)} = \frac{T_3}{\Delta t_p^{(m)}} = \frac{216}{m}, \tag{133}$$

$$g_p^{(m)} = S^{1/N_p^{(m)}} = S^{m/216}. \tag{134}$$

The nominal packet is the five-node choice

$$\Delta t_p^{(5)} = 5\Delta t_n = 320\,\mathrm{Myr}, \tag{135}$$

so the epoch contains

$$N_p^{(5)} = \frac{216}{5} = 43.2 \tag{136}$$

nominal intervals. Since 5 does not divide 216, the five-node packet does not tile the epoch into an integer number of equal cells. If forced into discrete cells, it corresponds to 43 complete 320 Myr packets plus one remaining 64 Myr node. That remainder is bookkeeping, not an additional physical epoch.

23 Central 720-Node Reconstruction

The calibrated central row is

$$N_h = 720, \qquad D_h = 46.08\,\mathrm{Gly}, \qquad S = \frac{720}{216} = \frac{10}{3}. \tag{137}$$

Distributing the total stretch geometrically over $N_p = 43.2$ nominal packets gives

$$\boxed{g_p = S^{1/N_p} = \exp\left(\frac{\ln(10/3)}{43.2}\right) = 1.028261735189.} \tag{138}$$

The associated constant-rate representation is a dimensional diagnostic:

$$H_p^{\mathrm{eq}} = \frac{\ln g_p}{\Delta t_p} = \frac{\ln S}{T_3} = 0.087092940128\ \mathrm{Gyr}^{-1}. \tag{139}$$

This quantity is not automatically the observed H_0, nor does it establish a constant physical expansion rate across the third leg. It is simply the unique smooth exponential rate that reproduces the selected endpoint ratio over the selected interval.

24 Alternative Packet Durations and Rebinning Invariance

The packet factor changes when the bin width changes even though the total endpoint stretch does not. Let

$$C_{64} = S^{1/216} \tag{140}$$

be the per-node factor. Then an m-node packet has

$$\boxed{g_m = C_{64}^m = S^{m/216}.} \tag{141}$$

Its nominal count is $N_m = 216/m$, and therefore

$$\boxed{(g_m)^{N_m} = S} \tag{142}$$

for every positive m. More importantly,

$$\boxed{\frac{\ln g_m}{m\Delta t_n} = \frac{\ln S}{216\Delta t_n} = \frac{\ln S}{T_3} \equiv H_{\mathrm{end}}^{\mathrm{eq}}.} \tag{143}$$

Thus $H^{\text{eq}}_{\text{end}}$ is invariant under rebinning, while g_m is not. The invariant content of the endpoint construction is S together with T_3; the value 1.0282617 appears because the chosen reporting interval is five nodes.

m	Δt_m (Myr)	$216/m$	g_m	gain (%)	epoch tiling
1	64	216.0	1.005589510	0.558951	exact
2	128	108.0	1.011210272	1.121027	exact
3	192	72.0	1.016862445	1.686244	exact
4	256	54.0	1.022546199	2.254620	exact
5	320	43.2	1.028261735	2.826174	coarse
6	384	36.0	1.034009221	3.400922	exact
8	512	27.0	1.045600733	4.560073	exact
9	576	24.0	1.051445134	5.144513	exact
10	640	21.6	1.057322197	5.732220	coarse
12	768	18.0	1.069175058	6.917506	exact
18	1152	12.0	1.105536855	10.553686	exact
24	1536	9.0	1.143135315	14.313531	exact
27	1728	8.0	1.162411351	16.241135	exact

The five-node value is numerically near 1.028, but nearby bin widths produce different factors. Solving for the continuous node width that gives exactly $g = 1.028$ yields

$$m_{1.028} = \frac{216\ln(1.028)}{\ln(10/3)} \approx 4.95433, \tag{144}$$

or

$$\Delta t_{1.028} \approx 317.077\,\text{Myr}. \tag{145}$$

This closeness explains why the five-node, 320 Myr choice produces 1.0282617, but it also demonstrates that the resemblance alone cannot select the packet duration. A physical promotion of 320 Myr requires an independent mechanism, prior empirical motivation, or held-out prediction that selects five nodes before comparison with the constant-w benchmark.

24.1 Duration-sensitivity decision rule

The nominal five-node packet may be called physically selected only if at least one of the following is derived independently of the 1.028 resemblance:

1. a covariant field equation or constitutive law generates a 320 Myr characteristic timescale;
2. a preregistered external dataset identifies the five-node scale and survives held-out testing;
3. a transition, resonance, or matching condition in Papers II–III fixes $m = 5$;
4. competing packet widths fail a common likelihood or predictive criterion without retuning the endpoint anchors.

Absent such selection, $m = 5$ remains an interpretable reporting scale rather than a new fundamental constant.

25 Approved Wide Stress Frame

The approved wide frame is

$$\boxed{678 \leq N_h \leq 762}. \tag{146}$$

It is exactly symmetric around 720 in integer-node space:

$$N_h = 720 + \Delta N, \qquad -42 \leq \Delta N \leq 42. \tag{147}$$

The corresponding endpoint ranges are

$$43.392\,\text{Gly} \leq D_h \leq 48.768\,\text{Gly}, \tag{148}$$
$$3.138888889 \leq S \leq 3.527777778, \tag{149}$$
$$1.026832114994 \leq g_p \leq 1.029612103797. \tag{150}$$

Relative to the central packet factor, the two endpoints move by only

$$-0.139033\% \quad \text{and} \quad +0.131325\%, \tag{151}$$

respectively. The compression follows from

$$\frac{d \ln g_p}{d \ln S} = \frac{1}{43.2} \approx 0.023148. \tag{152}$$

Thus a one-percent fractional change in S produces only about a 0.0231% fractional change in the inferred packet factor.

26 π-Proximity Probe

The continuous node count that would produce $S = \pi$ is

$$N_h^{(\pi)} = 216\pi = 678.584013175395. \tag{153}$$

Therefore π lies between the adjacent integer rows 678 and 679:

$$\frac{678}{216} < \pi < \frac{679}{216}. \tag{154}$$

The nearest integer-node row is

$$\boxed{N_h = 679, \quad D_h = 43.456\,\text{Gly}, \quad S = 3.143518518519, \quad g_p = 1.026867147672.} \tag{155}$$

Its relative difference from π is

$$\frac{|679/216 - \pi|}{\pi} \times 100\% = 0.061302185\%. \tag{156}$$

This row is retained for exploratory pattern analysis only. It was not used to choose the central 720-node calibration, and it does not establish circular geometry, a hidden symmetry, or a new cosmological law.

27 Developmental Exclusion Registry

The previously considered 2.304 Gyr candidate mega-interval is removed from the active Paper IV hierarchy. It has no parameter, equation, calibration, or interpretive role in the manuscript.

A separate developmental quantity remains in the source record:

$$\Delta t_\mu = \frac{T_3}{6000} = 2.304\,\mathrm{Myr} = 0.036(64\,\mathrm{Myr}) = 0.0072(320\,\mathrm{Myr}). \tag{157}$$

This micro-fold is retained only for provenance auditing. It is not part of the canonical 64 Myr node or 320 Myr packet hierarchy unless an independent physical derivation is supplied.

The repeated number 4.608 also remains unassigned. The identities

$$\frac{T_3}{3} = 4.608\,\mathrm{Gyr}, \qquad 46.08 - 3(13.824) = 4.608 \tag{158}$$

are arithmetic. The second expression suppresses the conversion by c and mixes a distance anchor with a time anchor unless every term is first converted into a common dimension. No transition duration, hidden leg, or physical residual is inferred from the repetition.

28 Continuous and Discrete Representations

For the endpoint mapping alone, the continuous interpolation

$$A(t;S) = \exp\left(\frac{\ln S}{T_3} t\right), \qquad 0 \le t \le T_3, \tag{159}$$

satisfies $A(0) = 1$ and $A(T_3) = S$. Sampling this curve at packet times $t_j = j\Delta t_p$ gives

$$A_j = g_p^j. \tag{160}$$

These samples do not imply physical stepwise jumps. A literal staircase model would be a different hypothesis and would require transition widths, conservation laws, and observational signatures.

29 Claim Hierarchy, Replacement Boundary, and Forbidden Identifications

1. **Identity:** arithmetic that follows from declared inputs and definitions.

2. **Calibration:** a parameter selected from or normalized to an adopted anchor.
3. **Correspondence:** a numerical or structural resemblance without a derived mechanism.
4. **Prediction:** a result fixed before comparison with held-out data.
5. **Validation:** survival of independent, cross-observable tests.

Paper IV does not claim

- $g_p = \Lambda$;
- $g_p = w$ or $g_p = -w$;
- g_p is automatically an inflationary e-fold;
- 46.08 Gly is an elapsed age;
- the 4.608 repetition has a physical meaning;
- the three legs have equal durations;
- the π proximity is evidence of a geometric mechanism; or
- packet arithmetic repairs a failed Paper II or Paper III result.

30 Reproducibility Package and Final Scope

The final release package preserves the numerical and interpretive trail needed to reproduce the analysis:

- the complete integer-node ledger for $678 \leq N_h \leq 762$;
- the source-provenance ledger separating adopted anchors, definitions, arithmetic identities, and excluded developmental claims;
- the packet-duration sensitivity grid demonstrating rebinning dependence;
- the controlled shape-function grid and analytic family summary;
- the equation-status ledger linking every inherited or constructed equation to its claim class; and
- the complete LaTeX source and final audit record.

The package is sufficient to regenerate the tables and verify every central number without assigning new physics to the packet construction. A later numerical implementation may solve the frozen Paper III GT1 equations and compare the resulting histories with data, but that is a separate research stage rather than a missing step in this paper.

31 Final Findings and Four-Paper Series Closure

The controlled numerical reconstruction yields

$$\boxed{\begin{aligned} T_3 &= 13.824\,\mathrm{Gyr}, \\ D_h &= 46.08\,\mathrm{Gly}, \\ S &= \frac{D_h}{cT_3} = \frac{10}{3}, \\ N_p &= \frac{T_3}{0.320\,\mathrm{Gyr}} = 43.2, \\ g_5 &= S^{1/N_p} = 1.028261735. \end{aligned}} \tag{161}$$

The due-diligence comparison is

$$\boxed{\begin{aligned} g_5 &= 1.028261735 && \text{packet arithmetic}, \\ w_{\mathrm{DE}} &= -1.028 \pm 0.031 && \text{constant-}w\text{ observational fit}. \end{aligned}} \tag{162}$$

Their magnitudes are close, but the quantities have different definitions, signs, physical meanings, and inference routes. The resemblance is therefore recorded as a correspondence rather than an equality.

The final conclusions are:

1. The central 720-node reconstruction is arithmetically exact once the adopted anchors are declared.
2. The broad frame $678 \le N_h \le 762$ shows that the packet factor remains near 1.028 and is not fragile to modest movement of the horizon-node anchor.
3. The $N_h = 679$ proximity to $S \simeq \pi$ is a real arithmetic observation but has no derived geometric meaning.
4. The value g_5 is not invariant under packet rebinning. The endpoint quantities S, $\ln S$, and $\ln S/T_3$ are the invariant content of the reconstruction.
5. The endpoints determine only $\int H dt = \ln S$. They do not determine a unique $a(t)$, $H(t)$, $q(t)$, $j(t)$, or equation of state.
6. The endpoint-equivalent linear control gives $\dot{H} = 0$, $q = -1$, and $w_{\mathrm{tot}} = -1$; it does not derive $w_{\mathrm{DE}} = -1.028$.
7. Any physical dark-energy interpretation must be reconstructed from the inherited Paper III composition and derivative equations, not inserted from the packet factor.
8. The minimum flat GT1 action requires $\dot{H} \le 0$ for nonnegative standard/scaffold densities and a canonical scalar, excluding total-phantom histories within that realization.
9. The total endpoint growth is 1.20397 e-folds. This number is not the complete three-leg causal history and therefore does not independently validate or falsify Paper II's inflation-replacement, horizon, flatness, matching, or perturbation counterclaims.
10. Packet arithmetic cannot repair or override a failed frozen realization in Papers I–III; conversely, failure of the physical packet interpretation cannot reject their independent claim families.

Accordingly, Paper IV closes at the strongest defensible statement:

$$\boxed{\begin{array}{c}\text{The third-leg packet construction is dimensionally auditable,}\\ \text{numerically reproducible, and stable across its declared stress frame.}\\ \text{Its proximity to } |w_{\mathrm{DE}}| \simeq 1.028 \text{ is noteworthy but unexplained.}\end{array}} \tag{163}$$

The completed series therefore has the following division of labor:

$$\boxed{\begin{array}{rl}\text{Paper I}: & \text{architecture and terminology,}\\ \text{Paper II}: & \text{causal, curvature, matching, perturbation, and observational tests,}\\ \text{Paper III}: & \text{covariant binding, scaffolding, and chasing dynamics,}\\ \text{Paper IV}: & \text{packet reconstruction, sensitivity, and correspondence audit.}\end{array}} \tag{164}$$

Internal consistency and arithmetic reproducibility do not constitute observational validation. The Triple Bounce framework remains a reciprocal, claim-compartmentalized research program: its replacement thesis, directional history, causal mechanisms, Gravity Triune realizations, and packet interpretation must each receive the verdict actually reached by their own frozen tests.

A Full Wide-Frame Numerical Ledger

Table 1: Integer-node stress frame. Δg is relative to the central $N_h = 720$ packet factor.

N_h	ΔN	D_h (Gly)	S	g_p	Δg (%)
678	-42	43.392	3.138889	1.026832115	-0.1390
679	-41	43.456	3.143519	1.026867148	-0.1356
680	-40	43.520	3.148148	1.026902130	-0.1322
681	-39	43.584	3.152778	1.026937062	-0.1288
682	-38	43.648	3.157407	1.026971944	-0.1254
683	-37	43.712	3.162037	1.027006776	-0.1220
684	-36	43.776	3.166667	1.027041559	-0.1187
685	-35	43.840	3.171296	1.027076291	-0.1153
686	-34	43.904	3.175926	1.027110974	-0.1119
687	-33	43.968	3.180556	1.027145608	-0.1085
688	-32	44.032	3.185185	1.027180193	-0.1052
689	-31	44.096	3.189815	1.027214728	-0.1018
690	-30	44.160	3.194444	1.027249215	-0.0985
691	-29	44.224	3.199074	1.027283653	-0.0951
692	-28	44.288	3.203704	1.027318042	-0.0918
693	-27	44.352	3.208333	1.027352383	-0.0884
694	-26	44.416	3.212963	1.027386675	-0.0851
695	-25	44.480	3.217593	1.027420919	-0.0818
696	-24	44.544	3.222222	1.027455115	-0.0784
697	-23	44.608	3.226852	1.027489263	-0.0751
698	-22	44.672	3.231481	1.027523363	-0.0718
699	-21	44.736	3.236111	1.027557416	-0.0685

Continued on next page

Table 1 – continued

N_h	ΔN	D_h (Gly)	S	g_p	Δg (%)
700	-20	44.800	3.240741	1.027591421	-0.0652
701	-19	44.864	3.245370	1.027625378	-0.0619
702	-18	44.928	3.250000	1.027659288	-0.0586
703	-17	44.992	3.254630	1.027693151	-0.0553
704	-16	45.056	3.259259	1.027726967	-0.0520
705	-15	45.120	3.263889	1.027760737	-0.0487
706	-14	45.184	3.268519	1.027794459	-0.0454
707	-13	45.248	3.273148	1.027828135	-0.0422
708	-12	45.312	3.277778	1.027861764	-0.0389
709	-11	45.376	3.282407	1.027895347	-0.0356
710	-10	45.440	3.287037	1.027928884	-0.0324
711	-9	45.504	3.291667	1.027962374	-0.0291
712	-8	45.568	3.296296	1.027995819	-0.0259
713	-7	45.632	3.300926	1.028029217	-0.0226
714	-6	45.696	3.305556	1.028062570	-0.0194
715	-5	45.760	3.310185	1.028095878	-0.0161
716	-4	45.824	3.314815	1.028129140	-0.0129
717	-3	45.888	3.319444	1.028162356	-0.0097
718	-2	45.952	3.324074	1.028195528	-0.0064
719	-1	46.016	3.328704	1.028228654	-0.0032
720	+0	46.080	3.333333	1.028261735	+0.0000
721	+1	46.144	3.337963	1.028294772	+0.0032
722	+2	46.208	3.342593	1.028327763	+0.0064
723	+3	46.272	3.347222	1.028360710	+0.0096
724	+4	46.336	3.351852	1.028393613	+0.0128
725	+5	46.400	3.356481	1.028426471	+0.0160
726	+6	46.464	3.361111	1.028459285	+0.0192
727	+7	46.528	3.365741	1.028492055	+0.0224
728	+8	46.592	3.370370	1.028524781	+0.0256
729	+9	46.656	3.375000	1.028557463	+0.0288
730	+10	46.720	3.379630	1.028590101	+0.0319
731	+11	46.784	3.384259	1.028622696	+0.0351
732	+12	46.848	3.388889	1.028655247	+0.0383
733	+13	46.912	3.393519	1.028687754	+0.0414
734	+14	46.976	3.398148	1.028720219	+0.0446
735	+15	47.040	3.402778	1.028752640	+0.0477
736	+16	47.104	3.407407	1.028785018	+0.0509
737	+17	47.168	3.412037	1.028817353	+0.0540
738	+18	47.232	3.416667	1.028849646	+0.0572
739	+19	47.296	3.421296	1.028881895	+0.0603
740	+20	47.360	3.425926	1.028914102	+0.0634
741	+21	47.424	3.430556	1.028946267	+0.0666
742	+22	47.488	3.435185	1.028978389	+0.0697
743	+23	47.552	3.439815	1.029010469	+0.0728
744	+24	47.616	3.444444	1.029042506	+0.0759

Continued on next page

Table 1 – continued

N_h	ΔN	D_h (Gly)	S	g_p	Δg (%)
745	+25	47.680	3.449074	1.029074502	+0.0790
746	+26	47.744	3.453704	1.029106456	+0.0822
747	+27	47.808	3.458333	1.029138368	+0.0853
748	+28	47.872	3.462963	1.029170238	+0.0884
749	+29	47.936	3.467593	1.029202067	+0.0914
750	+30	48.000	3.472222	1.029233854	+0.0945
751	+31	48.064	3.476852	1.029265600	+0.0976
752	+32	48.128	3.481481	1.029297304	+0.1007
753	+33	48.192	3.486111	1.029328968	+0.1038
754	+34	48.256	3.490741	1.029360590	+0.1069
755	+35	48.320	3.495370	1.029392171	+0.1099
756	+36	48.384	3.500000	1.029423712	+0.1130
757	+37	48.448	3.504630	1.029455212	+0.1161
758	+38	48.512	3.509259	1.029486671	+0.1191
759	+39	48.576	3.513889	1.029518090	+0.1222
760	+40	48.640	3.518519	1.029549468	+0.1252
761	+41	48.704	3.523148	1.029580806	+0.1283
762	+42	48.768	3.527778	1.029612104	+0.1313

References

[1] J. B. Fine, *The Triple Bounce Cosmological Framework: A Three-Leg Interpretation of Cosmic History*, Paper I, Version 1.1.0 replacement-thesis and claim-scope research manuscript (2026).

[2] J. B. Fine, *Inflation, Horizon, and Flatness as Incomplete-History Effects: Causal and Geometric Tests of the Triple Bounce Framework*, Paper II, Version 1.1.0 replacement-thesis and claim-scope research manuscript (2026).

[3] J. B. Fine, *The Gravity Triune: Binding, Scaffolding, and Chasing as Coupled Gravitational Functions in the Triple Bounce Cosmological Framework*, Paper III, Version 1.1.0 replacement-thesis and claim-scope research manuscript (2026).

[4] A. Einstein, "The Foundation of the General Theory of Relativity," *Annalen der Physik* **49**, 769–822 (1916).

[5] A. Friedmann, "On the Curvature of Space," *Zeitschrift fuer Physik* **10**, 377–386 (1922).

[6] A. Raychaudhuri, "Relativistic Cosmology. I," *Physical Review* **98**, 1123–1126 (1955).

[7] C.-P. Ma and E. Bertschinger, "Cosmological Perturbation Theory in the Synchronous and Conformal Newtonian Gauges," *Astrophysical Journal* **455**, 7–25 (1995), arXiv:astro-ph/9506072.

[8] M. Bartelmann and P. Schneider, "Weak Gravitational Lensing," *Physics Reports* **340**, 291–472 (2001), arXiv:astro-ph/9912508.

[9] D. W. Hogg, "Distance Measures in Cosmology," arXiv:astro-ph/9905116 (1999).

[10] M. Visser, "Jerk, Snap and the Cosmological Equation of State," *Classical and Quantum Gravity* **21**, 2603–2616 (2004), arXiv:gr-qc/0309109.

[11] Planck Collaboration, N. Aghanim et al., "Planck 2018 Results. VI. Cosmological Parameters," *Astronomy & Astrophysics* **641**, A6 (2020), arXiv:1807.06209.

[12] Planck Collaboration, Y. Akrami et al., "Planck 2018 Results. X. Constraints on Inflation," *Astronomy & Astrophysics* **641**, A10 (2020), arXiv:1807.06211.

[13] J. B. Fine, *320 Million Year Packets - Cosmological Constant*, internal developmental record and equation-history source (2025–2026).

[14] J. B. Fine, *YES - Packet Expansion Rate Equations*, internal developmental record and equation-history source (2025–2026).